EC8391	CONTROL SYSTEMS ENGINEERING	L	T	P	C
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OBJECTIVES:

- To introduce the components and their representation of control systems
- To learn various methods for analyzing the time response, frequency response and stability of the systems.
- To learn the various approach for the state variable analysis.

UNIT I SYSTEMS COMPONENTS AND THEIR REPRESENTATION 9

Control System: Terminology and Basic Structure-Feed forward and Feedback control theory-Electrical and Mechanical Transfer Function Models-Block diagram Models-Signal flow graphs models-DC and AC servo Systems-Synchronous -Multivariable control system

UNIT II TIME RESPONSE ANALYSIS 9

Transient response-steady state response-Measures of performance of the standard first order and second order system-effect on an additional zero and an additional pole-steady error constant and system- type number-PID control-Analytical design for PD, PI,PID control systems

UNIT III FREQUENCY RESPONSE AND SYSTEM ANALYSIS 9

Closed loop frequency response-Performance specification in frequency domain-Frequency response of standard second order system- Bode Plot - Polar Plot- Nyquist plots-Design of compensators using Bode plots-Cascade lead compensation-Cascade lag compensation-Cascade lag-lead compensation

UNIT IV CONCEPTS OF STABILITY ANALYSIS 9

Concept of stability-Bounded - Input Bounded - Output stability-Routh stability criterion-Relative stability-Root locus concept-Guidelines for sketching root locus-Nyquist stability criterion.

UNIT V CONTROL SYSTEM ANALYSIS USING STATE VARIABLE METHODS 9

State variable representation-Conversion of state variable models to transfer functions-Conversion of transfer functions to state variable models-Solution of state equations-Concepts of Controllability and Observability-Stability of linear systems-Equivalence between transfer function and state variable representations-State variable analysis of digital control system-Digital control design using state feedback.

TOTAL:45 PERIODS

OUTCOMES:

Upon completion of the course, the student should be able to:

- Identify the various control system components and their representations.
- Analyze the various time domain parameters.
- Analysis the various frequency response plots and its system.
- Apply the concepts of various system stability criterions.
- Design various transfer functions of digital control system using state variable models.

TEXT BOOK:

1. M.Gopal, "Control System – Principles and Design", Tata McGraw Hill, 4th Edition, 2012.

REFERENCES:

1. J.Nagrath and M.Gopal, "Control System Engineering", New Age International Publishers, 5[th] Edition, 2007.
2. K. Ogata, „Modern Control Engineering", 5th edition, PHI, 2012.
3. S.K.Bhattacharya, Control System Engineering, 3rd Edition, Pearson, 2013.
4. Benjamin.C.Kuo, "Automatic control systems", Prentice Hall of India, 7th Edition,1995

EC 8391 · CONTROL SYSTEM ENGINEERING

UNIT - I SYSTEMS COMPONENTS AND THEIR REPRESENTATION

BASIC ELEMENTS OF CONTROL SYSTEM

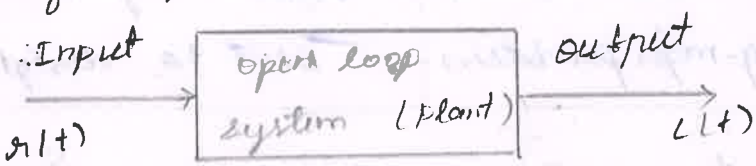
When a number of elements or components are connected in a sequence to perform a specific function, the group thus formed is called a system.

In a system when the output quantity is controlled by varying the input quantity, the system is called control system.

The o/p quantity is called controlled variable or response and i/p quantity is called command signal or excitation.

OPEN LOOP SYSTEM

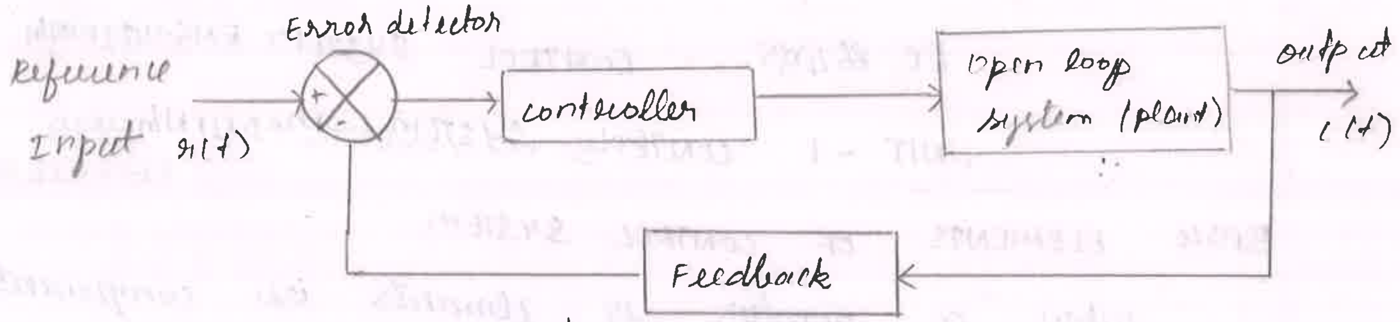
The control system in which the o/p quantity has no effect upon the i/p quantity are called open-loop control system (or) Any physical system which does not automatically correct the variation in its output, is called an open loop system.



open loop system

CLOSED LOOP SYSTEM:

Control system in which the o/p has an effect upon the i/p quantity in order to maintain the desired o/p value are called closed loop system.



closed loop system

The open loop system can be modified as closed loop system by providing a feedback. The provision of feedback automatically corrects the changes in output due to disturbances. Hence closed loop systems is also called automatic control system.

Advantages :

Accurate. accurate even in presence of non-linearities.

Less affected by nois.

System sensitivity may be made small to make system more stable.

Disadv :

Complex and costly.

Feedback may lead to oscillatory response.

Feedback reduces the overall gain of system.

Stability-major problem. Need to design a stable CLS more care.

Open loop system adv :

Simple and economical

easier to construct

Generally Stable.

disadv.

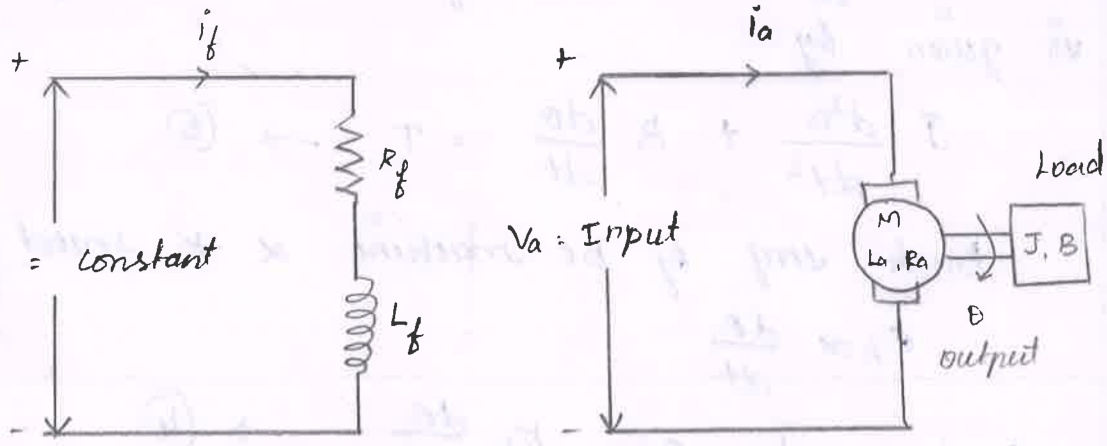
Inaccurate & Unreliable

Changes in o/p due to external disturbances are not corrected automatically.

TRANSFER FUNCTION OF ARMATURE CONTROLLED DC MOTOR.

The speed of DC motor is directly proportional to armature voltage and inversely proportional to flux in field winding. In armature controlled DC motor the desired speed is obtained by varying the armature voltage. This speed control system is an electro-mechanical control system.

The electrical system consists of the armature and field circuit but for analysis purpose, only the armature circuit is considered because the field is excited by constant voltage. The mechanical system consists of the rotating part of the motor and load connected to the shaft of the motor. The armature controlled DC motor speed control system is shown in fig.



Armature controlled DC motor

Let,

R_a - Armature resistance, Ω

L_a - Armature inductance, H

i_a - Armature current, A

V_a - Armature voltage, V

e_b - Back emf, V

K_t - Torque constant N-m/A.

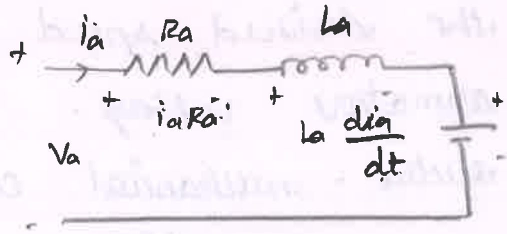
T - Torque developed by motor, N-m

θ - Angular displacement of shaft, rad

J - moment of inertia of motor and load N-m

K_b - Back emf constant, V (rad / sec). (rad / sec)

By KVL,



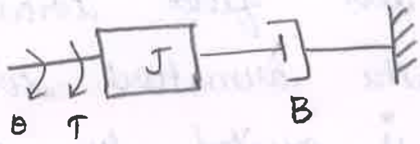
Eq. ckt. armature

$$i_a R_a + L_a \frac{di_a}{dt} + e_b = V_a \rightarrow ①$$

Torque of DC motor $\propto$ to product of flux and current.

flux - constant in this system

So. $T \propto i_a$ alone.



mech. system of motor.

$$\text{Torque } T = K_t \, i_a \rightarrow ②$$

The diff. eq. governing the mech. system of motor is given by

$$J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} = T \rightarrow ③$$

Back emf of DC machine $\propto$ to speed of shaft (angular velocity).

$$e_b \propto \frac{d\theta}{dt}$$

$$\text{Back emf } e_b = K_b \frac{d\theta}{dt} \rightarrow ④$$

Laplace transform of various time domain signals involved in this system are shown below.

$$L\{V_a\} = V_a(s)$$

$$L\{e_b\} = E_b(s)$$

$$L\{T\} = T(s)$$

$$L\{i_a\} = I_a(s)$$

$$L\{\theta\} = \theta(s)$$

The diff. eq. governing the armature controlled DC motor speed control systems are.

$$i_a R_a + L_a \frac{di_a}{dt} + e_b = V_a$$

$$T = K_t i_a$$

$$J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} = T$$

$$e_b = K_b \frac{d\theta}{dt}.$$

Taking L.T with zero initial conditions, we get,

$$I_a(s) R_a + L_a s I_a(s) + E_b(s) = V_a(s) \rightarrow (5)$$

$$T(s) = K_t I_a(s) \rightarrow (6)$$

$$Js^2\theta(s) + Bs\theta(s) = T(s) \rightarrow (7)$$

$$E_b(s) = K_b s\theta(s) \rightarrow (8)$$

on equating (6) & (7)

$$K_t I_a(s) = [Js^2 + Bs]\, \theta(s)$$

$$I_a(s) = \frac{Js^2 + Bs}{K_t} \theta(s) \rightarrow (9)$$

Eq. (5) can written as

$$(R_a + sL_a) I_a(s) + E_b(s) = V_a(s) \rightarrow (10)$$

Sub. (8) & (9) in (10)

$$(R_a + sL_a) \frac{(Js^2 + Bs)\theta(s)}{K_t} + K_b s\, \theta(s) = V_a(s)$$

$$\left[\frac{(R_a + sL_a)(Js^2 + Bs) + K_b K_t s}{K_t}\right] \theta(s) = V_a(s)$$

The required Transfer function is $\frac{\theta(s)}{V_a(s)}$

$$\frac{\theta(s)}{V_a(s)} = \frac{K_t}{(R_a + sL_a)(Js^2 + Bs) + K_b K_t s} \rightarrow (11)$$

$$= \frac{K_t}{R_a J s^2 + R_a B s + L_a J s^3 + L_a B s^2 + K_b K_t s}$$

$$= \frac{K_t}{s[JL_a s^2 + (JR_a + BL_a)s + (BR_a + K_b K_t)]}$$

$$= \frac{K_t / JL_a}{s\left[s^2 + \left(\frac{JR_a + BL_a}{JL_a}\right)s + \left(\frac{BR_a + K_b K_t}{JL_a}\right)\right]} \rightarrow (12)$$

Transfer fn. of armature controlled DC motor can be expressed in another std. form as shown below.

From eq. (12)

$$(11) \Rightarrow \frac{\theta(s)}{V_a(s)} = \frac{K_t}{(R_a + sL_a)(Js^2 + Bs) + K_b K_t s}$$

$$= \frac{K_t}{R_a\left(s\frac{L_a}{R_a} + 1\right) Bs\left(1 + \frac{Js^2}{Bs}\right) + K_b K_t s}$$

$$= \frac{K_t / R_a B}{s\left[(1 + sT_a)(1 + sT_m) + \frac{K_b K_t}{R_a B}\right]} \rightarrow (13)$$

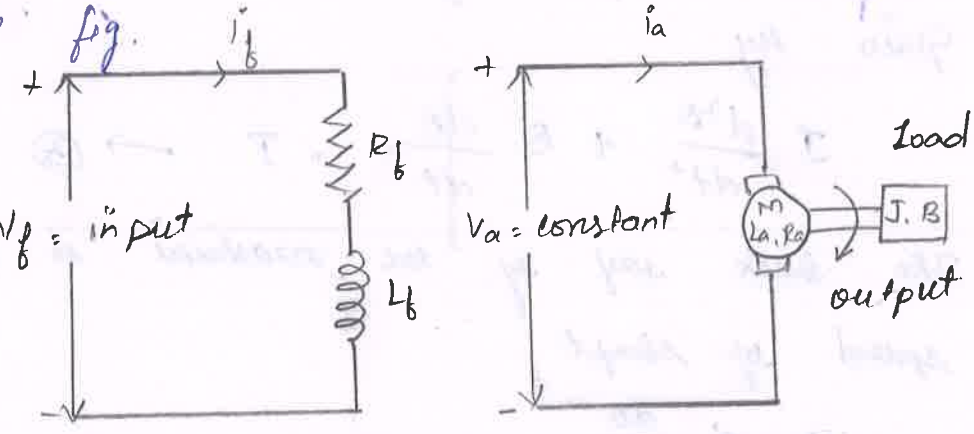
where $\frac{L_a}{R_a} = T_a$ = Electrical time constant

$\frac{J}{B} = T_m$ = Mechanical time constant.

TRANSFER FUNCTION OF FIELD CONTROLLED DC MOTOR

The speed of a DC motor is directly proportional to armature voltage and inversely proportional to field. In field controlled DC motor the armature voltage is kept constant and the speed is varied by varying the flux of the machine. Since flux is directly proportional to field current, the flux is varied by varying field current. The speed control system is an electromechanical control system.

The electrical system consist of armature and field circuit but for analysis purpose, only field circuit is considered because the armature is excited by a constant voltage. The mechanical system consist of the rotating part of the motor and the load connected to the shaft of the motor. The field controlled DC motor speed control system is shown in fig.



Field controlled DC motor

Let, R_f - Field resistance, Ω

L_f - Field inductance, H

i_f - Field current, A

V_f - Field Voltage, V

T - Torque developed by motor, N-m

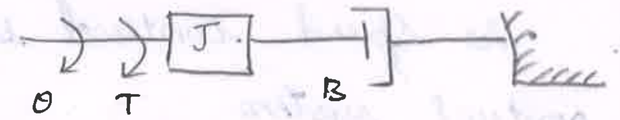
K_{tf} - Torque constant, N-m/A

J - Moment of inertia of rotor and load, kg-m²/rad

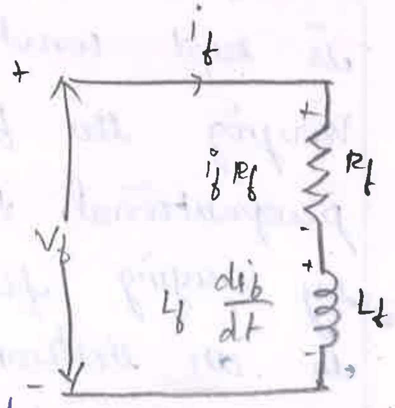
B - Frictional coefficient of rotor and load, N-m (rad/sec)

By KVL,

$$R_f i_f + L_f \frac{di_f}{dt} = V_f \longrightarrow (1)$$



mech. eqs. of motor



Eq. ckt of field

Torque of DC motor $\propto$ to product of flux and armature current.

i_a is constant. $T \propto$ flux but flux $\propto$ field current

So $T \propto i_f$

Torque, $T = K_{tf}\, i_f \longrightarrow (2)$

The diff. eq. governing the mech. system of motor is given by

$$J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} = T \longrightarrow (3)$$

The back emf of DC machine is proportional to speed of shaft.

$$e_b \propto \frac{d\theta}{dt}$$

Back emf, $e_b = K_b \frac{d\theta}{dt}$.

L.T of various time domain signals involved in this system are shown below.

$$L\{i_f\} = I_f(s)$$

$$L\{T\} = T(s)$$

$$L\{V_f\} = V_f(s)$$

$$L\{\theta\} = \theta(s)$$

The diff. eq. governing the field controlled DC motor are,

$$R_f i_f + L_f \frac{di_f}{dt} = V_f$$

$$T = K_{tf}\, i_f$$

$$J \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T$$

Taking L.T of above eqn. with zero initial conditions

$$R_f I_f(s) + L_f s I_f(s) = V_f(s) \rightarrow (4)$$

$$T(s) = K_{tf}\, I_f(s) \rightarrow (5)$$

$$J s^2 \theta(s) + B s \theta(s) = T(s) \rightarrow (6)$$

Equating (5) & (6), we get

$$K_{tf}\, I_f(s) = J s^2\theta(s) + B s\,\theta(s)$$

$$I_f(s) = s\frac{(Js+B)}{K_{tf}}\theta(s) \rightarrow (7)$$

Eq. (4) can be written as

$$(R_f + sL_f)\, I_f(s) = V_f(s) \rightarrow (8)$$

Sub (7) in (8), we get

$$(R_f + sL_f)\, s\, \frac{(Js + B)}{K_{tf}}\, \theta(s) = V_f(s)$$

$$\frac{\theta(s)}{V_f(s)} = \frac{K_{tf}}{s(R_f + sL_f)(B + sJ)}$$

$$= \frac{K_{tf}}{sR_f\left(1 + \frac{sL_f}{R_f}\right) B\left(1 + \frac{sJ}{B}\right)}$$

$$= \frac{K_m}{s(1 + sT_f)(1 + sT_m)} \longrightarrow (9)$$

where

$K_m = \frac{K_{tf}}{R_f B}$ = motor gain constant

$T_f = \frac{L_f}{R_f}$ = Field time constant

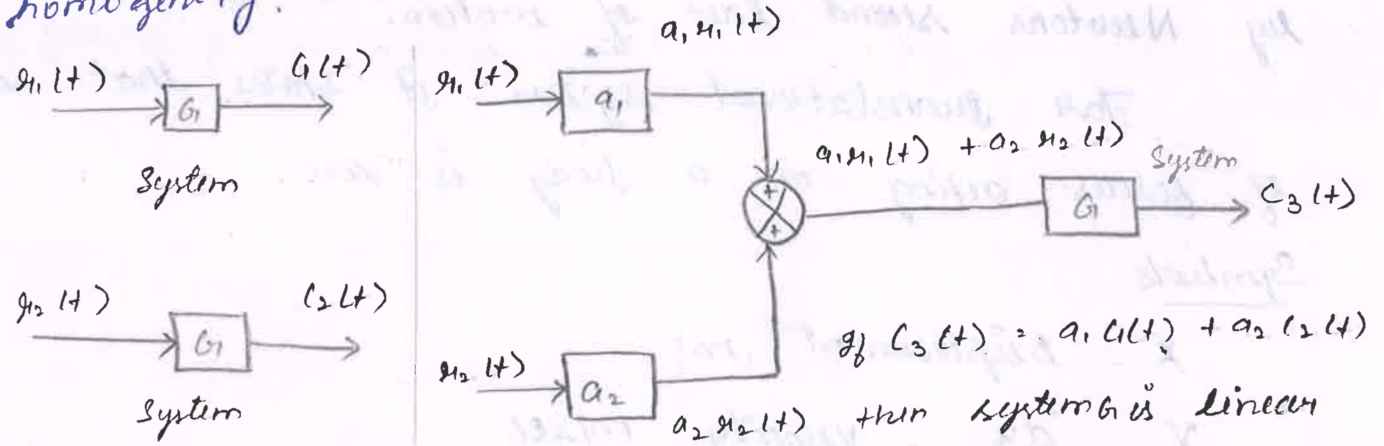
$T_m = \frac{J}{B}$ = mechanical time constant.

MATHEMATICAL MODEL OF CONTROL SYSTEMS

Control system is a collection of physical objects connected together to serve an objective.

The i/p o/p relations of various physical components of a system are governed by differential eqn. The mathematical model of a control system constitutes a set of differential eqn. The response or output of the system can be studied by solving diff. eqn. for various i/p conditions.

The mathematical model of a system is linear if it obeys the principle of superposition and homogenity.



Principle of liniarity and Superposition

mathematical model linear - if diff. eqn. has constant coefficient.

$$\text{Transfer function} = \left.\frac{\text{Laplace Transform of output}}{\text{Laplace Transform of input}}\right| \text{with zero initial Conditions}$$

MECHANICAL TRANSLATIONAL SYSTEMS

The model of mechanical translational system can be obtained by using three basic elements mass, spring and dash-pot.

Mass - weight of mechanical system

Spring - Elastic deformation of the body.

Dash - pot - Friction existing in rotating mech. system.

When a force is applied to a translational mechanical system, it is opposed by opposing force due to mass, friction and elasticity of a system. The force acting on a mechanical body are governed by Newton's second law of motion. (Sum of applied force is equal to sum of opposing force on a body)

For translational system it states that sum of forces acting on a body is zero.

Symbols :

x - Displacement, m

$v = \frac{dx}{dt}$ = Velocity m/sec

$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$ = Acceleration m/s^2

f = Applied force, N

f_m = opposing force offered by mass of body, N

f_k = " by elasticity of body (spring) N

f_b = " by friction " (dash-pot) N

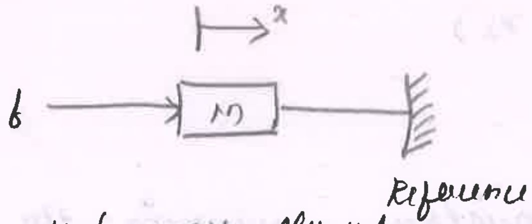
M = mass, kg , B = Viscous friction co-efficient N-s/m

K = Stiffness of spring, N/m

FORCE BALANCE EQ. OF IDEALIZED ELEMENTS

Ideal mass element - Negligible friction & elasticity.

Force is applied, mass opposing force $\propto$ to acceleration of body.



Ideal mass element

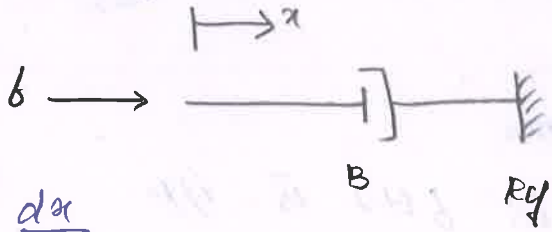
$$f_m \propto \frac{d^2x}{dt^2}$$

opp. force due to mass

By Newton's second law, $f = f_m = m\frac{d^2x}{dt^2}$

Ideal friction element dashpot. - Neg. mass & elasticity.

D.P offer opposing force $\propto$ to velocity of body.



Ideal dashpot with one end fixed to ref.

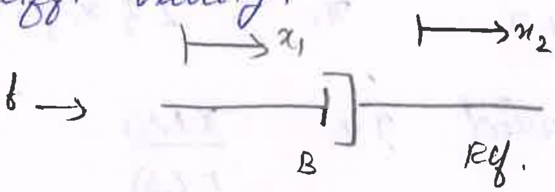
$$f_b \propto \frac{dx}{dt}$$

By Newton's 2nd law, $f_b = f = B\frac{dx}{dt}$.

Dashpot has displacement at both ends.

Opposing force $\propto$ diff. velocity.

$$f_b \propto \frac{d}{dt}(x_1 - x_2)$$

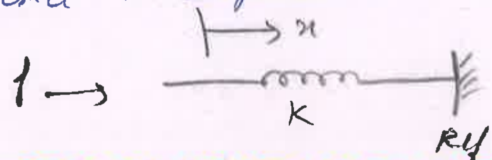


$$f = f_b = B\frac{d}{dt}(x_1 - x_2)$$

Ideal elastic element spring with one end fixed ref.

Spring offer opp. force $\propto$ displacement of body

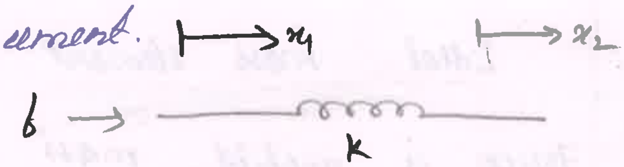
$$f_k \propto x$$



$$f = f_k = Kx.$$

Spring has displacement at both ends.

opp. force $\propto$ diff. displacement.

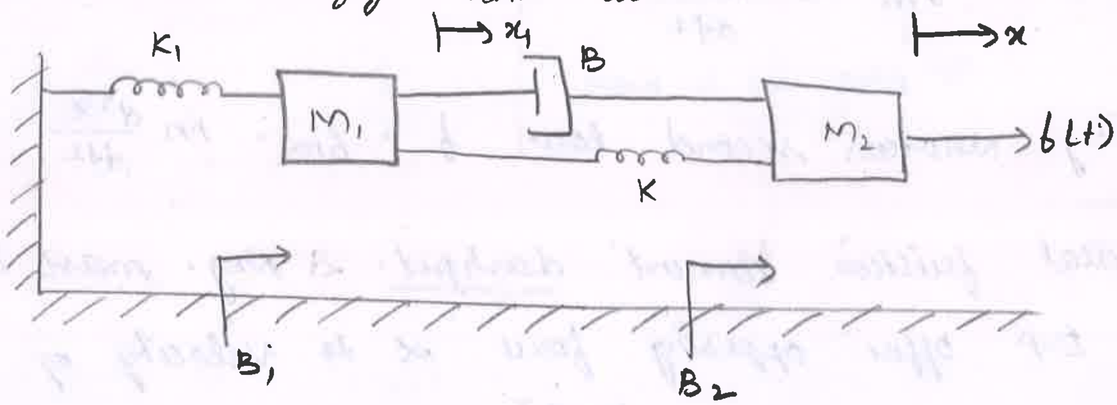


$f_k \propto (x_1 - x_2)$

$f = f_k = K(x_1 - x_2)$

problem 1:

write the diff. equations governing the mechanical system shown in fig. and determine the transfer function



Sol:

In given system

applied force $f(t)$ is i/p

displacement x is o/p.

Let L.T $f(x) = L\{f(x)\} = F(s)$

L.T $x = L\{x\} = X(s)$

L.T $x_1 = L\{x_1\} = X_1(s)$

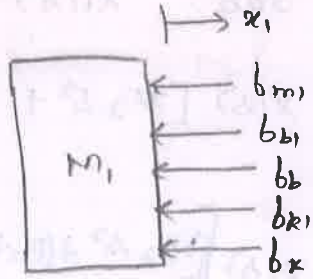
Required T.F $\dfrac{X(s)}{F(s)}$

2 nodes they are mass M_1 & M_2. the diff. eqs governing the system are given by force balance eq. at these nodes.

Let the displacement of mass M_1 be x_1. The free body diagram of mass M_1 is shown. The opposing force acting on mass M_1 are marked as f_{m1}, f_{b1}, f_b, f_{k1} & f_k

$$f_{m1} = M_1 \frac{d^2 x_1}{dt^2} \qquad f_{b1} = B_1 \frac{dx_1}{dt} \qquad f_{k1} = K_1 x_1$$

$$f_b = B\frac{d}{dt}(x_1 - x) \qquad f_k = k(x_1 - x)$$



By Newton's 2nd law

$$f_{m1} + f_{b1} + f_{k1} + f_b + f_k = 0$$

$$M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B\frac{d}{dt}(x_1 - x) + K_1 x_1 + k(x_1 - x) = 0$$

Take L.T with zero initial condition,

$$M_1 s^2 X_1(s) + B_1 s X_1(s) + Bs[X_1(s) - X(s)] + K_1 X_1(s) + k[X_1(s) - X(s)] = 0$$

$$X_1(s)[M_1 s^2 + (B_1 + B)s + (K_1 + k)] - X(s)[Bs + k] = 0$$

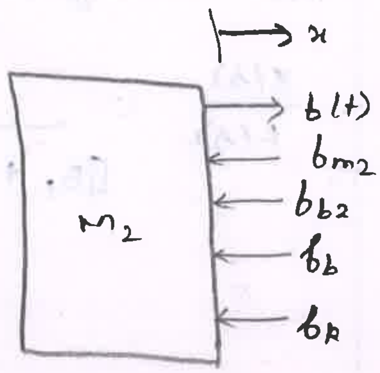
$$X_1(s)[M_1 s^2 + (B_1 + B)s + (K_1 + k)] = X(s)[Bs + k]$$

$$X_1(s) = X(s)\frac{Bs + K}{M_1 s^2 + (B_1 + B)s + (K_1 + k)} \longrightarrow (1)$$

The free body dig. of mass M_2. The opposing forces acting on M_2 are marked as f_{m2}, f_{b2}, f_b & f_k

$$f_{m2} = M_2 \frac{d^2 x}{dt^2} \qquad f_{b2} = B_2 \frac{dx}{dt}$$

$$f_b = B\frac{d}{dt}(x - x_1) \qquad f_k = k(x - x_1)$$



By Newton's 2nd law,

$$f_{m2} + f_{b2} + f_b + f_k = f(t)$$

$$M_2 \frac{d^2 x}{dt^2} + B_2 \frac{dx}{dt} + B\frac{d}{dt}(x - x_1) + k(x - x_1) = f(t)$$

L.T,

$$M_2 s^2 X(s) + B_2 s X(s) + Bs[X(s) - X_1(s)] + K[X(s) - X_1(s)] = F(s)$$

$$X(s)[M_2 s^2 + (B_2+B)s + K] - X_1(s)[Bs+K] = F(s) \longrightarrow (2)$$

Sub. $X_1(s)$ from (1) in (2),

$$X(s)[M_2 s^2 + (B_2+B)s + K] - X(s)\frac{(Bs+K)^2}{M_1 s^2 + (B_1+B)s + (K_1+K)} = F(s)$$

$$X(s)\left[\frac{[M_2 s^2 + (B_2+B)s + K][M_1 s^2 + (B_1+B)s + (K_1+K)] - (Bs+K)^2}{M_1 s^2 + (B_1+B)s + (K_1+K)}\right] = F(s)$$

$$\frac{X(s)}{F(s)} = \frac{M_1 s^2 + (B_1+B)s + (K_1+K)}{[M_1 s^2 + (B_1+B)s + (K_1+K)][M_2 s^2 + (B_2+B)s + K] - (Bs+K)^2}$$

Result:

The diff. eq. governing the system are,

1. $M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B\frac{d}{dt}(x_1 - x) + K_1 x_1 + K(x_1 - x) = 0$

2. $M_2 \frac{d^2x}{dt^2} + B_2 \frac{dx}{dt} + B\frac{d}{dt}(x - x_1) + K(x - x_1) = f(t)$

The transfer function of the system is,

$$\frac{X(s)}{F(s)} = \frac{M_1 s^2 + (B_1+B)s + (K_1+K)}{[(B_1+B)s + M_1 s^2 + (K_1+K)][M_2 s^2 + (B_2+B)s + K] - (Bs+K)^2}$$

MECHANICAL ROTATIONAL SYSTEMS

The model of rotational mechanical system can be obtained by using three elements.

moment of inertia [J] of mass. (weight)

dash-pot with rotational frictional coefficient [B] (friction existing)

torsional spring with stiffness [K] (elastic deformation of body)

When torque is applied to a rotational mech. system, it is opposed by opposing torque due to moment of inertia, friction and elasticity of system. Torque acting on rotational mech. body are governed by Newton's second law of motion. For rotational systems.

$\hookrightarrow$ Sum of applied torque = sum of opposing torque on body.

Symbols :

θ – Angular displacement, rad x – Displacement

$\frac{d\theta}{dt}$ – Angular Velocity, rad/sec. $v = \frac{dx}{dt}$ – Velocity

$\frac{d^2\theta}{dt^2}$ – Angular acceleration, rad/sec. $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$ – acceleration

T – Applied torque, N-m

J – Moment of inertia, kg-m²/rad

B – Rotational frictional coefficient, N-m/(rad/sec)

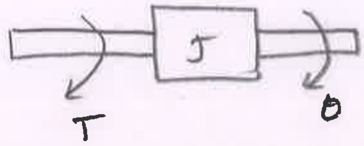
K – Stiffness of spring, N-m/rad.

TORQUE BALANCE EQ. OF IDEALISED ELEMENTS

Ideal mass element – Neg. friction & elasticity. (→ zero)

Opp. torque due to moment of inertia $\propto$ angular acceleration

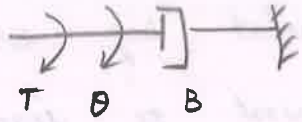
$T_j \propto \frac{d^2\theta}{dt^2}$



By Newton's 2nd law $T = T_j = \frac{d^2\theta}{dt^2}$

Ideal frictional element dash pot - req. moment of inertia & elasticity.
dash pot offer an opp. torque $\propto$ angular velocity.

$T_b \propto \frac{d\theta}{dt}$

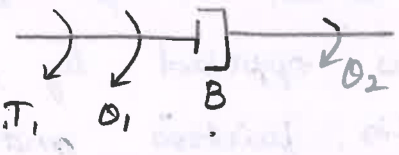


By Newton's 2nd law $T = T_b = B\frac{d\theta}{dt}$.

when dash pot has angular displacement at both ends.

opp. torque $\propto$ diff. angular velocity

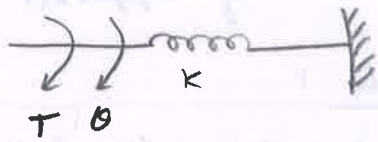
$T_b \propto \frac{d}{dt}(\theta_1 - \theta_2)$



$T = T_b = B\frac{d}{dt}(\theta_1 - \theta_2)$

Ideal elastic element, torsional spring, req. J & friction with one end fixed to ref.

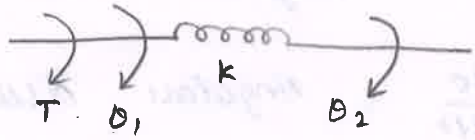
$T_k \propto \theta$



$T = T_k = K\theta$.

Spring has angular displacement at both ends.

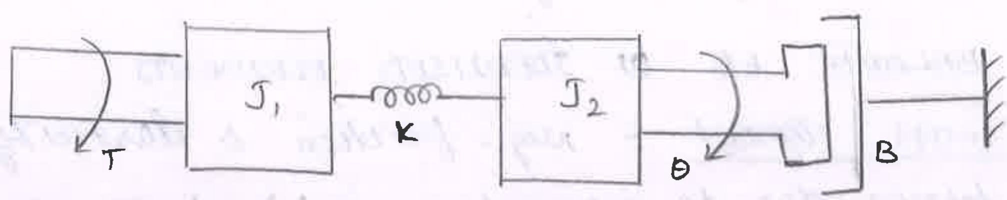
$T_k \propto (\theta_1 - \theta_2)$



$T = T_k = k(\theta_1 - \theta_2)$

Problem 1:

write the diff. eqn. governing the mech. rotational system shown in fig. Obtain the transfer function of the system



Solution:

Applied torque T - Input

angular displacement θ - O/P.

Let. L.T $T = L\{T\} = T(s)$

$\theta = L\{\theta\} = \theta(s)$

$\theta_1 = L\{\theta_1\} = \theta_1(s)$

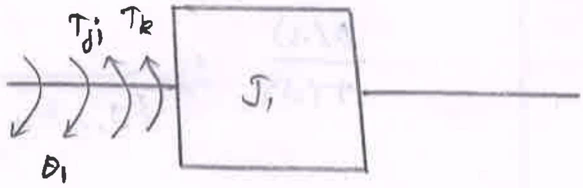
Required T.F $= \dfrac{\theta(s)}{T(s)}$

System has two nodes, mass with moment of inertia J_1 and J_2. The diff. eq. governing the system are given by torque balance eq. at these nodes.

Let. angular dis. of mass with moment of inertia J_1 be θ_1 opposing torque acting on J_1 are marked as T_{j1} and T_k.

$T_{j1} = J_1 \dfrac{d^2\theta_1}{dt^2}$ $\qquad T_k = K(\theta_1 - \theta)$

By Newton 2nd law



$T_{j1} + T_k = T$

$J_1 \dfrac{d^2\theta}{dt^2} + K(\theta_1 + \theta) = T$

$J_1 \dfrac{d^2\theta_1}{dt^2} + K\theta_1 - K\theta = T \longrightarrow (1)$

L.T of (1) with zero initial condition

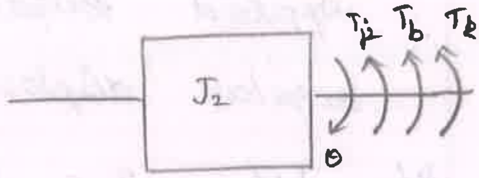
$J_1 s^2 \theta_1(s) + K\theta_1(s) - K\theta(s) = T(s)$

$(J_1 s^2 + K)\theta_1(s) - K\theta(s) = T(s) \longrightarrow (2)$

with moment of inertia J_2, opp. torque acting on J_2 are marked as T_{j2}, T_b & T_k

$$T_{j2} = J_2 \frac{d^2\theta}{dt^2} \qquad T_b = B\frac{d\theta}{dt} \qquad T_k = K(\theta - \theta_1)$$

$$T_{j2} + T_b + T_k = 0$$



$$J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K(\theta - \theta_1) = 0$$

$$J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta - K\theta_1 = 0$$

Taking L.T

$$J_2 s^2 \theta(s) + Bs\theta(s) + K\theta(s) - K\theta_1(s) = 0$$

$$(J_2 s^2 + Bs + K)\theta(s) - K\theta(s) = 0$$

$$\theta_1(s) = \frac{(J_2 s^2 + Bs + K)}{K}\theta(s) \longrightarrow ③$$

Sub $\theta_1(s)$ from ③ in ②

$$(J_1 s^2 + K)\frac{(J_2 s^2 + Bs + K)}{K}\theta(s) - K\theta(s) = T(s)$$

$$\left[\frac{(J_1 s^2 + K)(J_2 s^2 + Bs + K) - K^2}{K}\right]\theta(s) = T(s)$$

$$\frac{\theta(s)}{T(s)} = \frac{K}{(J_1 s^2 + K)(J_2 s^2 + Bs + K) - K^2}$$

Result :

The diff. eq. governing the system are,

1. $J_1 \frac{d^2\theta_1}{dt^2} + K\theta_1 - K\theta = T$

2. $J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta - K\theta_1 = 0$

The Transfer function of system is

$$\frac{\theta(s)}{T(s)} = \frac{K}{(J_1 s^2 + K)(J_2 s^2 + Bs + K) - K^2}$$

ELECTRICAL SYSTEMS

The model of electrical systems can be obtained by using resistor, capacitor and inductor. For modeling electrical system, the electrical network or eq. ckt. is formed by using R, L, C and V or I source.

The diff. eq. governing the electrical system can be formed by writing KCL by choosing nodes. or KVL by choosing various closed path in network.

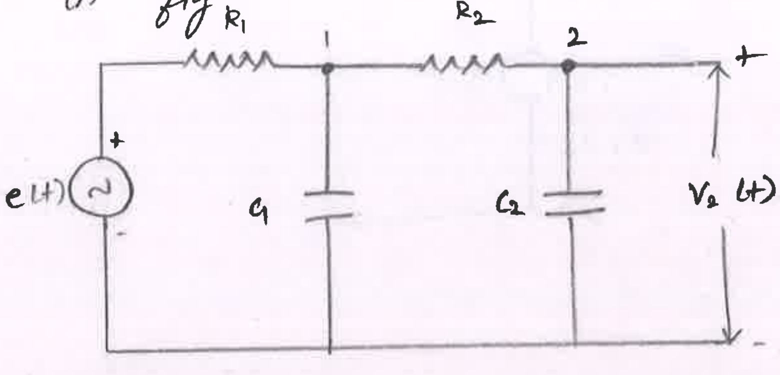
T.F = L.T and rearranging the ratio of o/p to i/p

CURRENT - VOLTAGE RELATION R, L and C.

Element	V across the element	current through element
$i(t)$ R, + $v(t)$ −	$v(t) = R\,i(t)$	$i(t) = \dfrac{v(t)}{R}$
$i(t)$ L, + $v(t)$ −	$v(t) = L\dfrac{d}{dt}i(t)$	$i(t) = \dfrac{1}{L}\int v(t)\,dt$
$i(t)$ C, + $v(t)$ −	$v(t) = \dfrac{1}{C}\int i(t)\,dt$	$i(t) = C\dfrac{dv(t)}{dt}$

problem 1 :

Obtain the transfer function of electrical network shown in fig



Sol:

In given network.

i/p - $e(t)$

o/p - $V_2(t)$

Let L.T of $e(t) = L\{e(t)\} = E(s)$

$V_2(t) = L\{V_2(t)\} = V_2(s)$

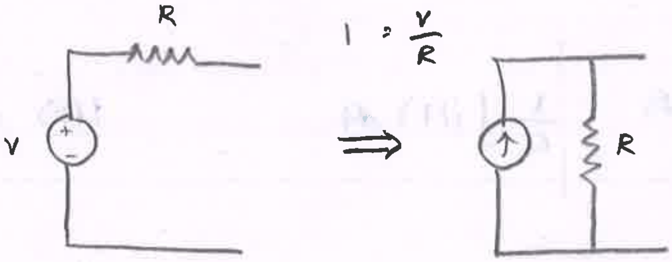
The T.F $= \dfrac{V_2(s)}{E(s)}$

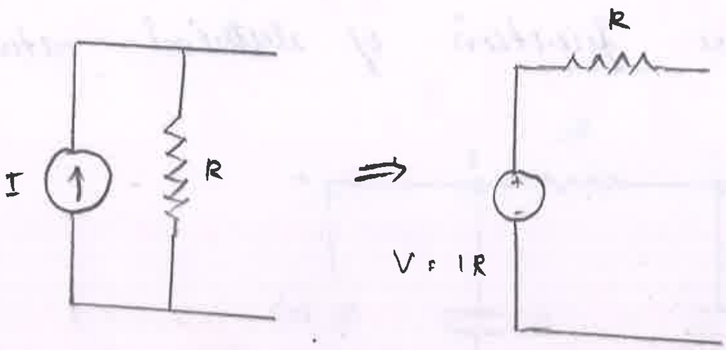
Transform the voltage source in series with resistance R_1 into eq. current source. The network has two nodes Node voltage V_1 and V_2.

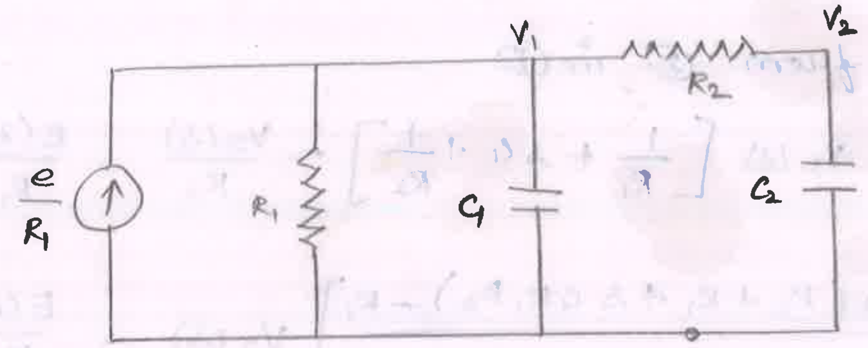
L.T $V_1 = V_1(s)$

$V_2 = V_2(s)$

The diff. eq. governing the network are given by KCL eq. at these nodes.

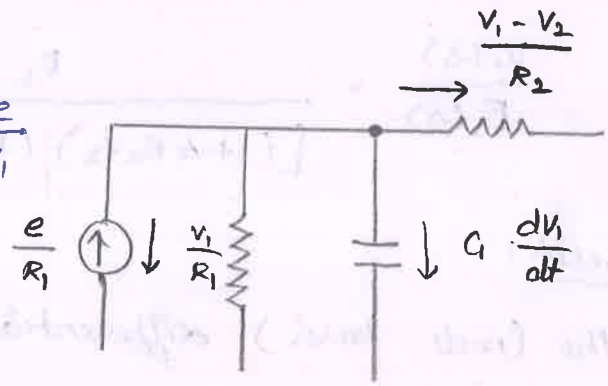






At node 1, by KCL

$$\frac{V_1}{R_1} + C_1 \frac{dV_1}{dt} + \frac{V_1 - V_2}{R_2} = \frac{e}{R_1}$$



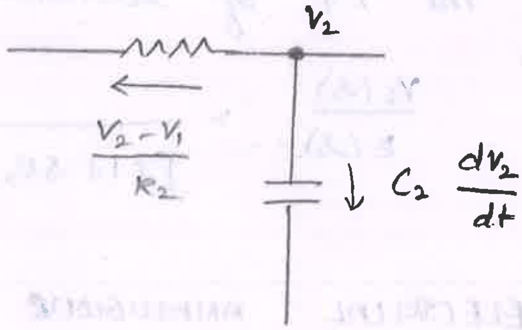
Taking L.T,

$$\frac{V_1(s)}{R_1} + C_1 s V_1(s) + \frac{V_1(s)}{R_2} - \frac{V_2(s)}{R_2} = \frac{E(s)}{R_1}$$

$$V_1(s)\left[\frac{1}{R} + s C_1 + \frac{1}{R_2}\right] - \frac{V_2(s)}{R_2} = \frac{E(s)}{R_1} \rightarrow (1)$$

At node 2 by KCL,

$$\frac{V_2 - V_1}{R_2} + C_2 \frac{dV_2}{dt} = 0$$



L.T, apply,

$$\frac{V_2(s)}{R_2} - \frac{V_1(s)}{R_2} + C_2 s V_2(s) = 0$$

$$\frac{V_1(s)}{R_2} = \frac{V_2(s)}{R_2} + C_2 s V_2(s) = \left[\frac{1}{R_2} + s C_2\right] V_2(s)$$

$$V_1(s) = [1 + s C_2 R_2]\, V_2(s). \rightarrow (2)$$

Sub $V_1(s)$ from ② in ①

$$[(1+sR_2C_2)\,V_2(s)]\left[\frac{1}{R_1}+sC_1+\frac{1}{R_2}\right]-\frac{V_2(s)}{R_2}=\frac{E(s)}{R_1}$$

$$\left[\frac{(1+sR_2C_2)(R_2+R_1+sC_1R_1R_2)-R_1}{R_1R_2}\right]V_2(s)=\frac{E(s)}{R_1}$$

$$\frac{V_2(s)}{E(s)}=\frac{R_2}{[(1+sR_2C_2)(R_1+R_2+sC_1R_1R_2)-R_1]}$$

Result :

The (node basis) differential eq. governing the electrical network are,

1. $\frac{V_1}{R_1}+C_1\frac{dV}{dt}+\frac{V_1-V_2}{R_2}=\frac{e}{R_1}$

2. $\frac{V_2-V_1}{R_2}+C_2\frac{dV_2}{dt}=0$

The T.F of electrical network is

$$\frac{V_2(s)}{E(s)}=\frac{R_2}{[(1+sR_2C_2)(R_1+R_2+sC_1R_1R_2)-R_1]}$$

ELECTRICAL ANALOGOUS CIRCUITS FOR TRANSLATIONAL AND ROTATIONAL SYSTEMS :

With respect to mechanical translational system, there are two analogy.

* Force - Voltage analogy (F-V)
* Force - current analogy (F-I).

The system remains analogous as long as the differential equations governing the system on the transfer functions are in identical form.

Transformation table in mechanical Translational system

MTS	F-V	F-I
Force (f)	Voltage source (V)	current source (I)
Velocity (v)	current (i)	Voltage (v)
mass (M)	Inductance (L)	capacitance (C)
Dashpot (B)	Resistance (R)	conductance ($G = 1/R$)
Spring (K)	Inverse of capacitance ($1/C$)	Inverse of Inductance ($1/L$)

Transformation table for mechanical rotational system.

MRS	T-V	T-I
Torque (T)	Voltage source (V)	current source (I)
Angular Velocity (W)	current (i)	Voltage (v)
Moment of inertia (J)	Inductance (L)	capacitance (C)
Dashpot (B)	Resistance (R)	conductance ($G = 1/R$)
Spring K	Inverse of capacitance ($1/C$)	Inverse of Inductance ($1/L$)

With respect to mechanical rotational system the two analogy are

* Torque Voltage analogy (T-V)
* Torque current analogy (T-I)

Force - Voltage and Torque - Voltage procedure :

procedure for developing Force - Voltage (F-V) and Torque voltage (T-V) circuits

1.) write the differential equations for all the independent masses in the system.

2.) change the differential equation in terms of velocity (v) for mechanical translational system and in angular velocity (ω) for mechanical rotational system by applying these changes.

Sub. $x = \int v\,dt$; $\frac{dx}{dt} = v$; $\frac{d^2x}{dt^2} = \frac{dv}{dt}$ for MTS

$\theta = \int \omega\,dt$; $\frac{d\theta}{dt} = \omega$; $\frac{d^2\theta}{dt^2} = \frac{d\omega}{dt}$ for MRS

3.) Then apply the change in transformation table in the differential equations in terms of velocity (for MTS) and angular velocity (for MRS)

4.) Apply KVL to draw the circuit of FV and T-V

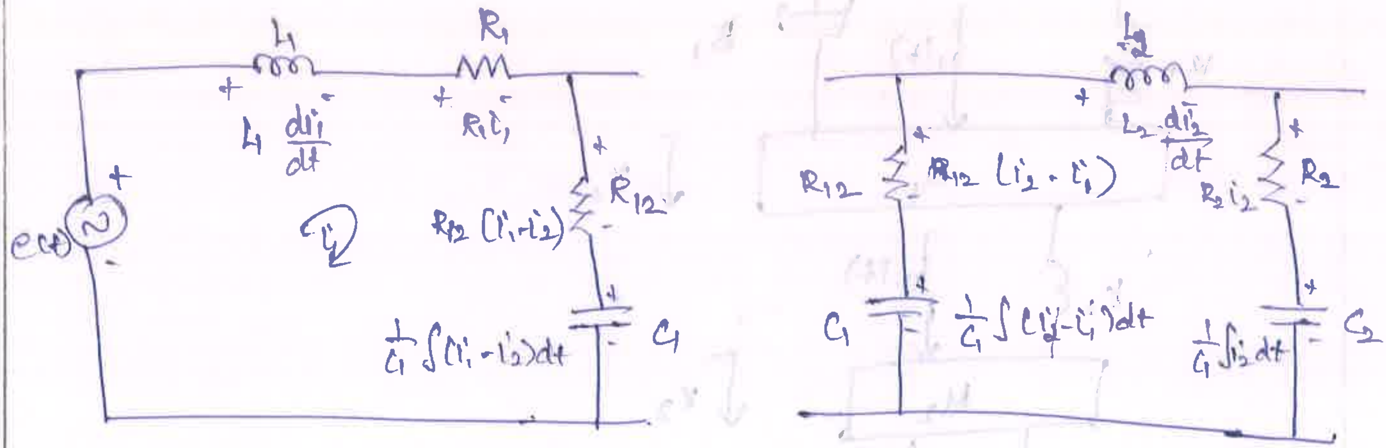
Force - current and Torque - current procedure,

The procedure for F-I & T-I is same as F-V and T-V, but in the last step apply KCL instead of KVL to draw the circuits of F-I and T-I analogy.

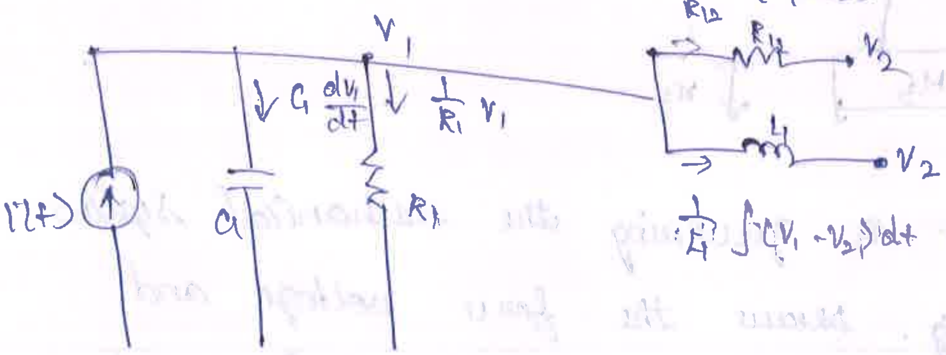
Problem 1:

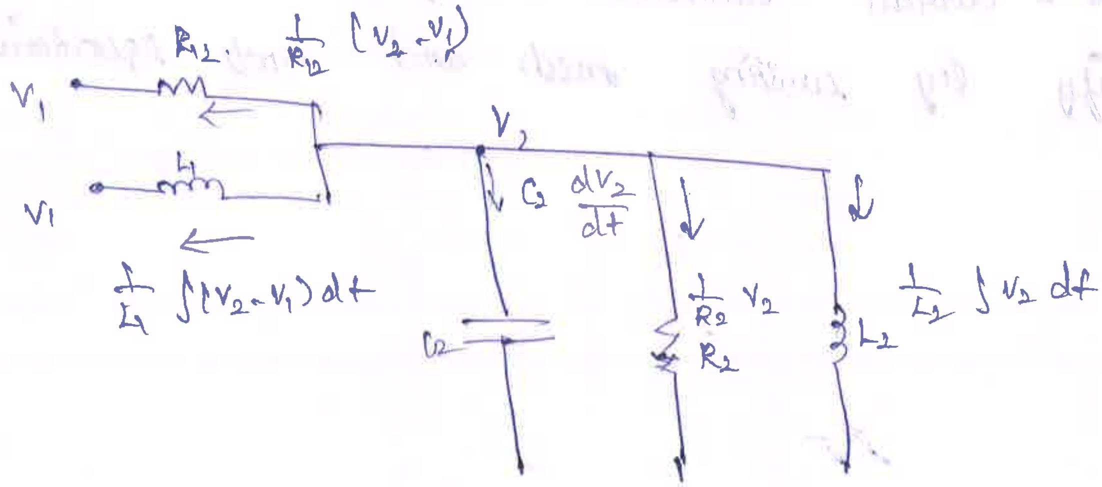
write the diff. eq. governing the mechanical system shown in fig. Draw the F-V and F-I electrical analogous circuits and verify for mesh and nodal equations.

Force-Voltage Analogous circuit:

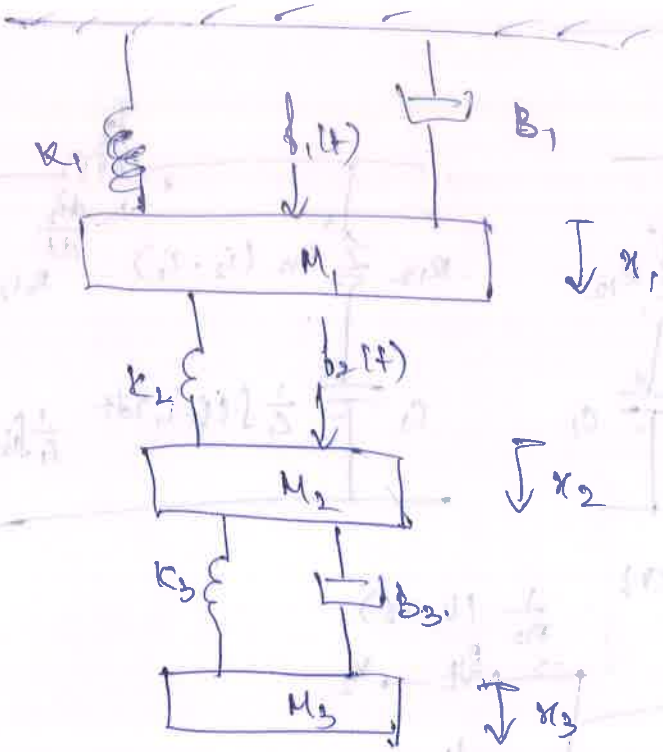


Mesh basis eq. using KVL.

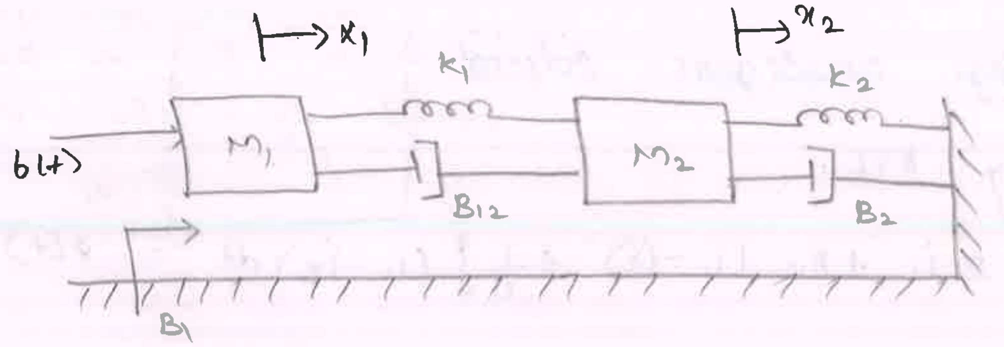




Tutorial:

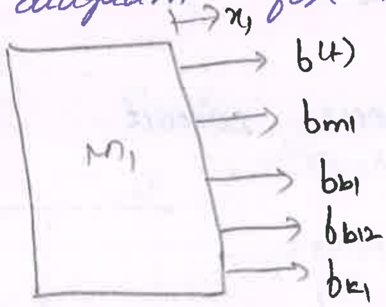


Write the diff. eq. governing the mechanical system shown in fig. Draw the force voltage and force - current electrical analogous circuits and Verify by writing mesh and node equations.



Sol:

Free body diagram for M_1.

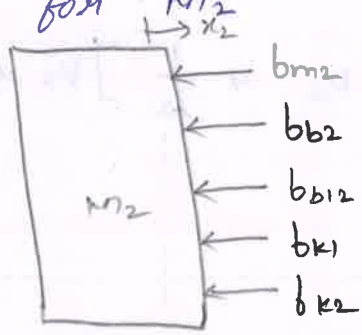


$$M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B_{12} \frac{d}{dt}(x_1 - x_2) + K_1(x_1 - x_2) = f(t) \rightarrow ①$$

Sub. $\frac{d^2x_1}{dt^2} = \frac{dv_1}{dt}$; $\frac{dx_1}{dt} = v_1$; $x_1 = \int v_1 \, dt$

$$M_1 \frac{dv_1}{dt} + B_1 v_1 + B_{12}(v_1 - v_2) + K_1 \int (v_1 - v_2)\, dt = f(t) \rightarrow ②$$

Free body diagram for M_2



$$M_2 \frac{d^2x}{dt^2} + B_2 \frac{dx}{dt} + K_2 x_2 + B_{12} \frac{d}{dt}(x_2 - x_1) + K_1(x_2 - x_1) = 0 \rightarrow ③$$

Sub. $\frac{d^2x_2}{dt^2} = \frac{dv_2}{dt}$; $\frac{dx_2}{dt} = v_2$; $x_2 = \int v_2 \, dt$

$$M_2 \frac{dv_2}{dt} + B_2 v_2 + K_2 \int v_2 \, dt + B_{12}(v_2 - v_1) + K_1 \int (v_2 - v_1)\, dt = 0 \rightarrow ④$$

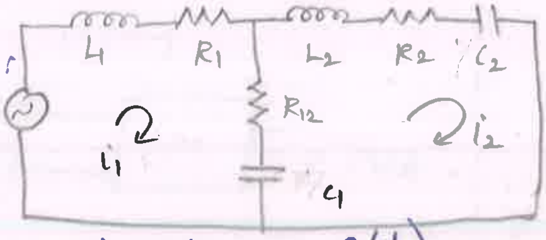
FVAC

$f(t) = e(t)$ $M_1 \rightarrow L_1$ $B_1 \rightarrow R_1$ $K_1 : 1/C_1$

$v_1 \rightarrow i_1$ $M_2 \rightarrow L_2$ $B_2 \rightarrow R_2$ $K_2 : 1/C_2$

$v_2 \rightarrow i_2$ $B_{12} \rightarrow R_{12}$

Force Voltage analogous circuit :

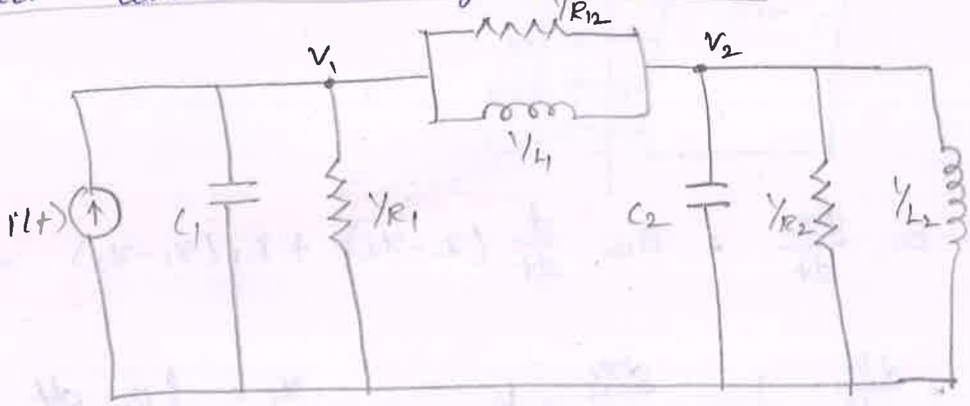


On applying KVL,

$$L_1 \frac{di_1}{dt} + R_1 i_1 + R_{12}(i_1 - i_2) + \frac{1}{C_1}\int (i_1 - i_2)\,dt = e(t)$$

$$L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt + R_{12}(i_2 - i_1) + \frac{1}{C_1}\int (i_2 - i_1) = 0$$

Force current analogous circuit



$f(t) = i(t)$ $M_1 \to C_1$

$M_2 \to C_2$

$V_1 \to V_1$

$V_2 \to V_2$ $B_{12} \to 1/R_{12}$

$B_1 \to 1/R_1$ $K_1 \to 1/L_1$

$B_2 \to 1/R_2$ $K_2 \to 1/L_2$

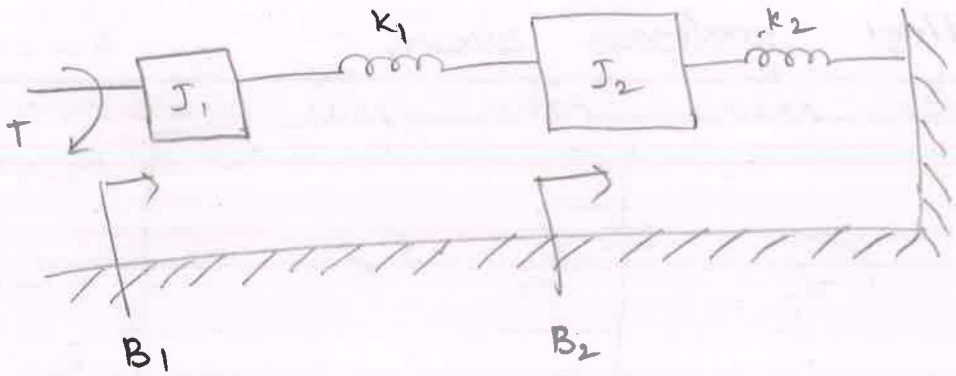
On applying KCL,

$$C_1 \frac{dV_1}{dt} + \frac{1}{R_1} V_1 + \frac{1}{R_{12}}(V_1 - V_2) + \frac{1}{L_1}\int (V_1 - V_2)\,dt = i(t)$$

$$C_2 \frac{dV_2}{dt} + \frac{1}{R_2} V_2 + \frac{1}{L_2}\int V_2\,dt + \frac{1}{R_{12}}(V_2 - V_1) + \frac{1}{L_2}\int (V_2 - V_1)\,dt = 0 .$$

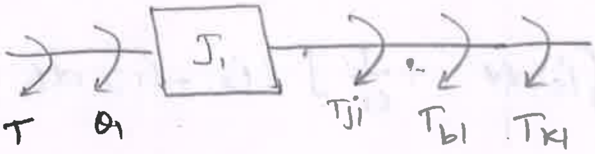
Problem 2 :

write the diff. eq. governing the mechanical rotational system shown in fig. Draw the T.V and T.I electrical analogous circuit and verify by writing mesh and nodal equations.



Sol :

Free body diagram for J_1

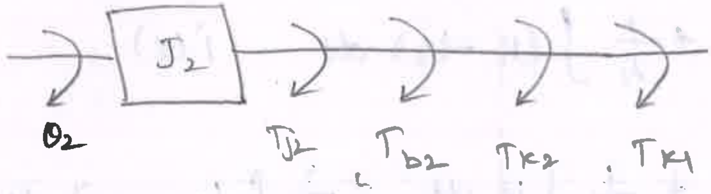


$$J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + K_1(\theta_1 - \theta_2) = T$$

sub. $\dfrac{d^2\theta_1}{dt^2} = \dfrac{d\omega}{dt}$; $\dfrac{d\theta_1}{dt} = \omega_1$; $\theta_1 = \int \omega_1 dt$

$$J_1 \frac{d\omega_1}{dt} + B_1\omega_1 + K_1 \int (\omega_1 - \omega_2)\, dt = T$$

~~Free~~ Free body diagram for J_2

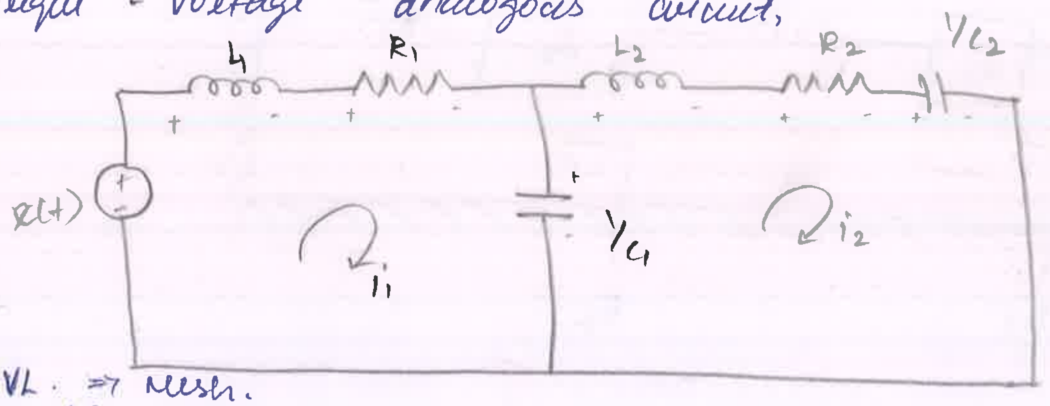


$$J_2 \frac{d^2\theta_2}{dt^2} + K_2\theta_2 + B_2 \frac{d\theta_2}{dt} + K_1(\theta_2 - \theta_1) = 0$$

Sub $\dfrac{d^2\theta_2}{dt^2} = \dfrac{d\omega_2}{dt}$; $\dfrac{d\theta_2}{dt} = \omega_2$; $\theta_2 = \int \omega_2 dt$

$$J_2 \frac{d\omega_2}{dt} + K_2 \int \omega_2 dt + B_2\omega_2 + K_1 \int (\omega_2 - \omega_1)\, dt = 0$$

Torque - Voltage analogous circuit.



$T \to e(t)$ $\quad J_1 \to L_1$

$\omega_1 \to i_1$ $\quad J_2 \to L_2$

$\omega_2 \to i_2$

$B_1 \to R_1$ $\quad K_1 \to 1/C_1$

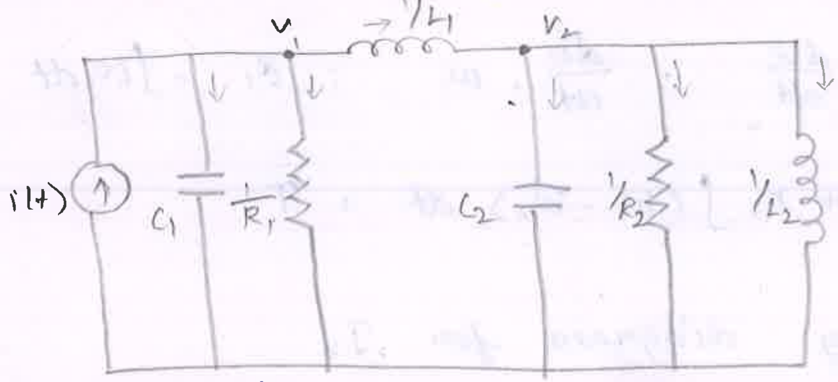
$B_2 \to R_2$ $\quad K_2 \to 1/C_2$

KVL $\Rightarrow$ Mesh.

$$L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int (i_1 - i_2)\,dt = e(t)$$

$$L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt + \frac{1}{C_1}\int (i_2 - i_1)\,dt = 0$$

Torque - current analogous circuit



$T \to i(t)$ $\quad \omega_1 \to v_1$

$\omega_2 \to v_2$

$B_1 \to 1/R_1$ $\quad J_1 \to C_1$

$B_2 \to 1/R_2$ $\quad J_2 \to C_2$

$K_1 \to 1/L_1$

$K_2 \to 1/L_2$

KCL $\to$ Nodal.

$$C_1 \frac{dv_1}{dt} + \frac{1}{R_1} v_1 + \frac{1}{L_1}\int (v_1 - v_2)\,dt = i(t)$$

$$C_2 \frac{dv_2}{dt} + \frac{1}{R_2} v_2 + \frac{1}{L_2}\int v_2\,dt + \frac{1}{L_1}\int (v_2 - v_1)\,dt = 0$$

MRS	T-V	T-I
T	v, e	i
ω	i	v
J	L	C
B	R	1/R
K	1/C	1/L

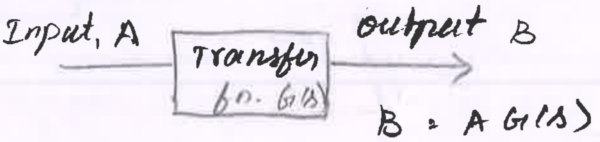
BLOCK DIAGRAMS:

A control system consists of no. of components. In control engg. to show the fun. performed by each component, we commonly use a diagram called block diagram.

=> pictorial representation of fun. performed by each component and of flow of signals. Elements are block, branch point & summing point.

BLOCK:

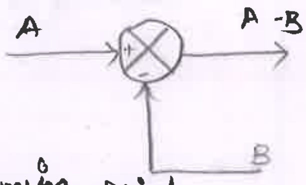
All system variables are linked to each other through fn. blocks. Block is a symbol for math. operation on i/p signal to block that produces output.



Functional block

SUMMING POINT:

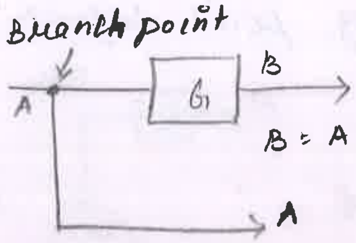
To add two or more signals in the system.



Summing point

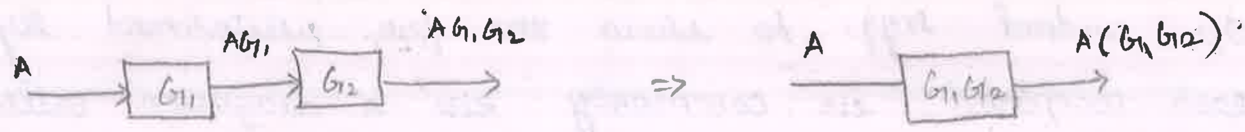
BRANCH POINT:

It is a point from which the signal from a block goes concurrently to other blocks or summing points.

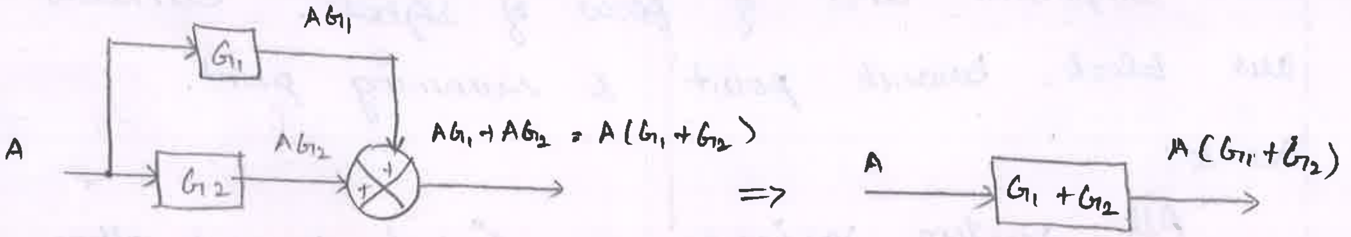


BLOCK DIAGRAM REDUCTION

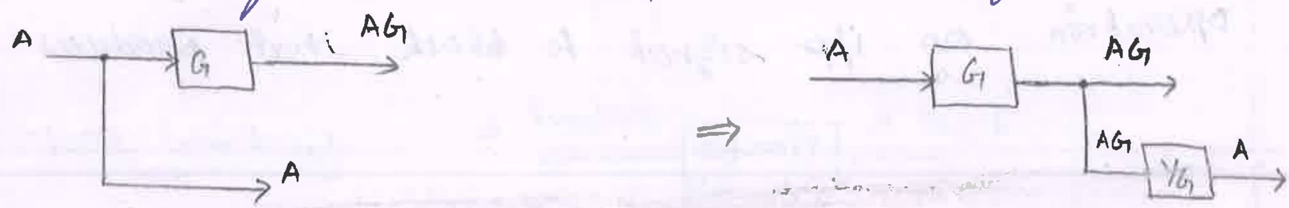
Rule 1: combining blocks in cascade



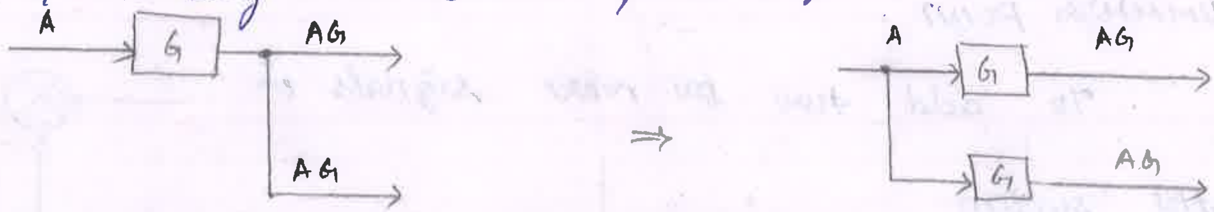
Rule 2: Combining parallel blocks (comb. feed forward path)



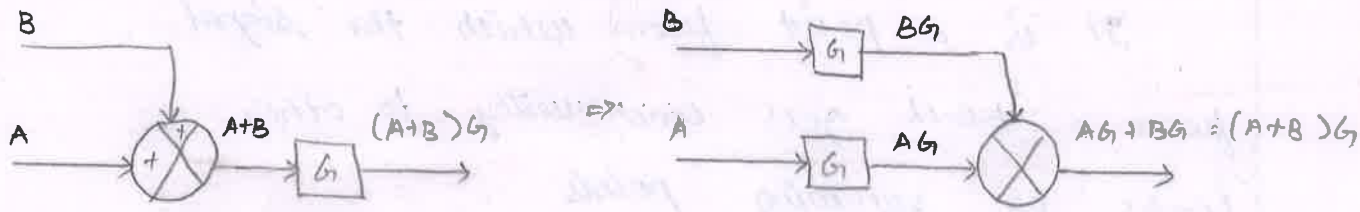
Rule 3: Moving the branch point ahead of block



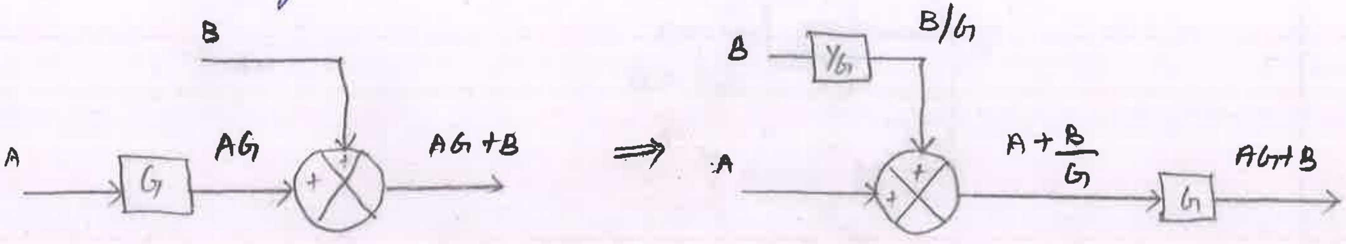
Rule 4: Moving the branch point before the block



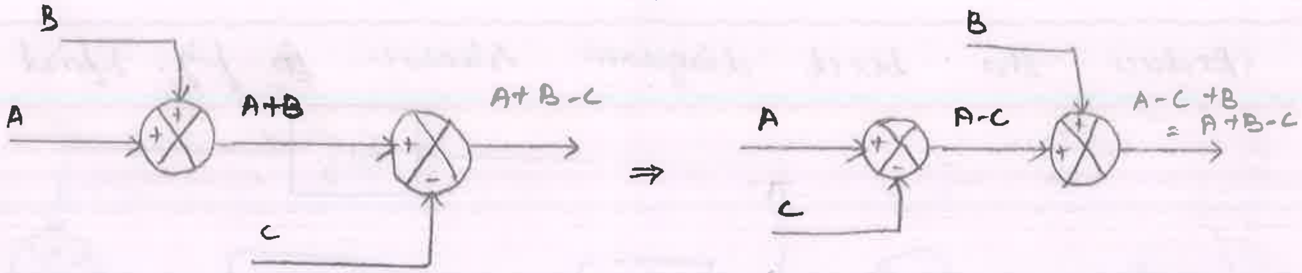
Rule 5: Moving the summing point ahead of block



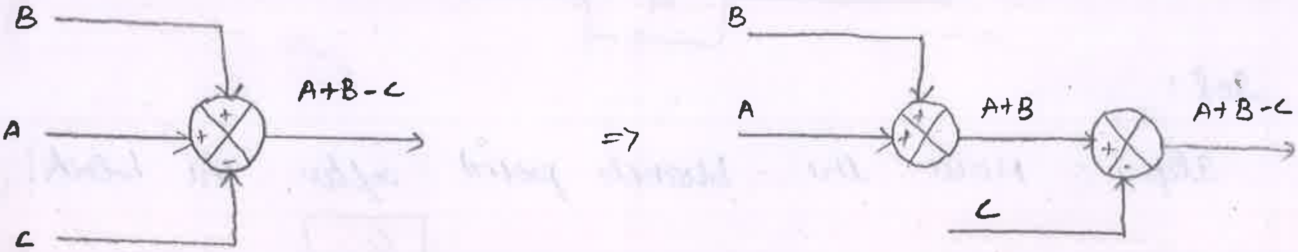
Rule 6: Moving the summing point before the block



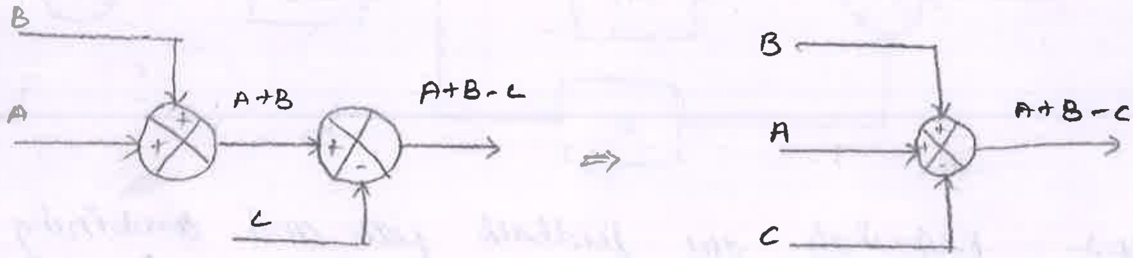
Rule 7 : Interchanging summing point



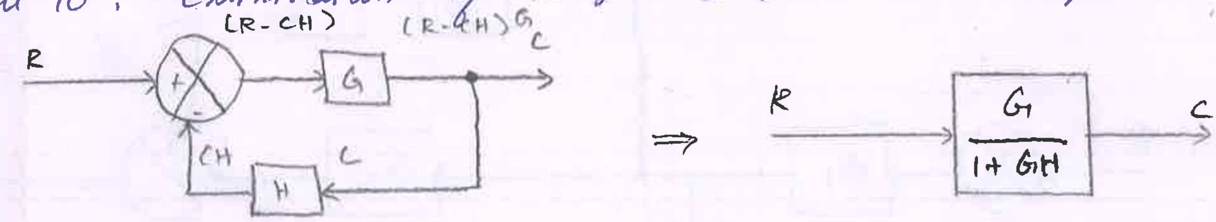
Rule 8 : Splitting summing points



Rule 9 : combining summing points



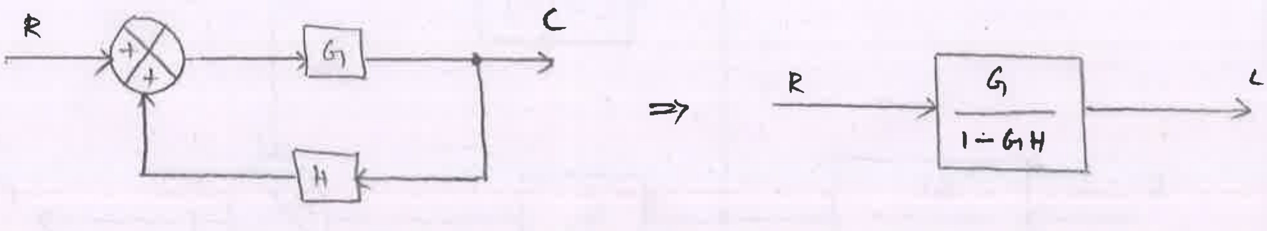
Rule 10 : Elimination of (negative) feedback loop



PROOF : $C = (R - CH)G \Rightarrow C = RG - CHG \Rightarrow C + CHG = RG$

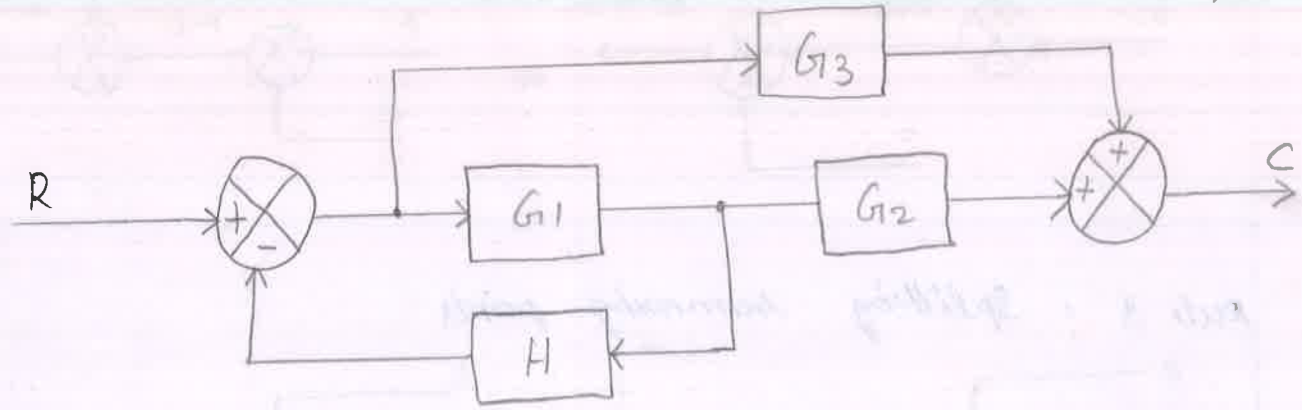
$\therefore C(1 + HG) = RG \Rightarrow \dfrac{C}{R} = \dfrac{G}{1+GH}$

Rule 11 : Elimination of (positive) feedback loop.



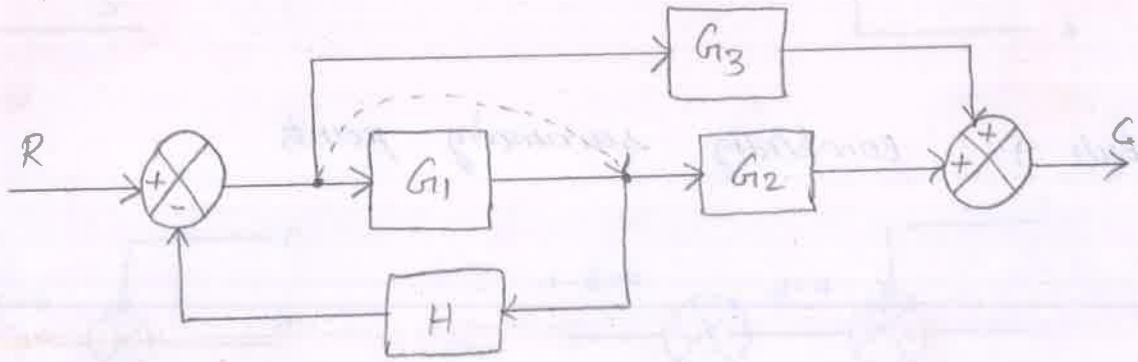
problem 1:

Reduce the block diagram shown in fig. & find $\frac{C}{R}$

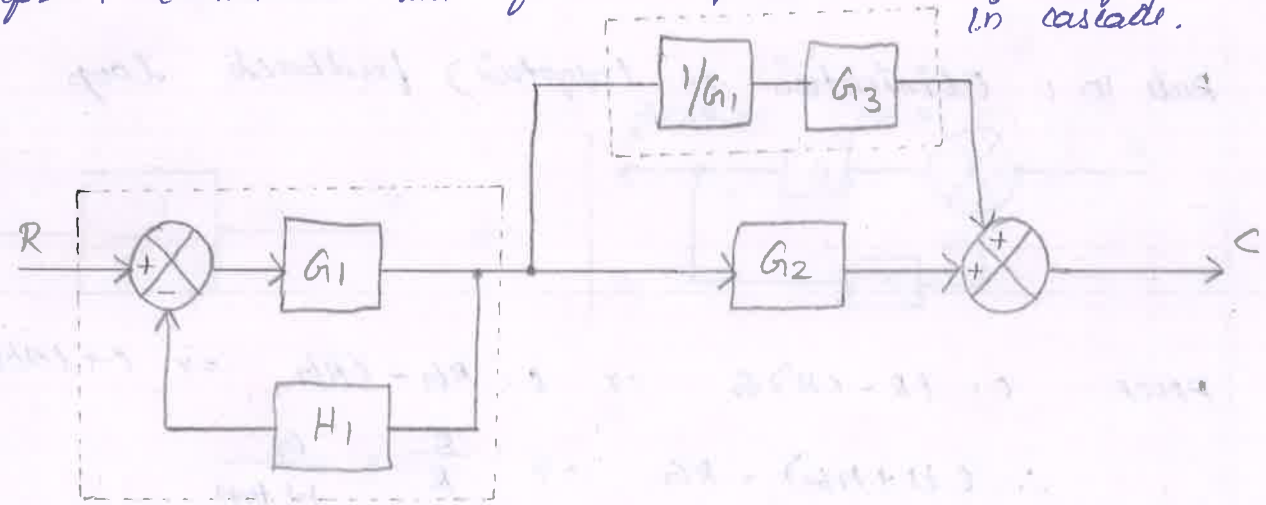


Sol:

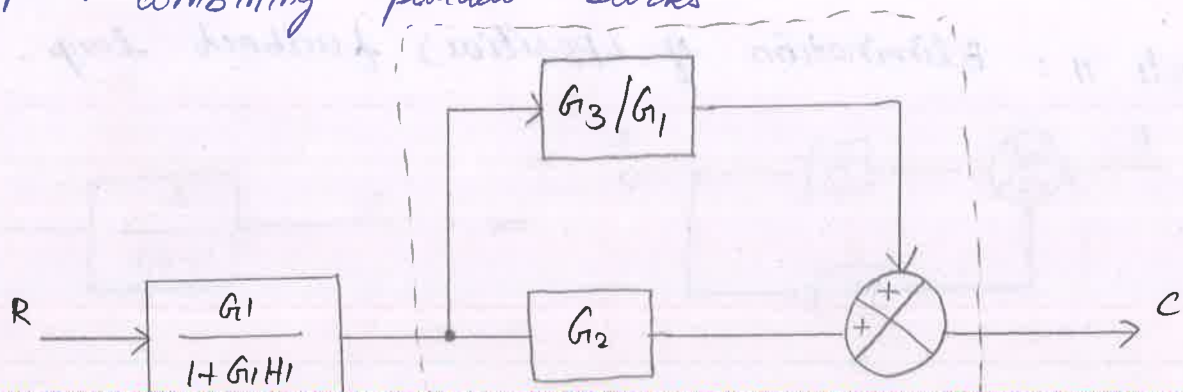
Step 1: Move the branch point after the block



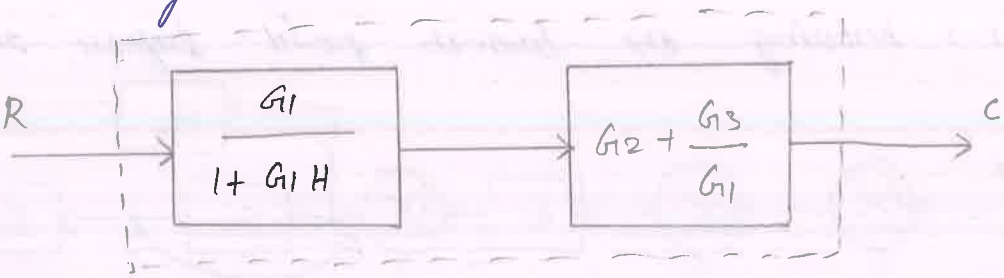
Step 2: Eliminate the feedback path and combining blocks in cascade.



Step 3: combining parallel blocks



Step 4 : combining blocks in cascade.



$$\frac{C}{R} = \left(\frac{G_1}{1+G_1H}\right)\left(G_2 + \frac{G_3}{G_1}\right)$$

$$= \left(\frac{G_1}{1+G_1H}\right)\left(\frac{G_1G_2 + G_3}{G_1}\right)$$

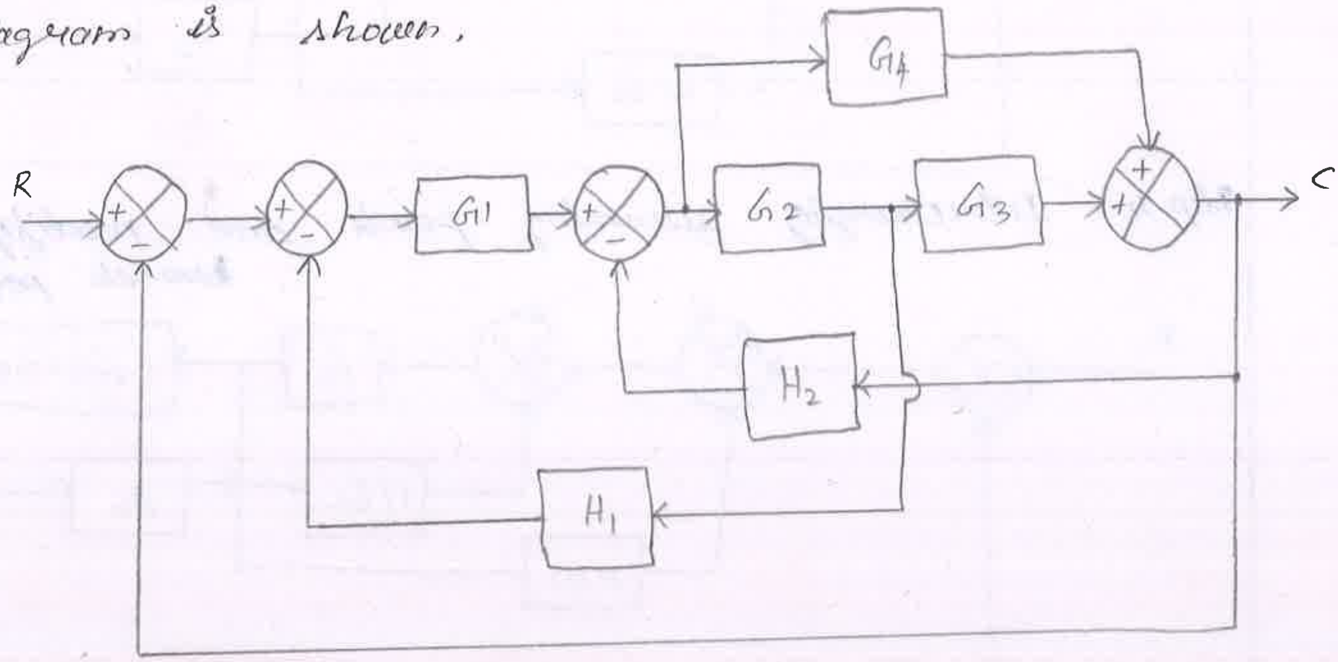
$$= \frac{G_1G_2 + G_3}{1+G_1H_1}$$

Result : The overall T.F of the system,

$$\frac{C}{R} = \frac{G_1G_2 + G_3}{1+G_1H}.$$

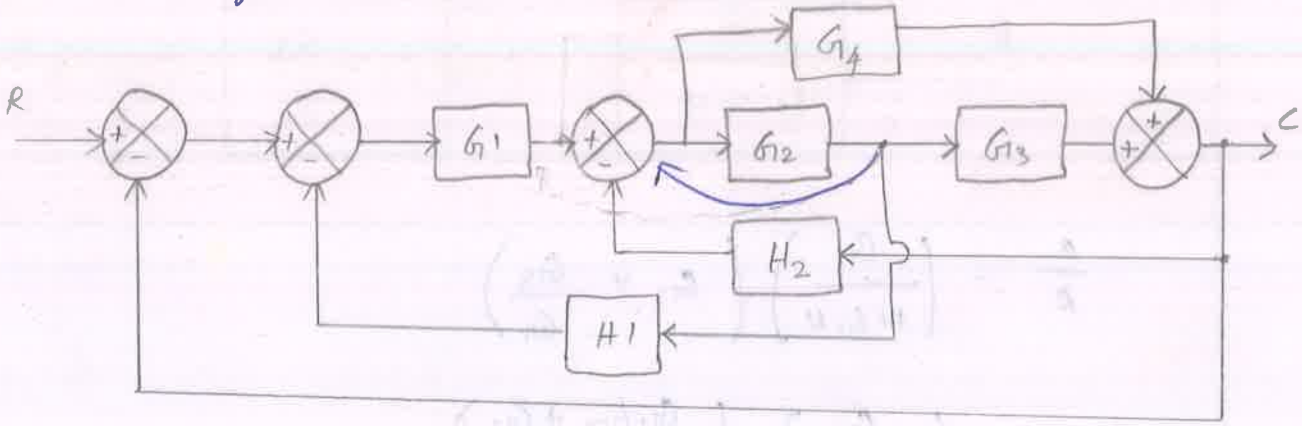
Problem 2 :

Using the block diagram reduction tech. find closed path loop transfer fn. of the system whose block diagram is shown.

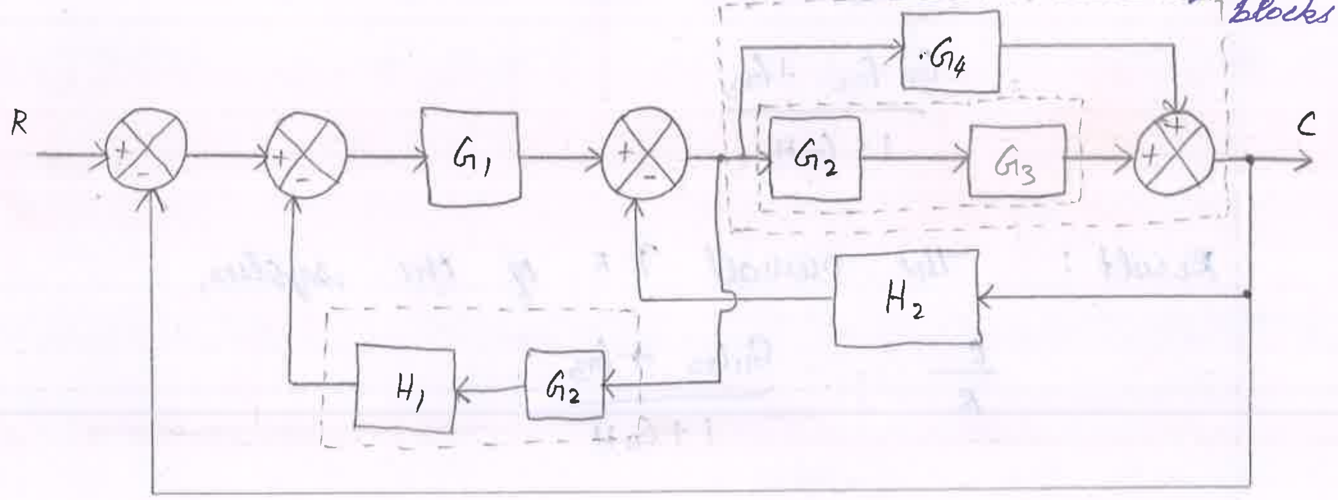


Sol :

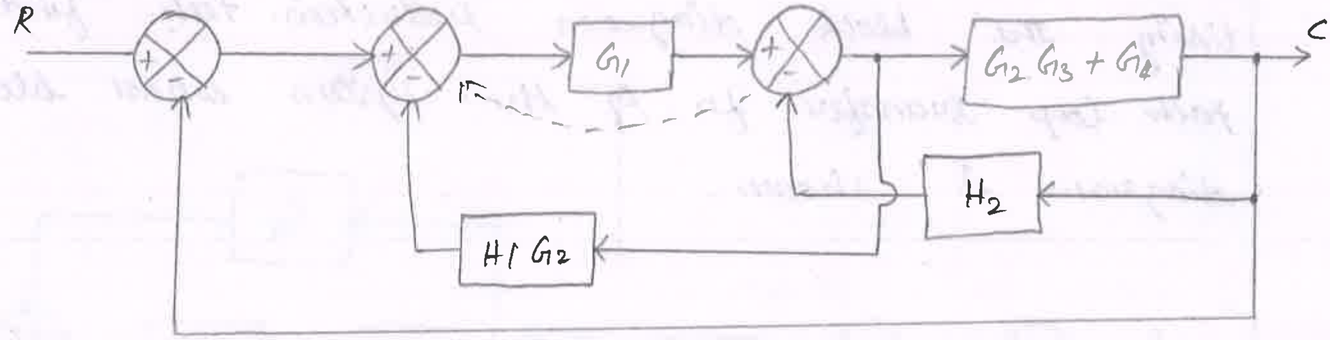
Step 1 : moving the branch point before the block.



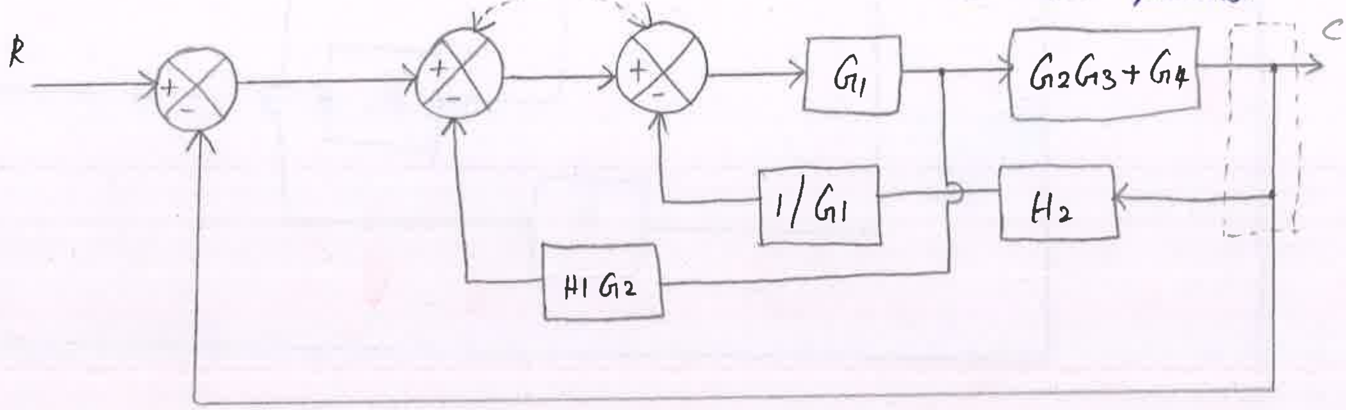
Step 2 : combining the blocks in cascade and eliminating ||el blocks



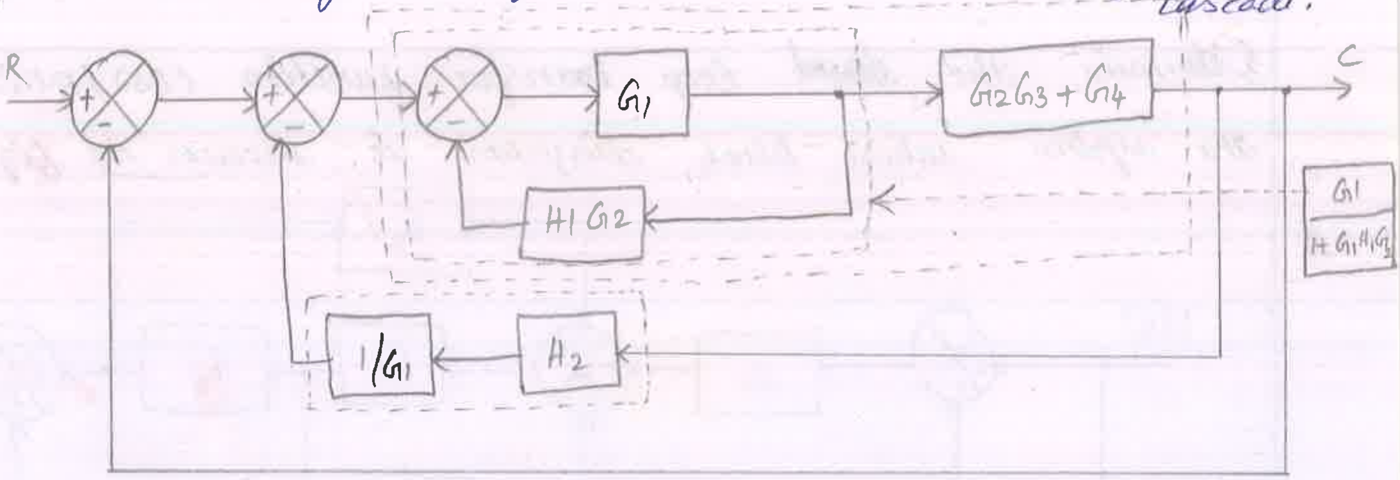
Step 3 : moving summing point before the block.



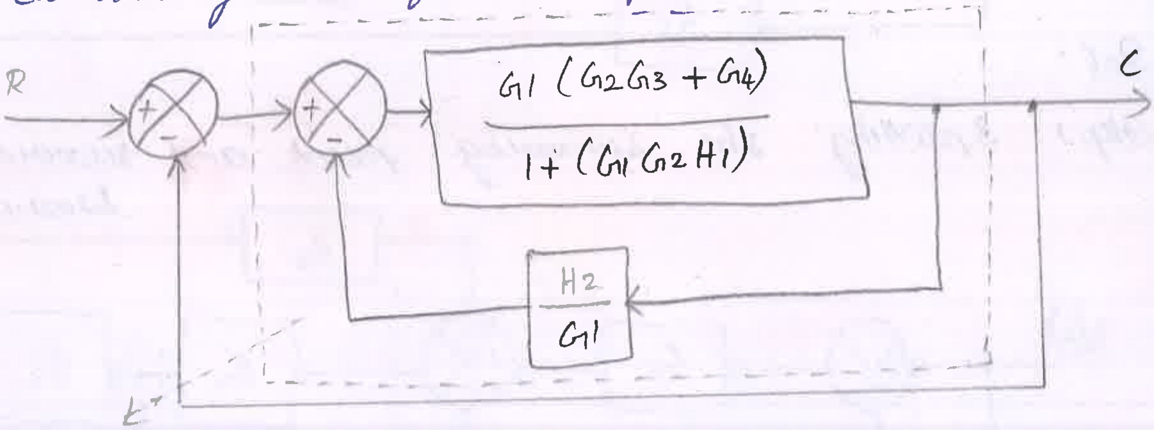
Step 4 : Interchanging summing points and modifying branch points.



Step 5 : Eliminating the feedback path and combining block in cascade.



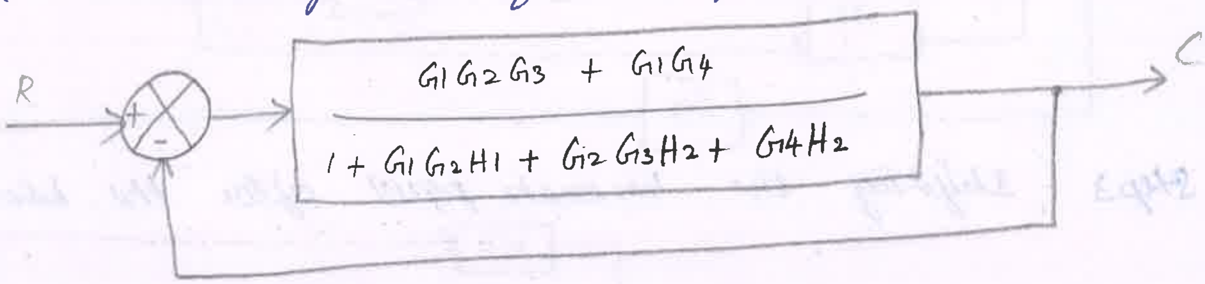
Step 6 : Eliminating the feedback path



$$\frac{\dfrac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1}}{1+\dfrac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1}\cdot\dfrac{H_2}{\cancel{G_1}}} = \frac{\dfrac{G_1G_2G_3+G_1G_4}{\cancel{1+G_1G_2H_1}}}{\dfrac{(1+G_1G_2H_1)+G_2G_3H_2+G_4H_2}{\cancel{1+G_1G_2H_1}}}$$

$$= \frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}$$

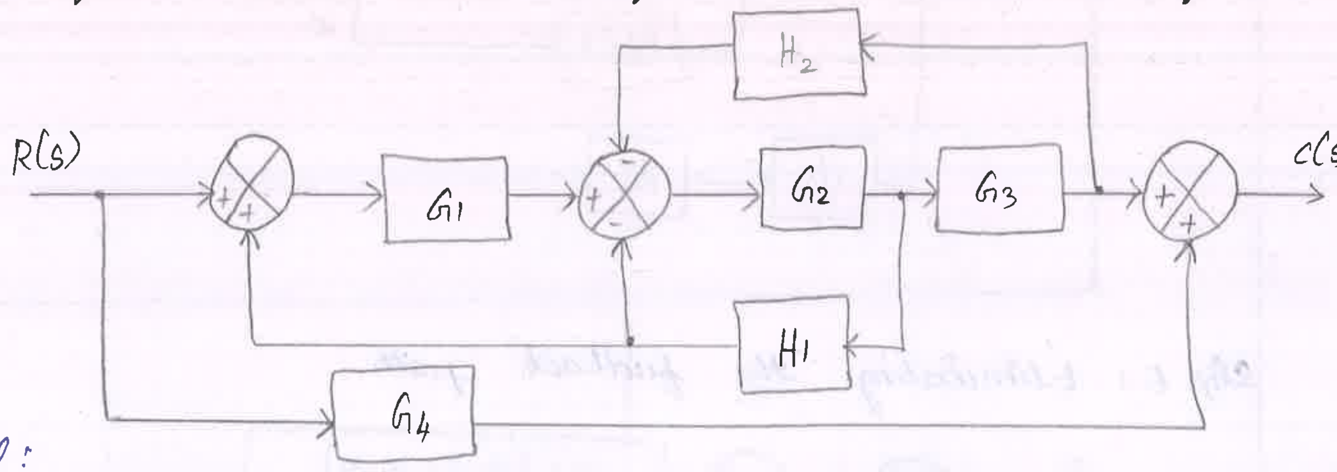
Step 7 : Eliminating the feedback path



$$\frac{C}{R} = \frac{\dfrac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}}{1+\dfrac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}} = \frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2+G_1G_2G_3+G_1G_4}$$

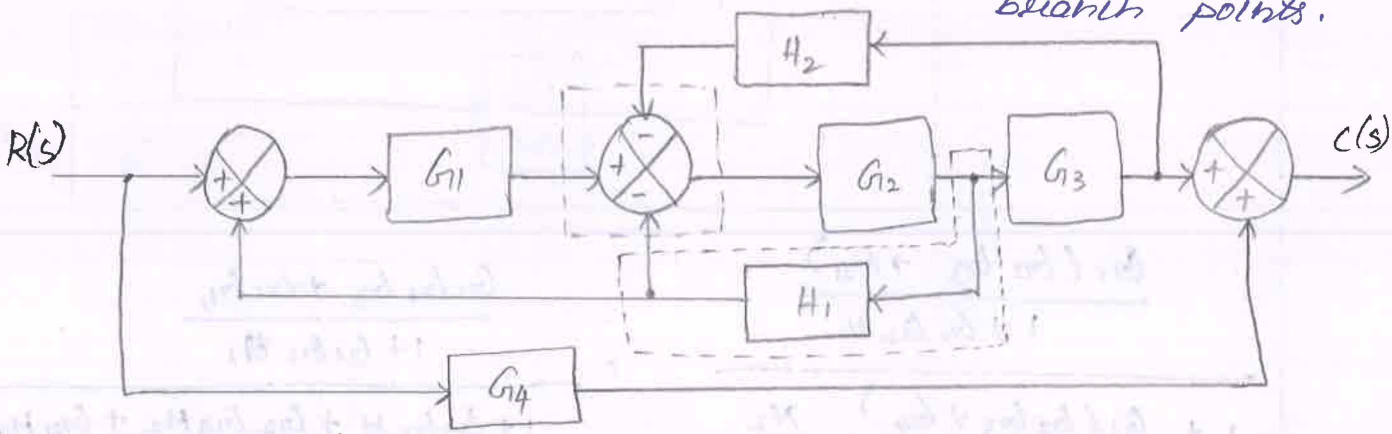
Problem 3 :

Determine the ~~closed~~ loop transfer function C(s)/R(s) of the system whose block diagram is shown in fig.

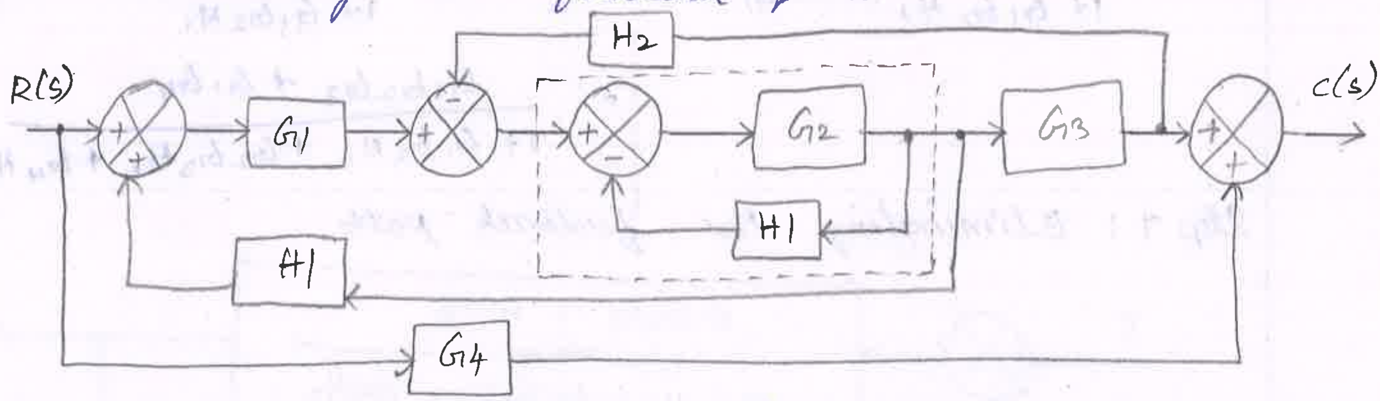


Sol :

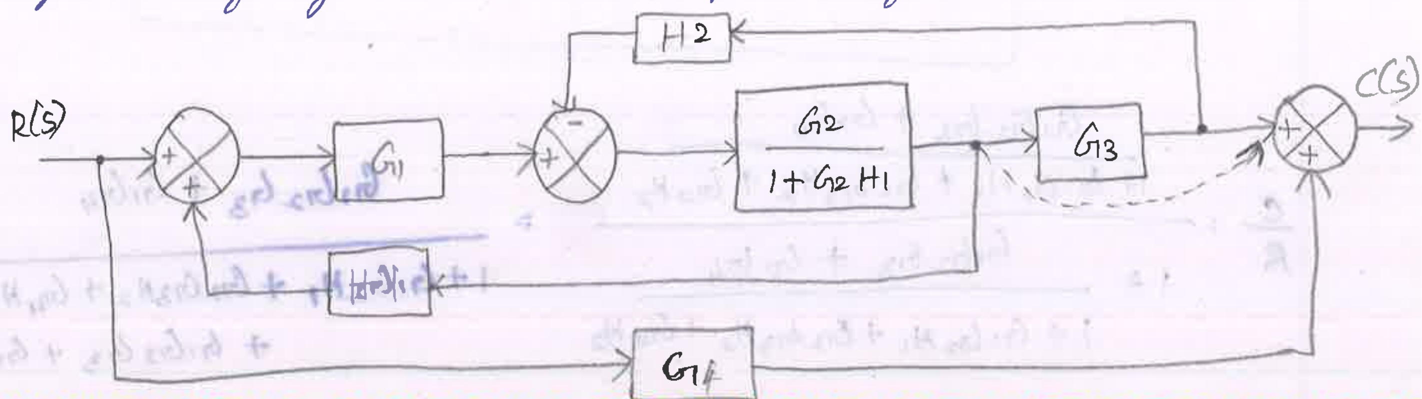
Step 1 : Splitting the summing point and rearranging the branch points.



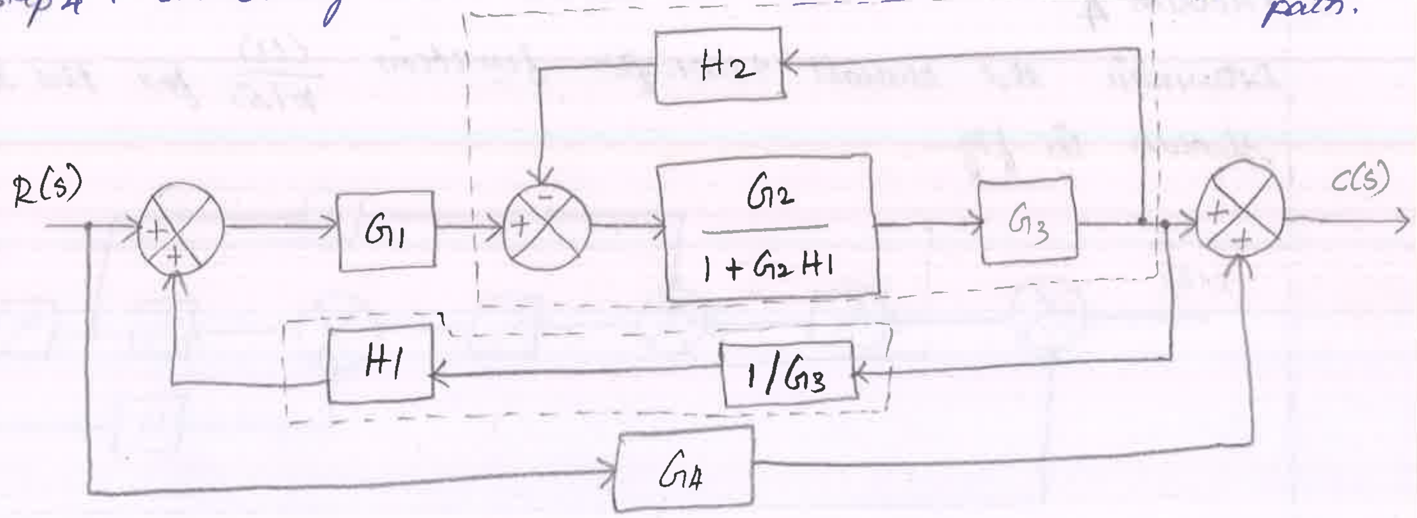
Step 2 : Eliminating the feedback path.



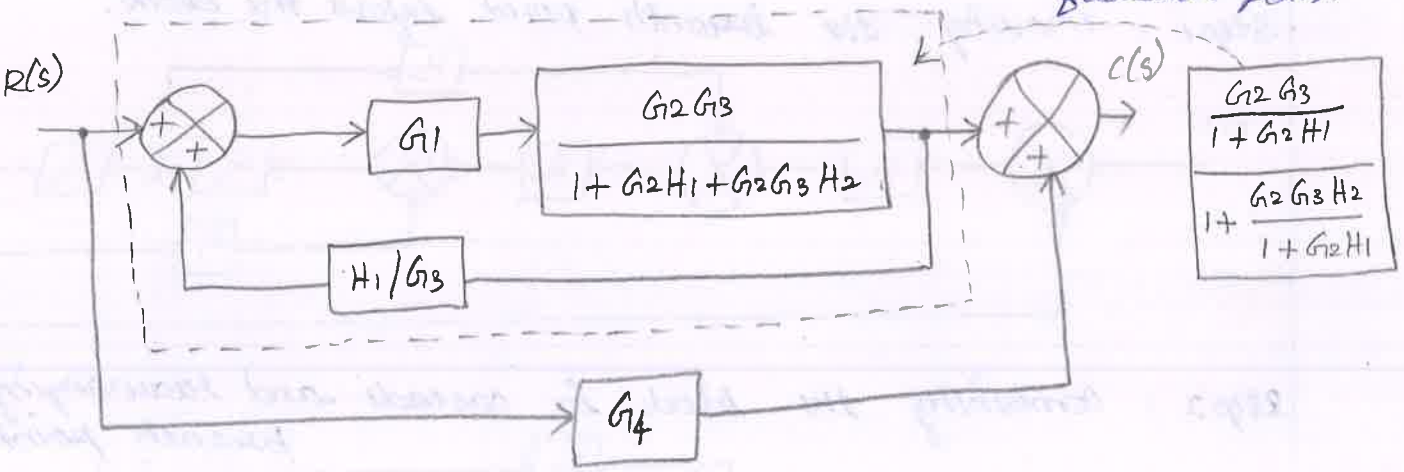
Step 3 : Shifting the branch point after the block.



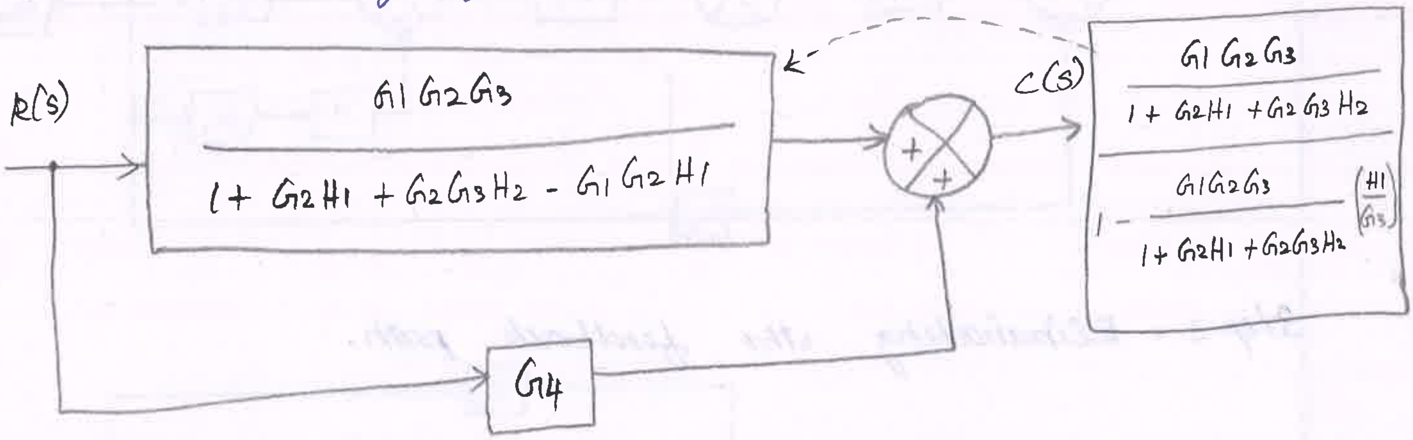
Step 4 : combining the blocks in cascade & eliminating feedback path.



Step 5 : combining the blocks in cascade & eliminating feedback path.



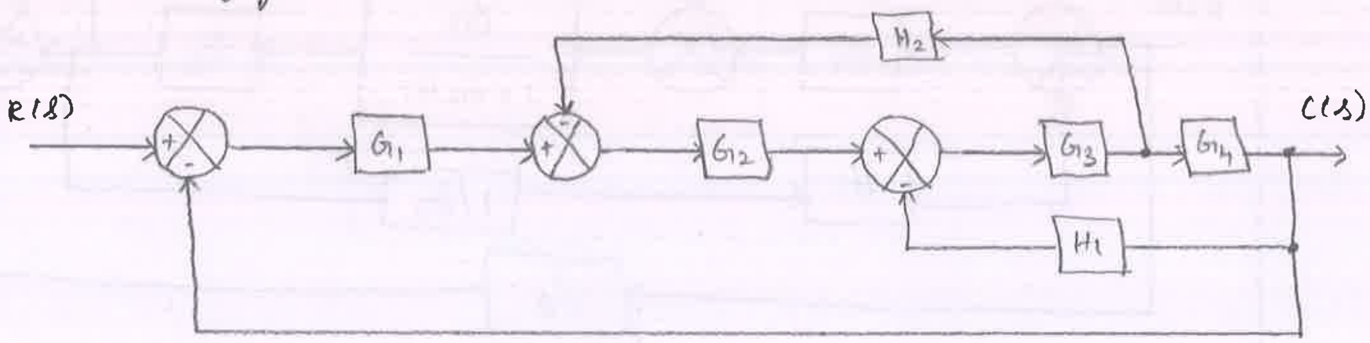
Step 6 : Eliminating forward path



$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3}{1 + G_2 H_1 + G_2 G_3 H_2 - G_1 G_2 H_1} + G_4$$

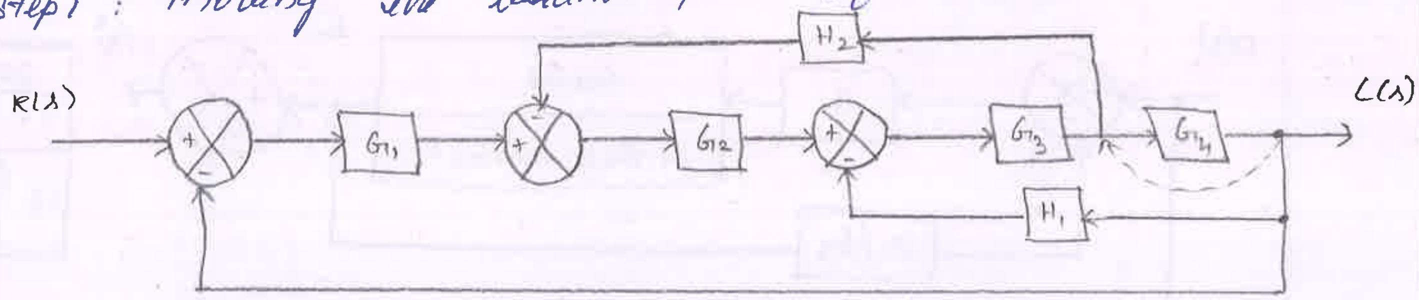
Problem 4:

Determine the overall transfer function $\frac{C(s)}{R(s)}$ for the system shown in fig.

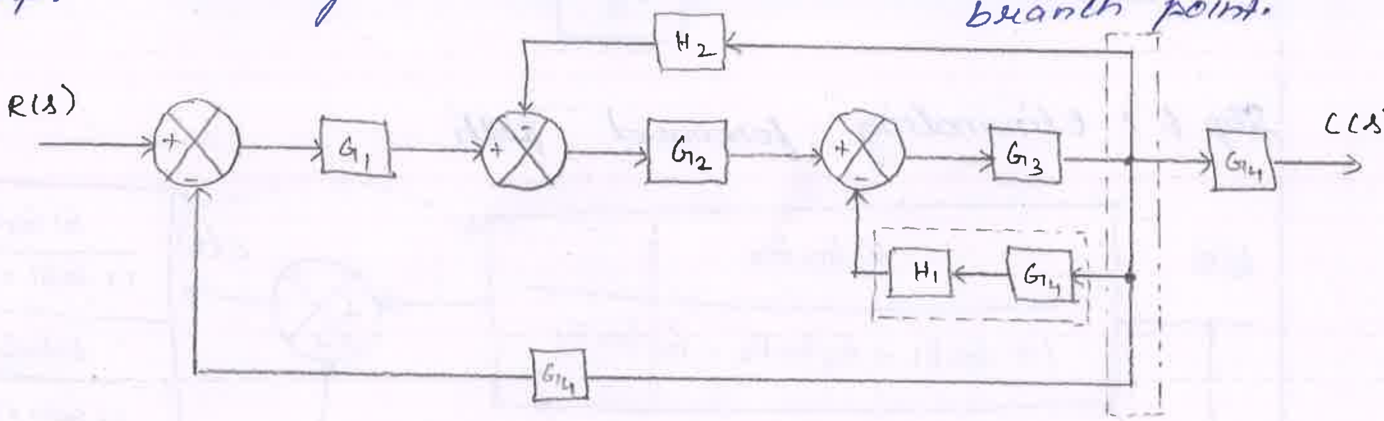


Sol:

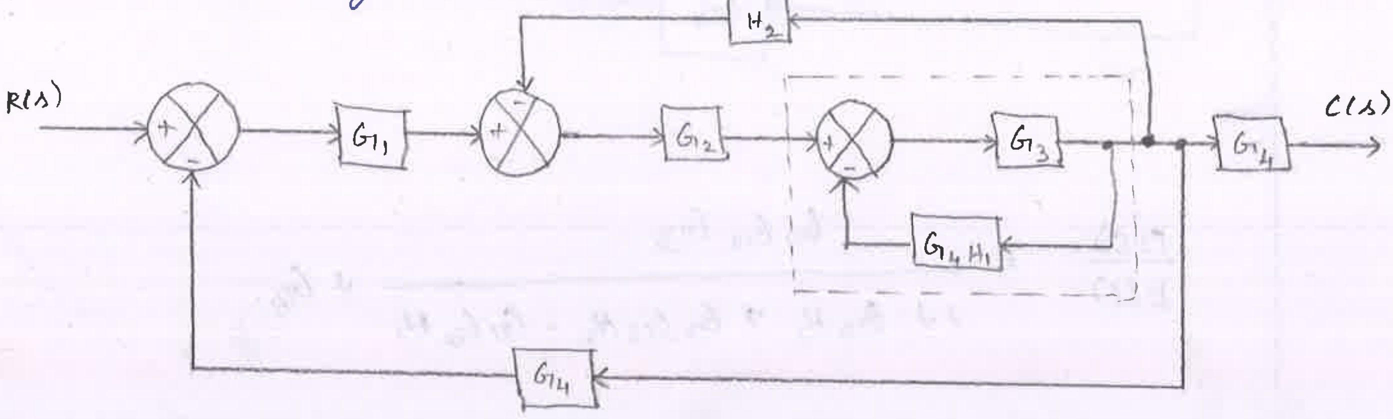
Step 1: moving the branch point before the block.



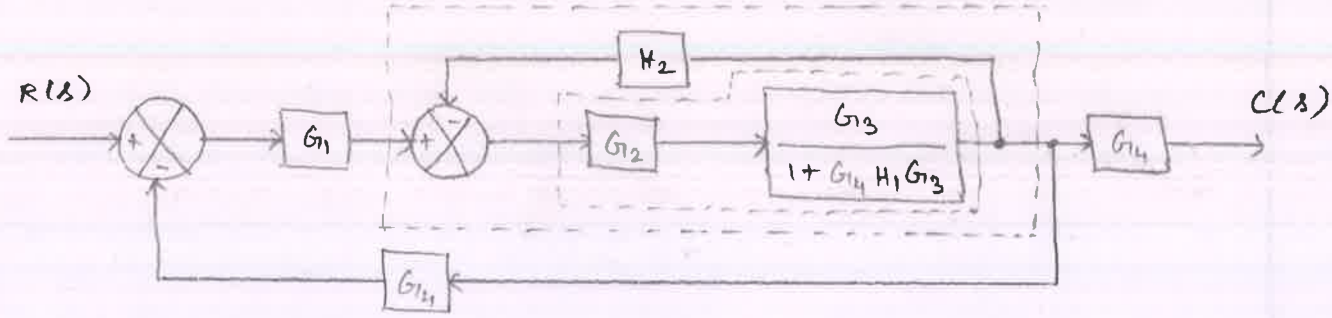
Step 2: Combining the block in cascade and rearranging the branch point.



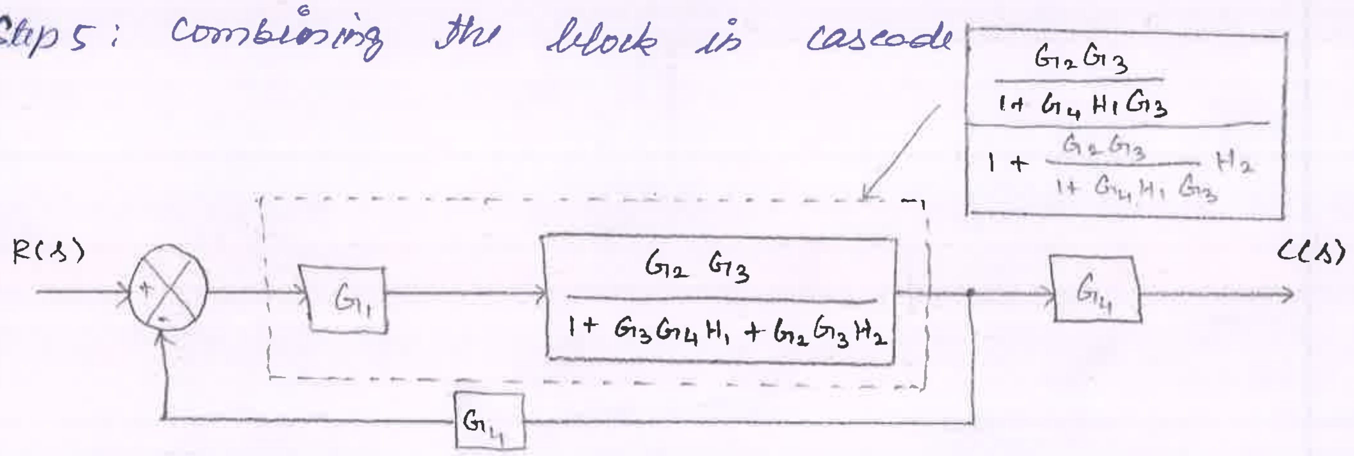
Step 3: Eliminating the feedback path.



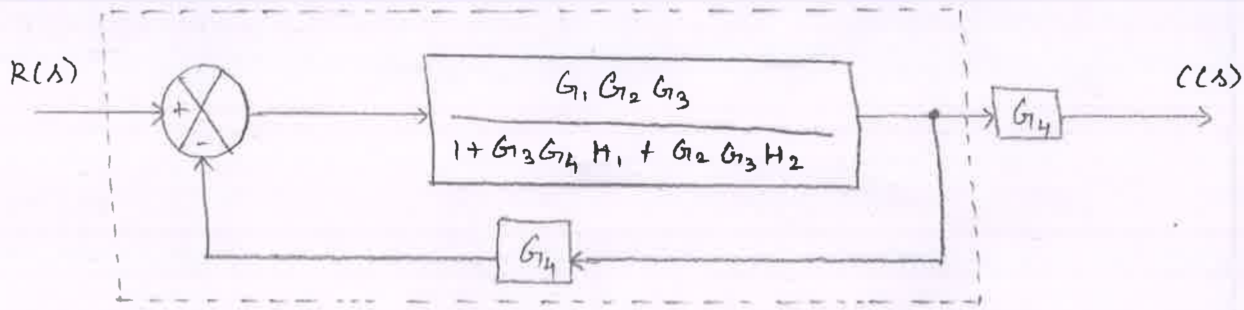
Step 4: Combining the block in cascade and eliminating feedback path.



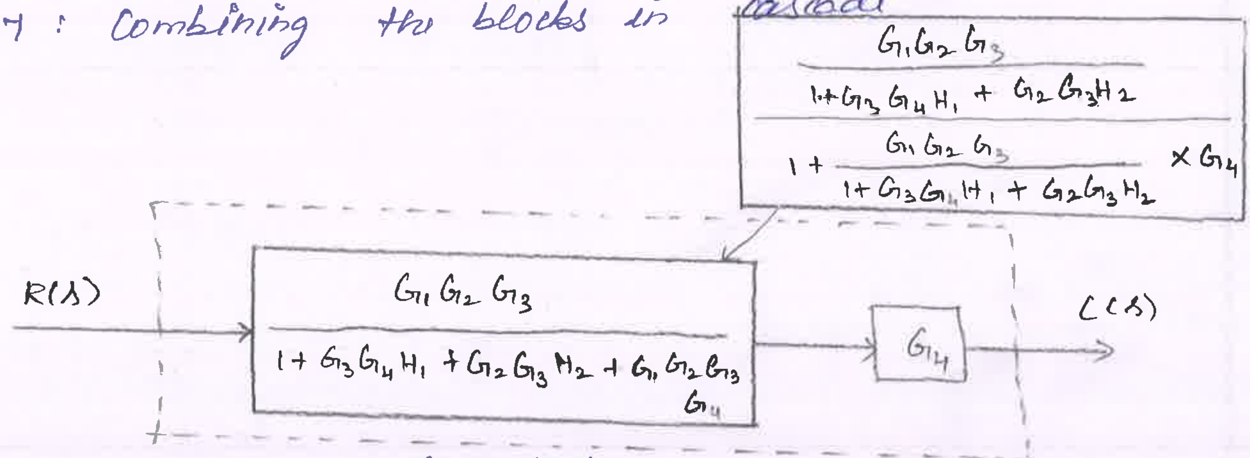
Step 5: Combining the block in cascade



Step 6: Eliminating the feedback path



Step 7: Combining the blocks in cascade



$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{1 + G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4}$$ //

SIGNAL FLOW GRAPH

It is used to represent the control system graphically and it was developed by S.J. Mason. It is a diagram that represent a set of simultaneous linear algebraic eqn. By L.T, time domain diff. eq. governing a control system can be transferred to set of algebric eq. in S-domain.

By using Mason's gain (multiplication factor) formula - overall gain of system can be computed easily. Signal flows in only one direction.

TERMS EXPLANATION:

Node - It is point rep. a Variable or signal

Branch - Directed line segment joining two nodes.

Transmittance - Gain acquired by signal when it travels from one node to another.

Input node (source) - Node has only outgoing branches

Output node (Sink) - Node has only incoming branches

Mixed node - Node has both incoming & outgoing branches.

path - Traversal of connected branches in direction of branch arrows.

Open path - Starts at a node, ends at another node

closed path - starts & ends at same node

Forward path - path from i/p to o/p node.

Forward path gain - product of branch transmittances (gain) forward path.

Individual loop - closed path start from node & arriving at same node.

Loop gain - product of gain of loop

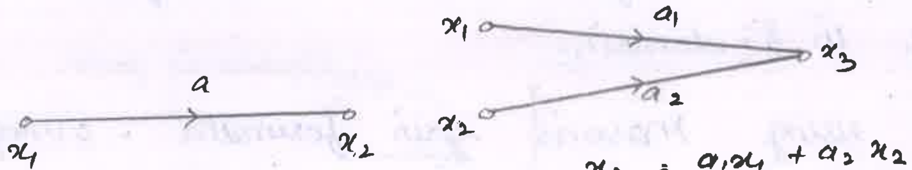
Non - touching loop - loop does not have common node.

SIGNAL FLOW GRAPH ALGEBRA

It can be reduced to obtain T.F of the system using the following rules

Rule 1: Incoming signal to a node through a branch is given by product of signal at previous node and the gain of branch.

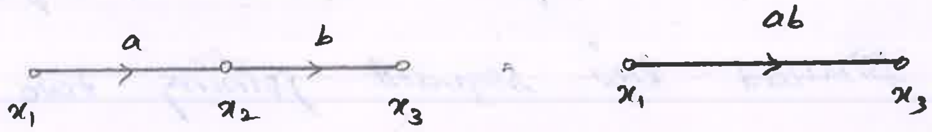
Eg:



$\therefore x_2 = a x_1$ $\qquad$ $x_3 = a_1 x_1 + a_2 x_2$

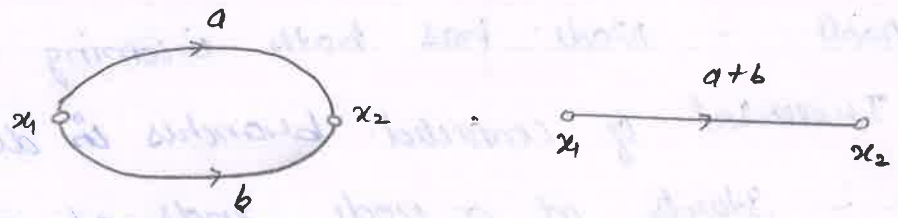
Rule 2: Cascaded branches can be combined to give a single branch whose transmittance is equal to product of indv. branch transmittance.

Eg:



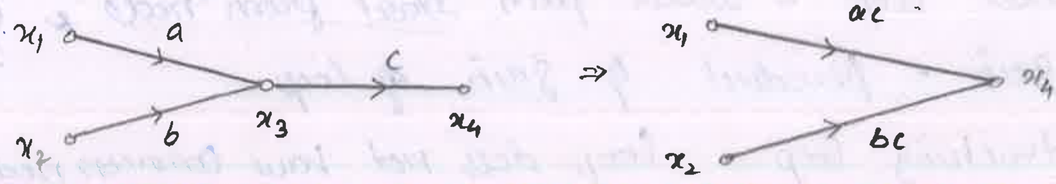
Rule 3: Parallel branches may be rept. by single branch whose transmittance is the sum of individual branch transmittances.

Eg:



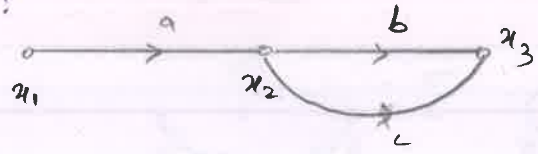
Rule 4: A mixed node can be eliminated by multiplying the transmittances of outgoing branch (from the mixed node) to transmittance of all incoming branches to mixed node.

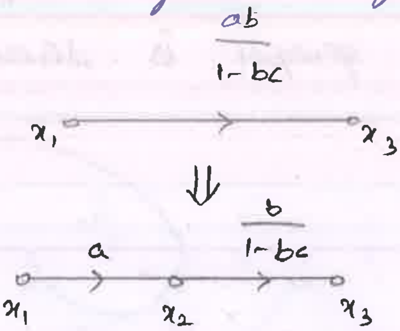
Eg.



Rule 5: A loop may be eliminated by writing eq. at i/p and o/p node and rearranging the eq. to find the ratio of o/p to i/p. This ratio gives the gain of resultant branch.

Eg:





proof:

$$x_2 = ax_1 + cx_3 \quad ; \quad x_3 = bx_2$$

Put $x_2 = ax_1 + cx_3$ in x_3

$$x_3 = b(ax_1 + cx_3) \Rightarrow abx_1 + bcx_3$$

$$x_3 - bcx_3 = abx_1 \Rightarrow x_3(1-bc) = abx_1$$

$$\frac{x_3}{x_1} = \frac{ab}{1-bc}$$

MASON'S GAIN FORMULA:

Used to determine the T.F of system from the signal flow graph of system.

$$T.F \quad T(s) = \frac{C(s)}{R(s)} = \frac{\text{output}}{\text{Input}}$$

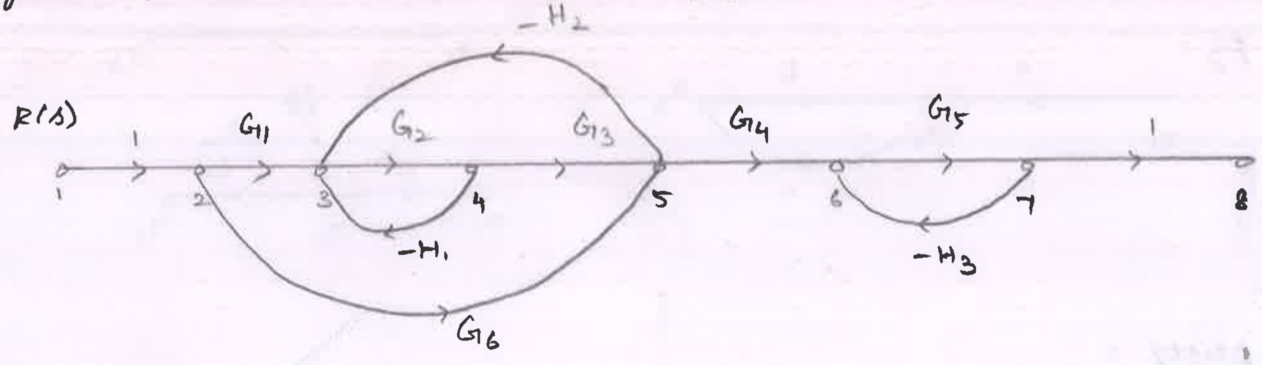
Mason's gain formula states the overall gain of the system as follows

$$\text{Overall gain}, \; \overset{S(s) \to T.F}{T} = \frac{1}{\Delta} \sum_{\substack{K \\ \to \text{No. of for. path}}} \overset{\to \text{Forward path gain}}{P_K} \Delta_K \to \text{Non touching } K^{th} \text{ forward path.}$$

Δ = 1 − Sum of individual loop gain + (Sum of gain product of all possible comb. of two non-touching loop) − (Sum of gain products of all possible comb. of 3 Non-touching

Problem 1:

Find the overall transfer fn. of the system whose signal flow graph is shown in fig.

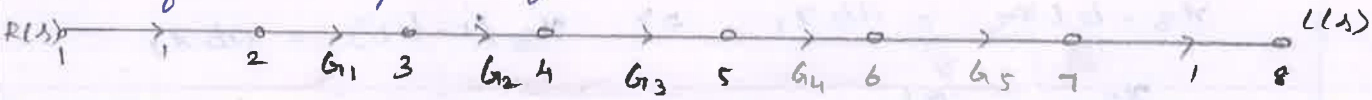


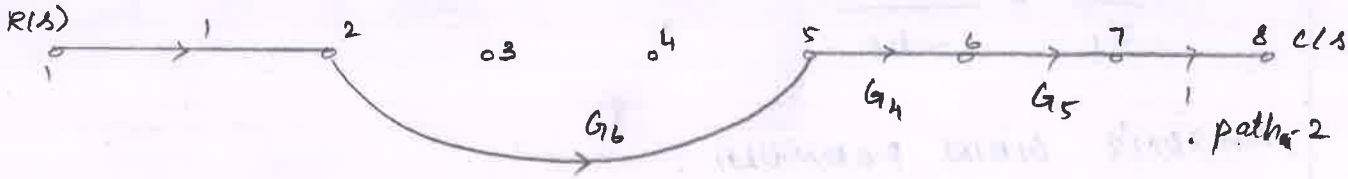
Sol :

1. Forward path Gains

There are two forward path $\therefore K = 2$

Let forward path gains be P_1 and P_2



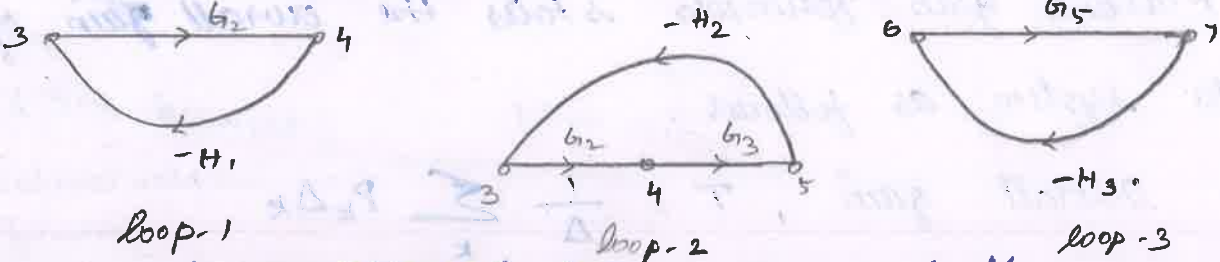


Grain of forward path - 1 $P_1 = G_1 G_2 G_3 G_4 G_5$

Grain of forward path - 2 $P_2 = G_4 G_5 G_6$

II. Individual loop gain

There are 3 individual loops. $\Rightarrow P_{11}, P_{21}$ & P_{31}



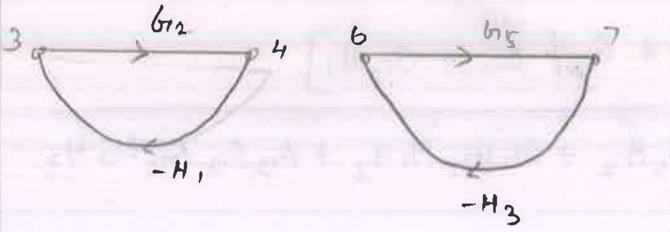
Loop gain of individual loop - 1 $P_{11} = -G_2 H_1$

loop - 2, $P_{21} = -G_2 G_3 H_2$

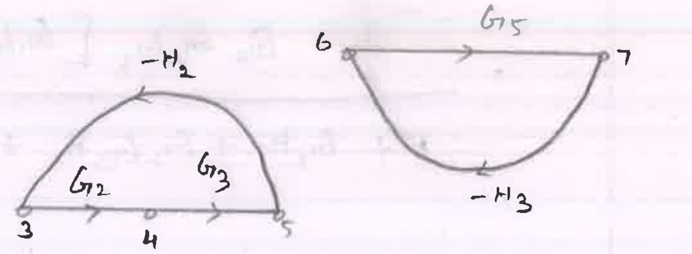
loop - 3, $P_{31} = -G_5 H_3$

III : Gain products of Two Non-touching loops

Two comb. of two non-touching loops. i.e. P_{12} & P_{22}



1^{st} comb.



2^{nd} comb.

Gain product of 1^{st} comb. of two non touching loops $\}$ $P_{12} = P_{11} P_{31} = (-G_2 H_1)(-G_5 H_3)$

$= G_2 G_5 H_1 H_3$

Gain product of 2^{nd} comb. of two non touching loop $\}$ $P_{22} = P_{21} P_{31} = (-G_2 G_3 H_2)(-G_5 H_3)$

$= G_2 G_3 G_5 H_2 H_3$

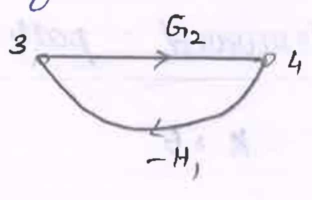
IV : Calculation of Δ and Δ_k

$$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + (P_{12} + P_{22})$$

$$= 1 - (-G_2 H_1 - G_2 G_3 H_2 - G_5 H_3) + (G_2 G_5 H_1 H_3 + G_2 G_3 G_5 H_2 H_3)$$

$$= 1 + G_2 H_1 + G_2 G_3 H_2 + G_5 H_3 + G_2 G_5 H_1 H_3 + G_2 G_3 G_5 H_2 H_3$$

$\Delta_1 = 1$, since no part of graph which is non touching with 1st forward path



$$\Delta_2 = 1 - P_{11} = 1 - (-G_2 H_1)$$

$$= 1 + G_2 H_1$$

V : Transfer function, T

By Mason's gain formula T.F, T is given by

$$T = \frac{1}{\Delta} \sum_k P_k \Delta_k = \frac{1}{\Delta}(P_1 \Delta_1 + P_2 \Delta_2)$$ [∵ No. of forward path = 2, k = 2]

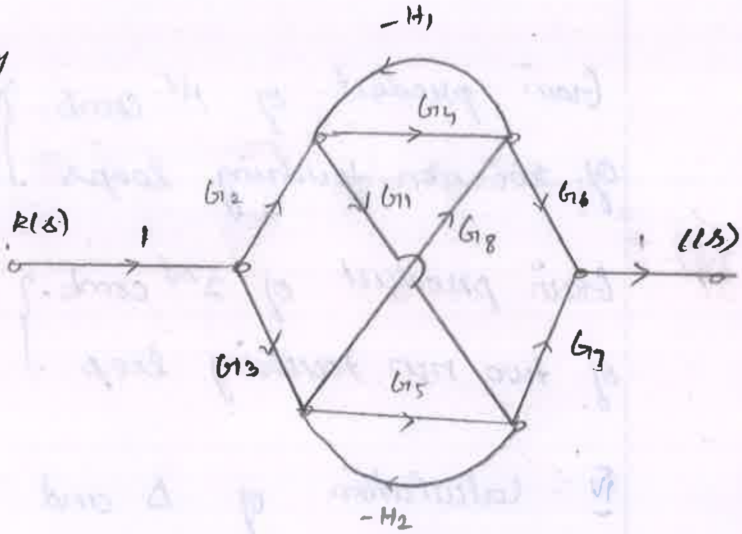
$$= \frac{G_1 G_2 G_3 G_4 G_5 + G_4 G_5 G_6 (1 + G_2 H_1)}{1 + G_2 H_1 + G_2 G_3 H_2 + G_5 H_3 + G_2 G_5 H_1 H_3 + G_2 G_3 G_5 H_2 H_3}$$

$$= \frac{G_1 G_2 G_3 G_4 G_5 + G_4 G_5 G_6 + G_2 G_4 G_5 G_6 H_1}{1 + G_2 H_1 + G_2 G_3 H_2 + G_5 H_3 + G_2 G_5 H_1 H_3 + G_2 G_3 G_5 H_2 H_3}$$

$$= \frac{G_2 G_4 G_5 \left[G_1 G_3 + G_6 / G_2 + G_6 H_1 \right]}{1 + G_2 H_1 + G_2 G_3 H_2 + G_5 H_3 + G_2 G_5 H_1 H_3 + G_2 G_3 G_5 H_2 H_3}$$

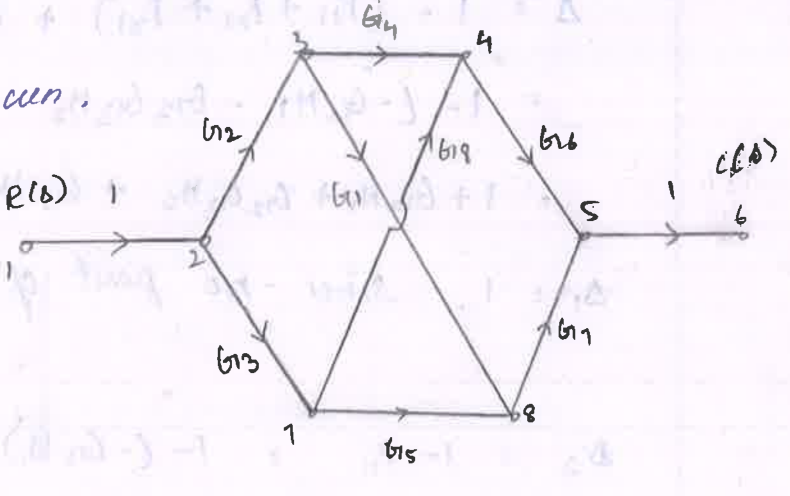
Problem 2:

Find the overall gain of the system whose signal flow graph is shown in fig.



Sol:

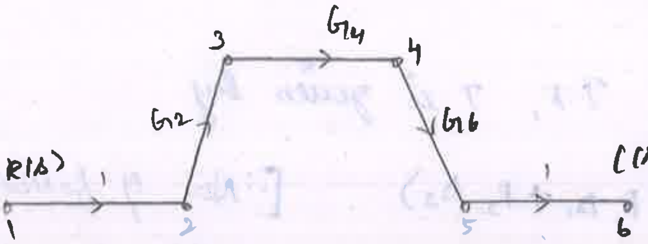
No. of nodes are shown.

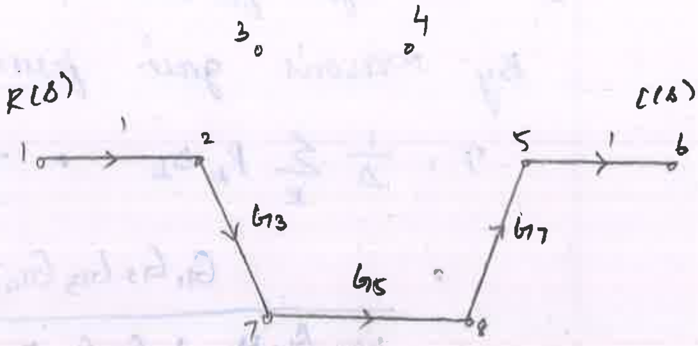


I : Forward path gain

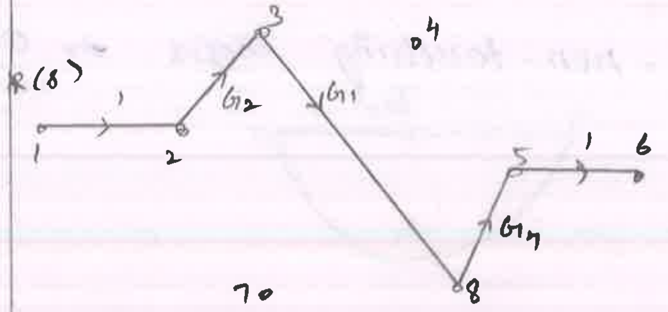
$K = 6$

Forward path gain be P_1 P_2 P_3 P_4 P_5 and P_6.

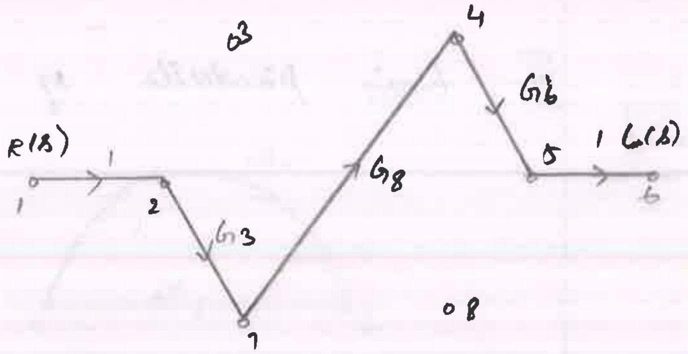




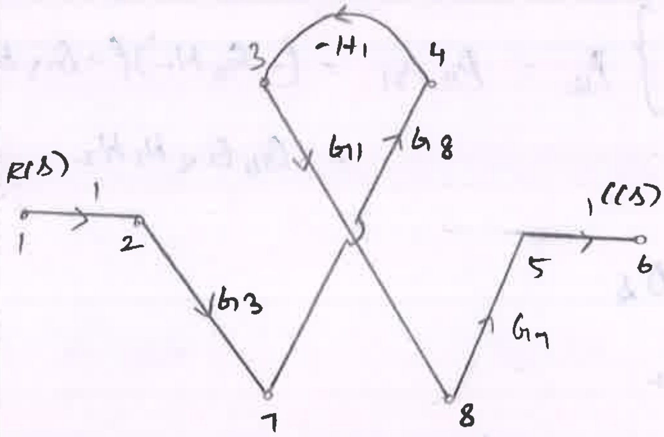
Forward path 2



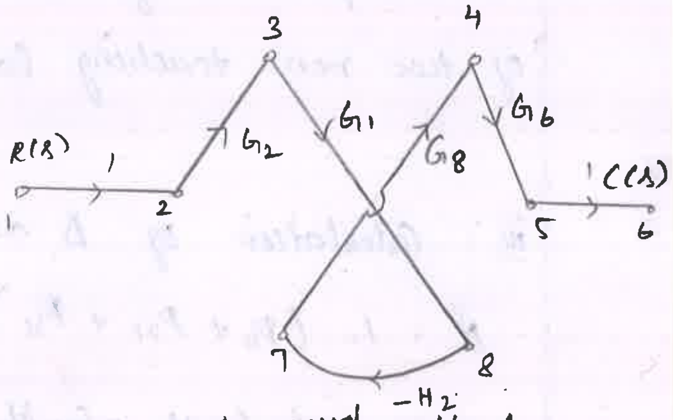
Forward path - 3



Forward path - 4



Forward path - 5



Forward path - 6

Gain of forward path - 1, $P_1 = G_2 G_4 G_6$

path - 2, $P_2 = G_3 G_5 G_7$

path - 3, $P_3 = G_1 G_2 G_7$

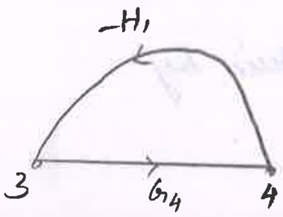
path - 4, $P_4 = G_3 G_8 G_6$

path - 5, $P_5 = -G_1 G_3 G_7 G_8 H_1$

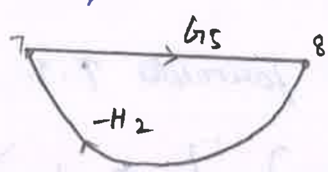
path - 6, $P_6 = -G_1 G_2 G_6 G_8 H_2$

II : Individual loop Gain

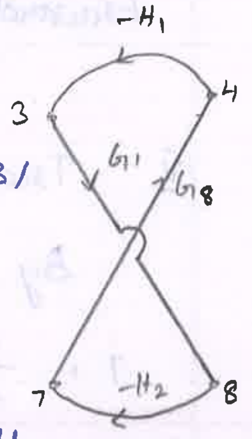
3 Individual loops. => P_{11}, P_{21} and P_{31}



loop 1



loop . 2



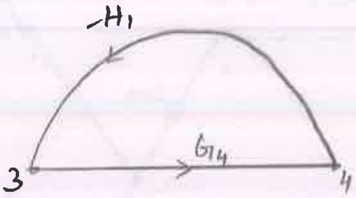
loop - 3

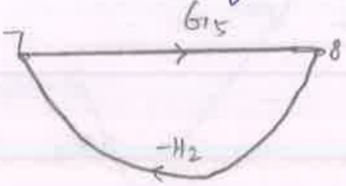
Loop gain of individual loop - 1 $P_{11} = -G_4 H_1$

loop . 2 $P_{21} = -G_5 H_2$

loop - 3 $P_{31} = G_1 G_8 H_1 H_2$

III : Gain products of Two - non - touching loops. → one.





Comb. of 2 non-touching loops.

Gain product of 1^{st} comb. of two non-touching loop $\Bigg\}$ $P_{12} = P_{11} P_{21} = (-G_4 H_1)(-G_5 H_2)$

$= G_4 G_5 H_1 H_2$

IV : Calculation of Δ and Δ_k

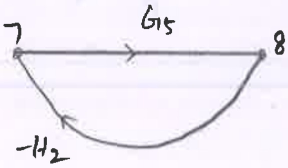
$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + P_{12}$

$= 1 - (-G_4 H_1 - G_5 H_2 + G_1 G_8 H_1 H_2) + G_4 G_5 H_1 H_2$

$= 1 + G_4 H_1 + G_5 H_2 - G_1 G_8 H_1 H_2 + G_4 G_5 H_1 H_2$

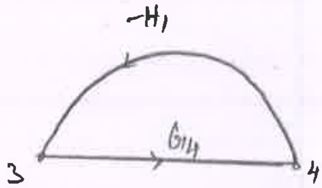
Forward path - 1 $\quad \Delta_1 = 1 - (-G_5 H_2)$

$= 1 + G_5 H_2$



Forward path - 2 $\quad \Delta_2 = 1 - (-G_4 H_1)$

$= 1 + G_4 H_1$



Forward path - 3, 4, 5, 6 $\quad \Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$

V : Transfer function, T

By Mason's gain formula T.F, T is given by

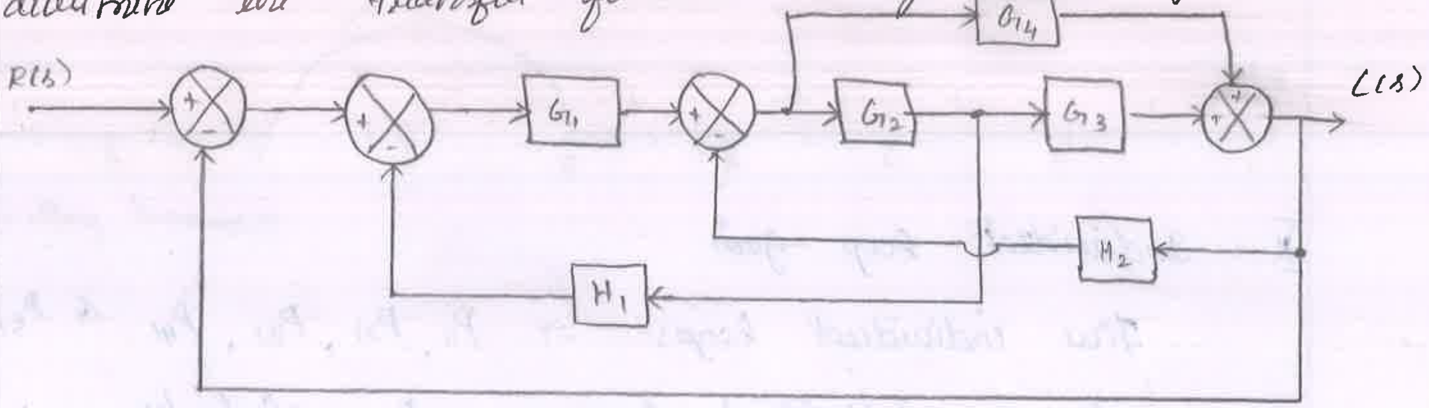
$T = \frac{1}{\Delta} \left[\sum_k P_k \Delta_k \right] \quad (\because k = 6)$

$= \frac{1}{\Delta} \left[P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3 + P_4 \Delta_4 + P_5 \Delta_5 + P_6 \Delta_6 \right]$

$$\therefore \frac{G_2 G_4 G_6 (1 + G_5 H_2) + G_3 G_5 G_7 (1 + G_4 H_1) + G_1 G_2 G_7 + G_3 G_6 G_8 - G_1 G_3 G_7 G_8 H_1 - G_1 G_2 G_6 G_8 H_2}{1 + G_4 H_1 + G_5 H_2 - G_1 G_8 H_1 H_2 + G_4 G_5 H_1 H_2}$$

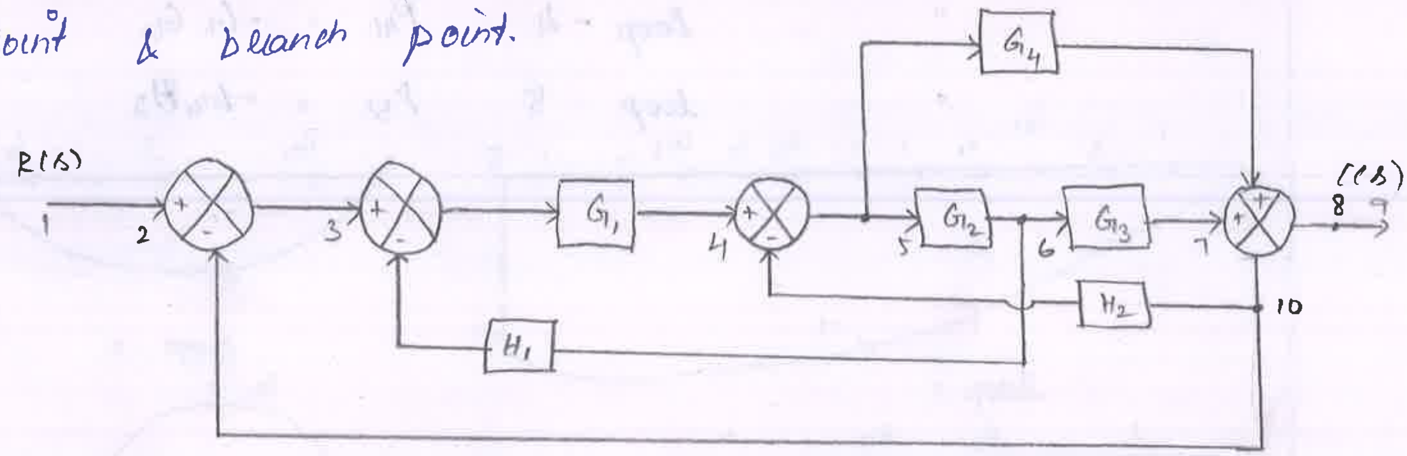
Problem 3 :

Convert the block diagram to signal flow graph and determine the transfer function using masons gain formula.

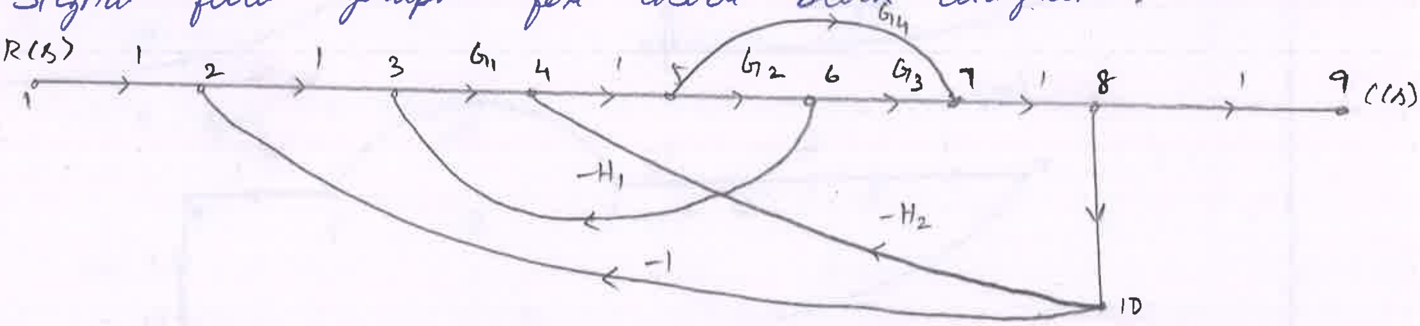


Sol :

Nodes are assigned at i/p, o/p, at every summing point & branch point.



Signal flow graph for above block diagram.

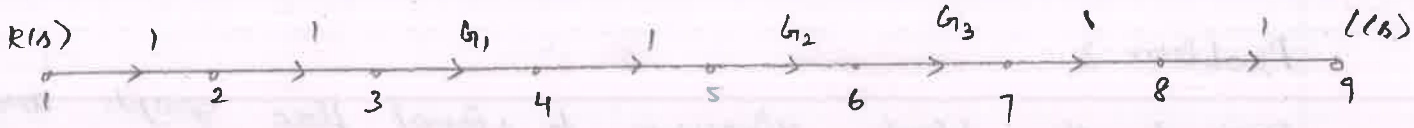


I. Forward path Gain :

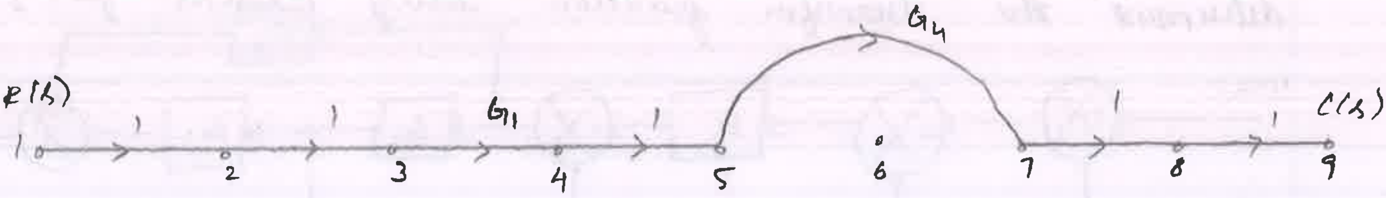
$K = 2 \Rightarrow P_1$ and P_2

Gain forward path-1 $P_1 = G_1 G_2 G_3$

" path 2 $P_2 = G_1 G_4$



Forward path-1



Forward path-2

II : Individual loop gain

Five individual loops => P_{11}, P_{21}, P_{31}, P_{41} & P_{51}

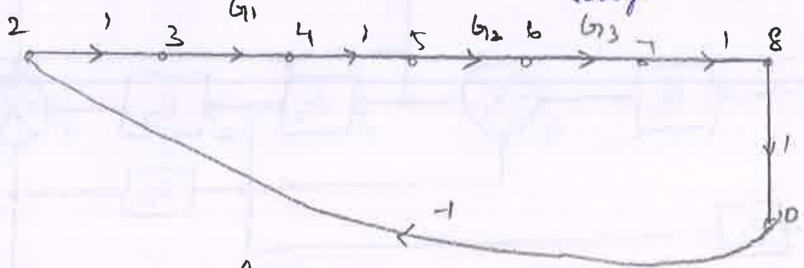
Loop gain of individual loop-1 $P_{11} = -G_1 G_2 G_3$

" loop-2 $P_{21} = -G_2 G_1 H_1$

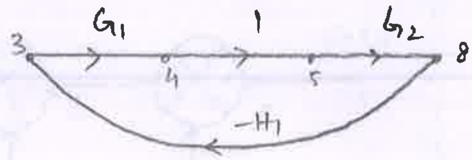
" loop-3 $P_{31} = -G_2 G_3 H_2$

" loop-4 $P_{41} = -G_1 G_4$

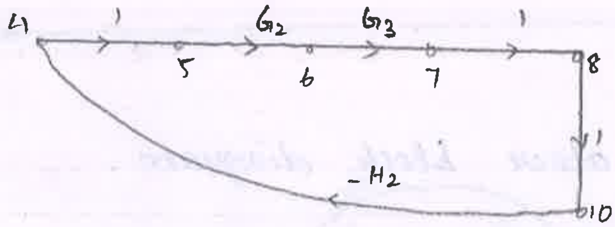
" loop-5 $P_{51} = -G_4 H_2$



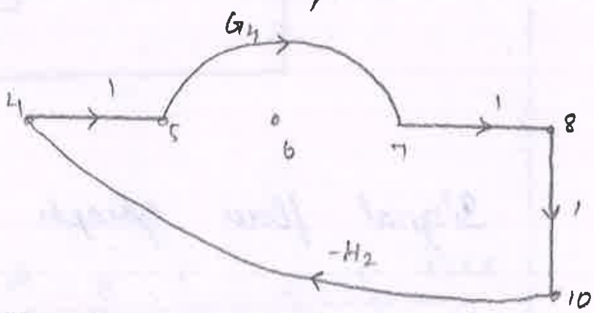
loop-1



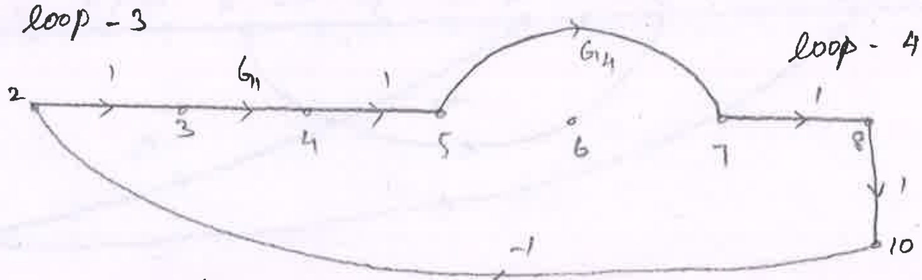
loop-2



loop-3



loop-4



loop-4

III : Gain products of Two non-touching loops.

No possible comb. of two, three, etc. non-touching loops.

iv : calculation of Δ and Δ_k

$$\Delta = 1 - [P_{11} + P_{21} + P_{31} + P_{41} + P_{51}]$$

$$= 1 + G_1 G_2 G_3 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_4 + G_4 H_2$$

Non part of graph is non - touching with for. path 1 & 2.

$$\Delta_1 = \Delta_2 = 1$$

v : Transfer function, T

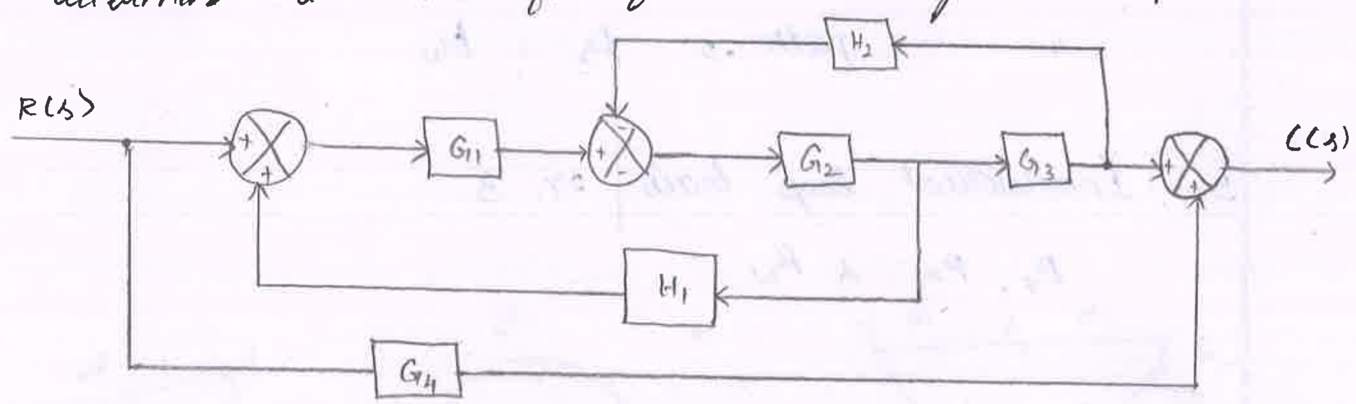
$$T = \frac{1}{\Delta} \sum_K P_k \Delta_k$$

$$= \frac{1}{\Delta} [P_1 \Delta_1 + P_2 \Delta_2]$$

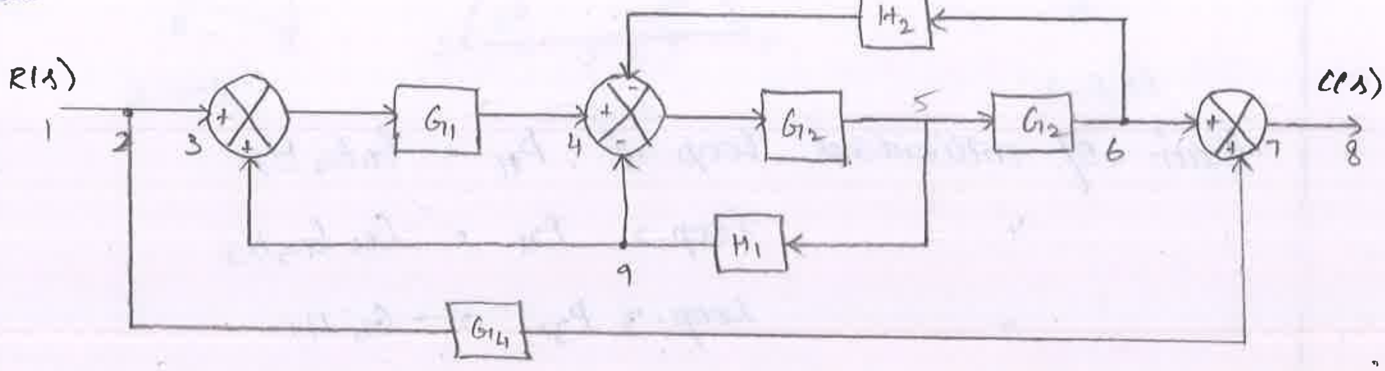
$$= \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 G_2 G_3 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_4 + G_4 H_2}$$

problem 4 :

Convert the given block diagram to signal flow graph and determine the transfer function using Mason's formula.

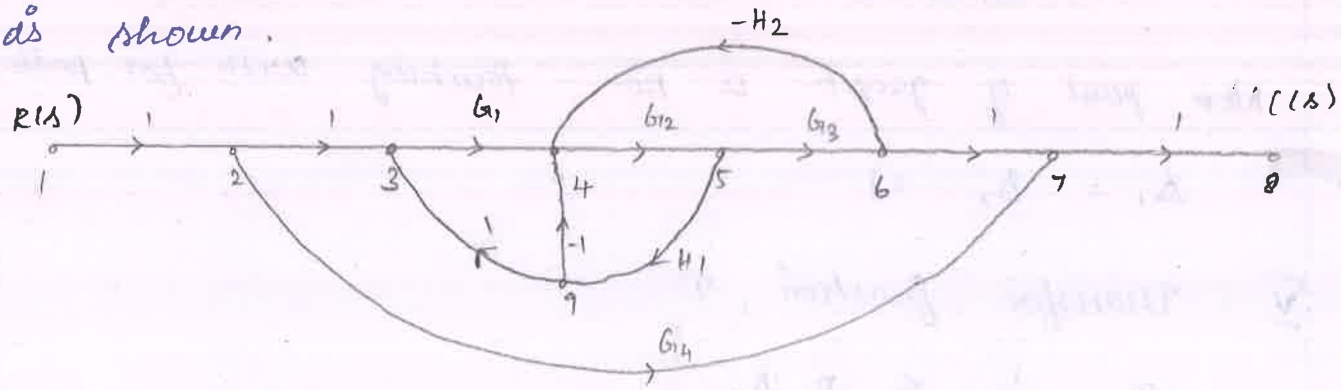


Sol :



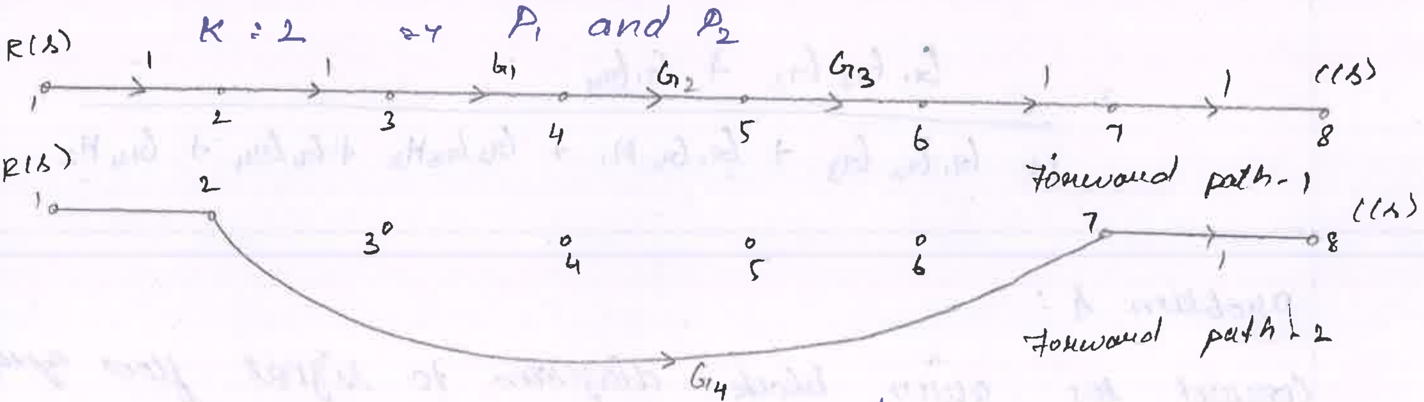
The nodes are assigned as i/p, o/p at every summing and branch point as shown.

The signal flow graph for the above block diagram as shown.



I : Forward path Gain

K = 2 ⇒ P_1 and P_2

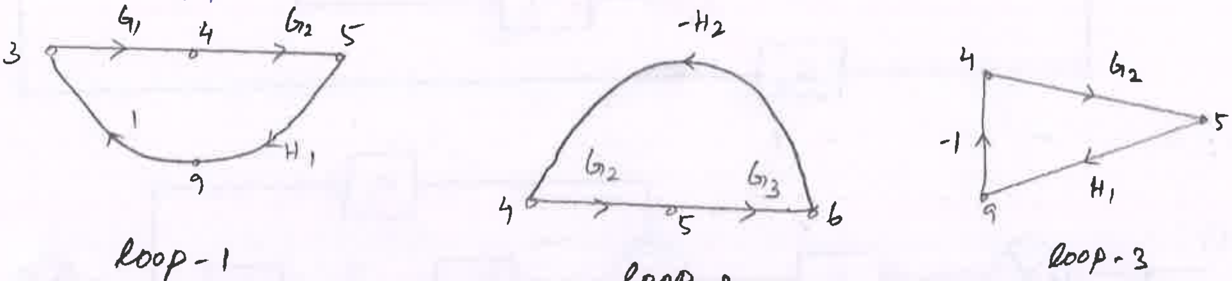


Gain forward path-1 $P_1 = G_1 G_2 G_3$

" path-2 $P_2 = G_4$

II : Individual loop Gain ⇒ 3

P_{11}, P_{21} & P_{31}



Gain of individual loop-1 $P_{11} = G_1 G_2 H_1$

" loop-2 $P_{21} = -G_2 G_3 H_2$

" loop-3 $P_{31} = -G_2 H_1$

III : Gain product of two-non touching loops.

No possible combination of two, three, etc. non touching loops

IV : Calculation of Δ and Δ_k

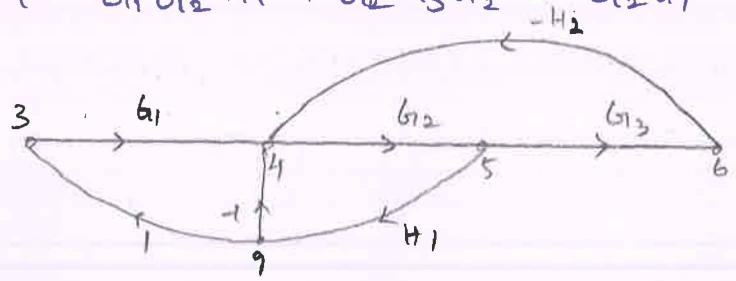
$$\Delta = 1 - [P_{11} + P_{21} + P_{31}]$$

$$= 1 - G_1 G_2 H_1 + G_2 G_3 H_3 + G_2 H_1$$

$$\Delta_1 = 1$$

$$\Delta_2 = 1 - [G_1 G_2 H_1 - G_2 G_3 H_2 - G_2 H_1]$$

$$= 1 - G_1 G_2 H_1 + G_2 G_3 H_2 + G_2 H_1$$



V : Transfer function, T

By mason's gain formula, T.F, T is given by

$$T = \frac{1}{\Delta} \sum_k P_k \Delta_k$$

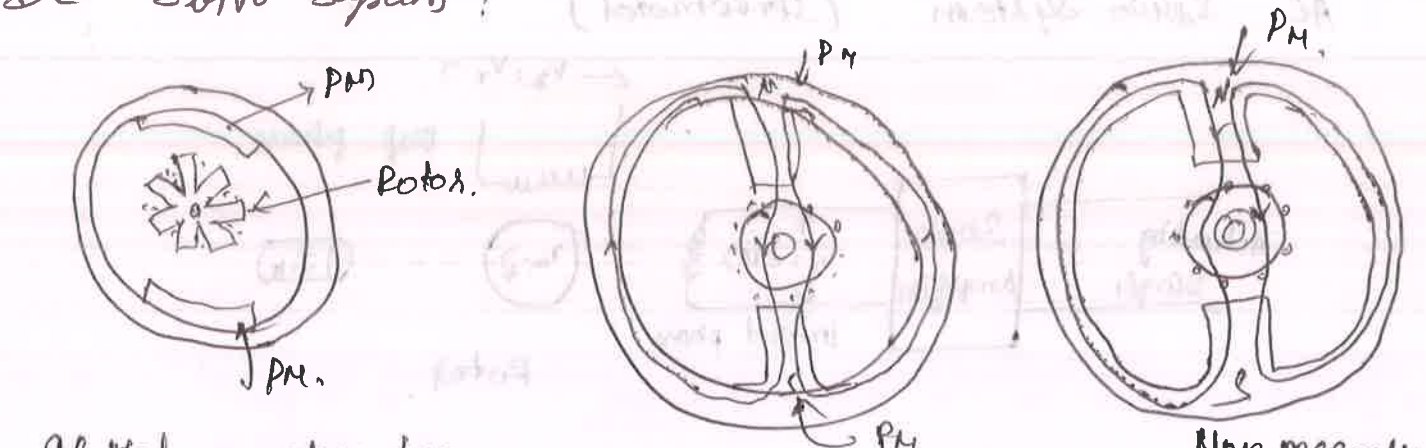
$$= \frac{1}{\Delta} [P_1 \Delta_1 + P_2 \Delta_2] \quad (\because k = 2)$$

$$= \frac{1}{\Delta} [G_1 G_2 G_3 + G_4 (1 - G_1 G_2 H_1 + G_2 G_3 H_2 + G_2 H_1)]$$

$$= \frac{1}{\Delta} [G_1 G_2 G_3 + G_4 - G_1 G_2 G_4 H_1 + G_2 G_3 G_4 H_2 + G_2 G_4 H_1$$

$$= \frac{G_1 G_2 G_3 + G_4 - G_1 G_2 G_4 H_1 + G_2 G_3 G_4 H_2 + G_2 G_4 H_1}{1 - G_1 G_2 H_1 + G_2 G_3 H_2 + G_2 H_1.}$$

DC Servo System :



Slotted armature type

Surface wound iron core.

An armature voltage controlled dc servomotor similar to a conventional dc motor with fixed excitation.

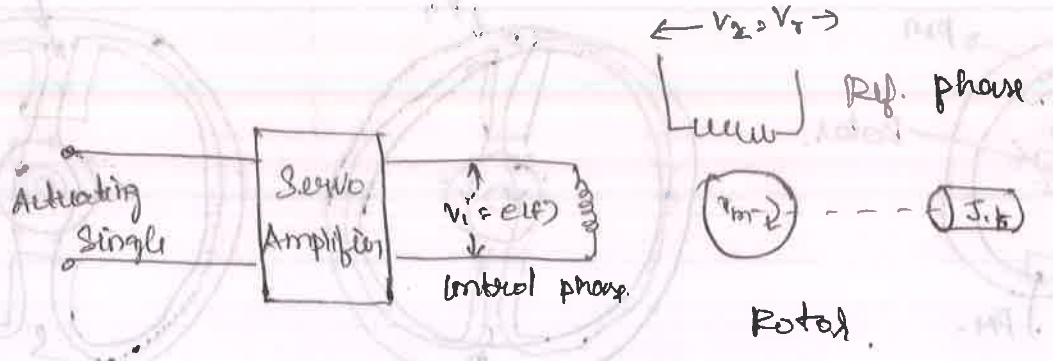
Due to high residual flux density and coercivity (PM) permanent magnets are used in construction of dc servomotors.

This property gives a higher torque - inertia ratio and high efficiency. The speed of a PM dc motor is directly proportional to the armature voltage given load torque. A PM dc motor has much more flatter speed torque characteristics than a field wound motor which has severe armature reaction effects.

There are three types of dc servomotors.

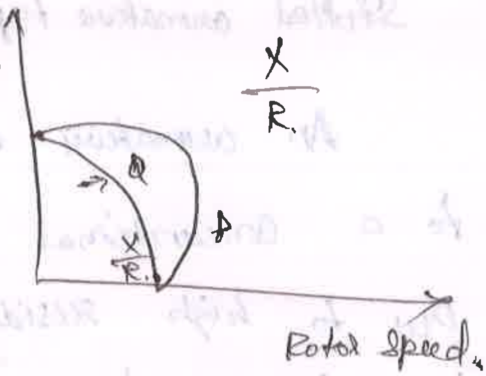
* Slotted armature type.
* Surface wound iron core type.
* Surface wound non magnetic core type

AC Servo System (Servomotor)



It is a two phase, induction Torque motor with little modifications.

the case V_1 and V_2 are equal in magnitude but they are 90° out of phase.



the two phase windings are producing a rotating magnetic feild. the direction of rotation depends upon the phase shift between V_1 and V_2. the rotor is placed in the rotating magnetic feild and so emf is induced in it, causing a current in the short circuited rotor.

mathematical modelling of AC Servomotor:

Speed - θ and error signal e.

$$T_m = f(\theta, e)$$

By using Taylor's series

$$T_m = T_m(0) + \frac{\partial T_m}{\partial e}\Big|_{e = e(0)} (e(t) - e(0)) + \dots + \frac{\partial T_m}{\partial \theta}\Big|_{\theta = \theta(0)} (\theta(t) - \theta(0)) + \dots$$

$$T_m = k[e(t) - e(0)] - f(\theta(t) - \theta(0))$$

where $$k = \left.\frac{\partial T_m}{\partial e}\right|_{e(t) = e(0)}$$

$$f = \left.\frac{-\partial T_m}{\partial \theta}\right|_{\theta = \theta(0)}$$

$$T_m = K\,e(t) - f\,\theta(t)$$

Mechanical relations for a motor is

$$T_m = J\ddot{\theta} + B\dot{\theta}$$

$$K\,e(t) - f\,\theta(t) = J\,\ddot{\theta}(t) + B\,\dot{\theta}(t)$$

L.T on both side,

$$(Js^2 + Bs)\theta(s) = KE(s) - f\,s\theta(s)$$

$$\frac{\theta(s)}{E(s)} = \frac{K}{Js^2 + (B+f)s}$$

$$= \frac{K}{s[Js + B + f]}$$

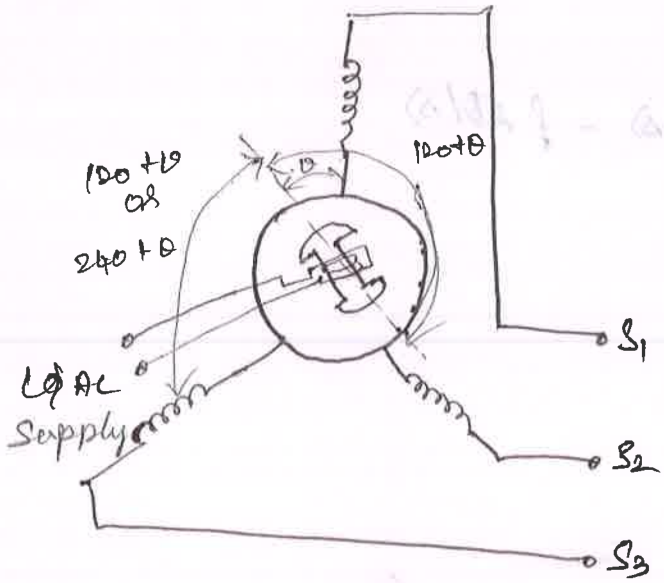
$$= \frac{K_m}{s[T_m s + 1]}$$

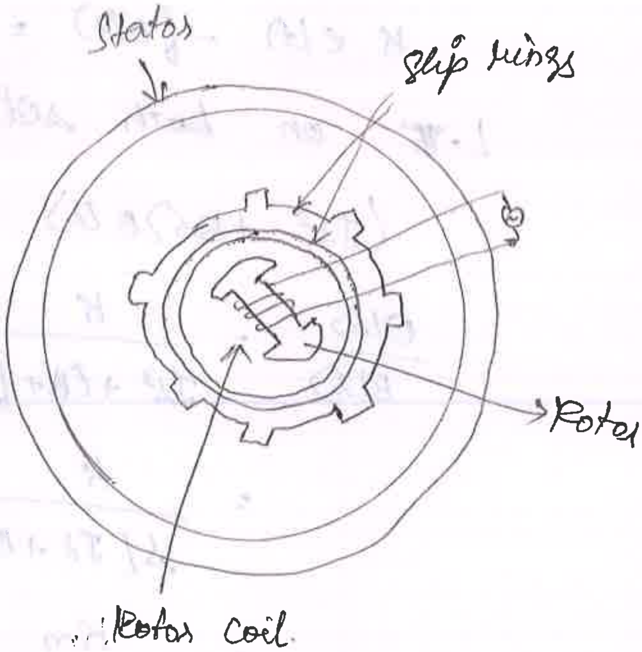
SYNCHROS :

The other names for synchros are selsyn and autosyn. It is an electromagnetic transducer that produces an output voltage depending upon the angular displacement. It consists of two devices called synchro transmitter and synchro receiver. It is mostly used as an error detector in control systems.

The synchro pair measures and compares two angular displacements and produces an output Voltage which is approximately linear with the angular difference.

Synchro transmitter is a basic system similar to a Y connected 3 phase alternator. The stator windings are concentric coils displaced 120° apart.





$$V_r(t) = A \sin \omega t$$

Phase Voltages induced in Stator coils S_1, S_2 & S_3.

$$V_{S1} = KA \sin \omega t \cos \theta$$

$$V_{S2} = KA \sin \omega t \cos(120° + \theta)$$

$$V_{S3} = KA \sin \omega t \cos(240 + \theta).$$

Corresponding line Voltages are

$$V_{L1} = V_{S1} V_{S2} = V_{S2} - V_{S1}$$

$$= KA \sin \omega t \left(\cos(120° + \theta) - \cos \theta\right).$$

$= KA \sin \omega t \left[2 \sin (60^\circ + \theta) \sin 60^\circ \right]$

$= KA \sin \omega t \left[\sqrt{3} \sin (60^\circ + \theta) \right]$

$V_{L2} = V_{S2, S3} = V_{S3} - V_{S2}$

$= KA \sin \omega t \left[\cos (240^\circ + \theta) - \cos (120^\circ + \theta) \right]$

$= 2 KA \sin \omega t \left[\sin (180^\circ + \theta) \sin 60^\circ \right]$

$= \sqrt{3} KA \sin \omega t \sin (180^\circ + \theta)$

$V_{L3} = V_{S3, S1} = V_{S1} - V_{S3}$

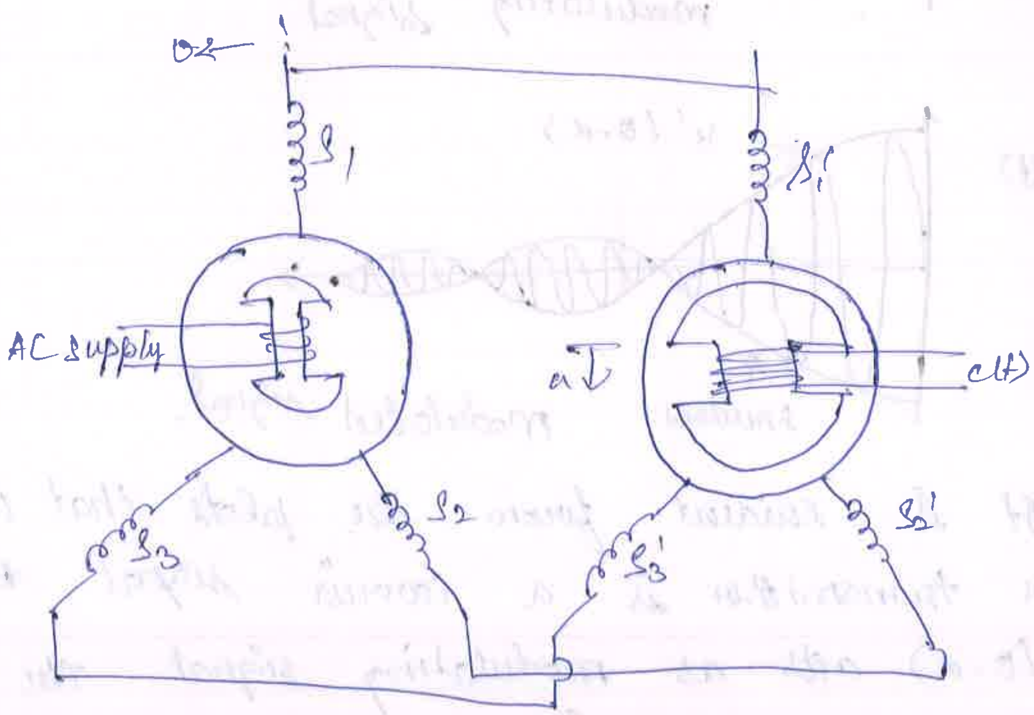
$= KA \sin \omega t \left[\cos \theta - \cos (240 + \theta) \right]$

$= -2 KA \sin \omega t \left[\sin (120 + \theta) \sin (120^\circ) \right]$

$= \sqrt{3} KA \sin \omega t \sin (300^\circ + \theta)$.

when $\theta = 0$, E_Y $V_{S1} = KA \sin \omega t$

$e(t) = K' A \sin \omega t \cos \phi$



Let initial position of rotors is 90° out of phase.

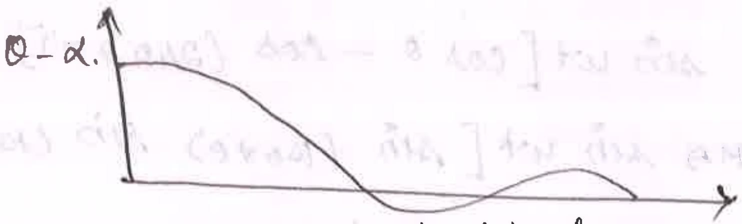
$e(t) = K' A \sin \omega t \cos 90 = 0.$

The net angle displacement b/w the rotors is $(90° + \theta - \alpha)$

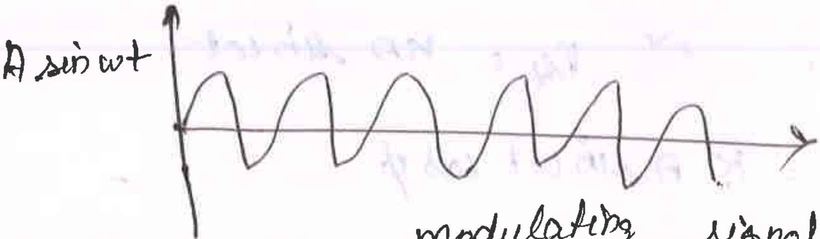
$e(t) = K' A \sin \omega t \cos (90° + \theta - \alpha)$

$= K' A \sin \omega t \sin (\theta - \alpha)$

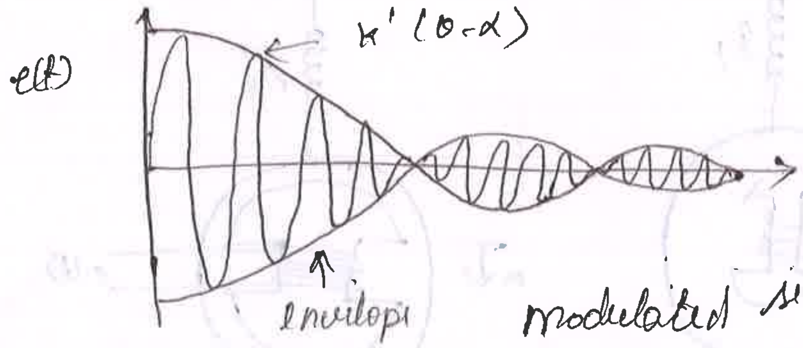
$e(t) = K' A (\theta - \alpha) \sin \omega t.$



error signal.



modulating signal.

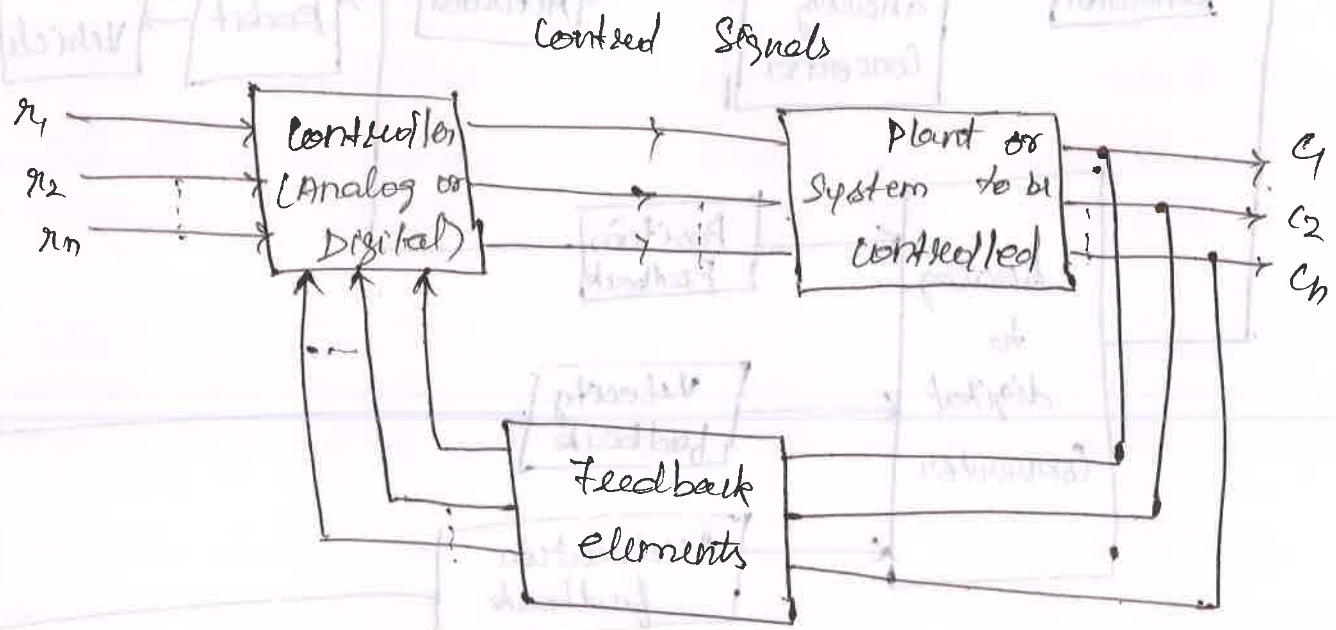


modulated signal.

It is evident from the plots that input to the transmitter is a carrier signal. The error $(\theta - \alpha)$ acts as modulating signal. The error signal $e(t)$ is a modulating signal. Modulated error signals are called suppressed carrier modulated signals.

MULTIVARIABLE CONTROL SYSTEM :

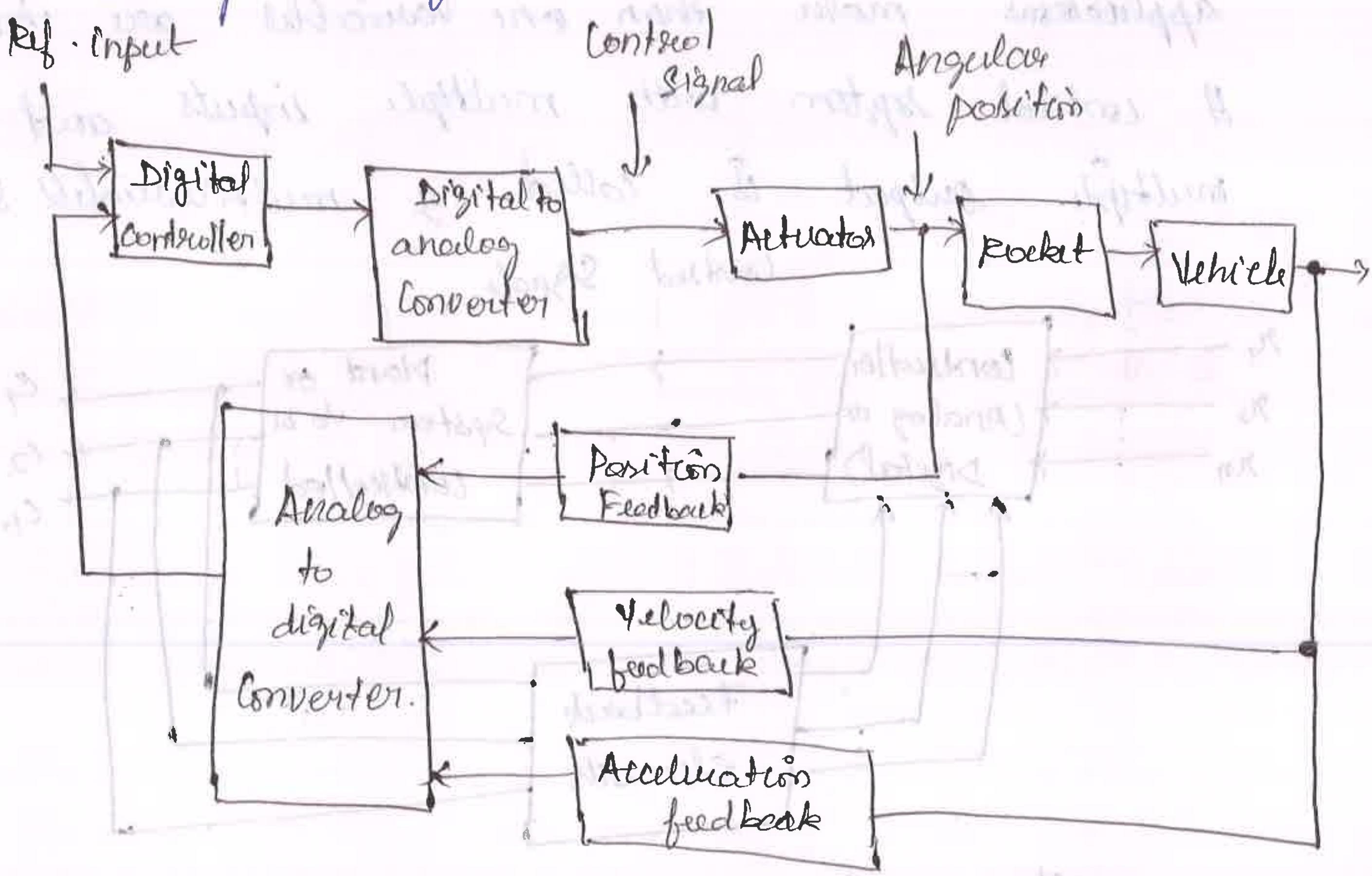
The control system in which there is only one output of the interest is called single variable system. But in many practical applications more than one variables are involved. A control system with multiple inputs and multiple output is called a multivariable system.



The part of the system which is required to be controlled is called plant. The controller provides proper controlling action depending on the reference inputs. There are n reference inputs r_1, r_2 ... r_n.

There are n output variables $c_1(t)$, $c_2(t)$... $c_n(t)$. The values of these variables represent the performance of the plant. The control signals produced by the controller are applied to the plant.

In case of multivariables systems, it is observed that a single input considerably affects more than one outputs. The system is said to be having strong interactions or coupling.



The interactions inherently present between inputs and outputs can be cancelled by designing a decoupling controller. In multivariable linear control system, each input is independently considered.

Air craft, space crafts are other examples whose movement is controlled by various inputs. Power generators, atomic reactors and jet engines are some of other examples of multivariable systems.

TWO MARKS :

1. What is system ?

when a number of elements or components in a sequence to perform a specific function, the group thus formed is called a system.

2. What is control system?

A system consists of a number of components connected together to perform a specific function. In a system when the o/p quantity is controlled by varying the i/p quantity, then the system is called control system.

The output quantity is called controlled variable or response and i/p quantity is called command signal or excitation.

3. What are the two major type of control system ?

The two type of control systems are open loop and closed loop system.

4. Define open loop system

The control system in which the o/p quantity has no effect upon the i/p quantity are called open loop control system. This means that the output is not feedback to the input for correction.

5. Define closed loop system:

The control systems in which the output has an effect upon the input quantity in order to maintain the desired output value are called closed loop control system.

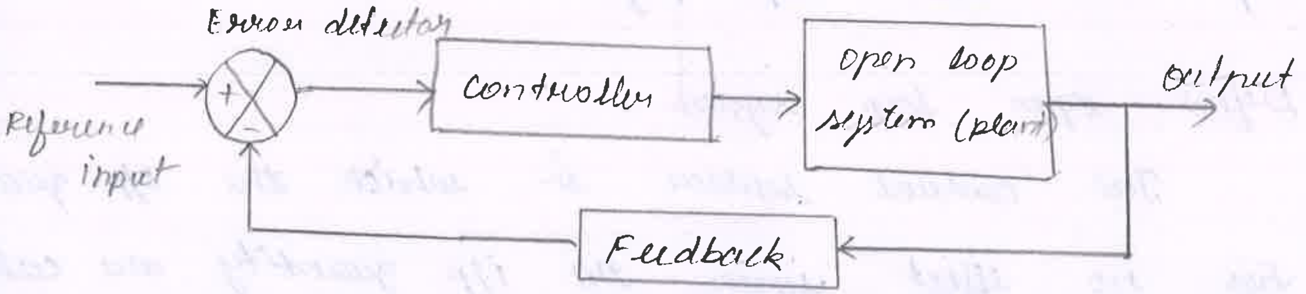
6. What is feedback? What type of feedback control system? is employed in control system?

The feedback is a control action in which the output is sampled and a proportional signal is given to input for automatic correction of any changes in desired output.

Negative feedback is employed in control system.

7. What are the components of feedback control system?

The components of feedback control system are plant, feedback path elements, error detector and controller.



8. Why negative feedback is invariably preferred in a closed loop system?

The negative feedback resulted in better stability in steady state and rejects any disturbance signals.

Problem 4 :

Consider a unity feedback system with a closed loop T.F $\frac{C(s)}{R(s)} = \frac{Ks+b}{s^2+as+b}$. Determine open loop T.F $G(s)$.

Show that steady state error with unit ramp input is given by $\frac{(a-K)}{b}$.

Sol :

For unity feedback system, $H(s) = 1$

The closed loop T.F, $M(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$

$$= \frac{G(s)}{1+G(s)}$$

$$\frac{G(s)}{1+G(s)} = M(s)$$

On cross mult. of above eq., we get,

$$G(s) = M(s)[1+G(s)] = M(s)+M(s)\,G(s)$$

$$G(s) - M(s)\,G(s) = M(s)$$

$$G(s)\,[1-M(s)] = M(s)$$

$$M(s) = \frac{Ks+b}{s^2+as+b}$$

Open loop T.F,

$$G(s) = \frac{M(s)}{1-M(s)} = \frac{\frac{Ks+b}{s^2+as+b}}{1-\frac{Ks+b}{s^2+as+b}}$$

$$= \frac{Ks+b}{(s^2+as+b)-(Ks+b)} = \frac{Ks+b}{s^2+as+b-Ks-b}$$

To find steady state error:

The error signal in s-domain, $E(s) = \dfrac{R(s)}{1+G(s)H(s)}$

Given that, $r(t) = a_0 + a_1 t + \dfrac{a_2}{2} t^2$

$$G(s) = \frac{10}{s(0.1s+1)} \; ; \; H(s) = 1$$

L.T, $r(t) \Rightarrow R(s)$

$$R(s) = \frac{a_0}{s} + \frac{a_1}{s^2} + \frac{a_2}{2}\frac{2!}{s^3} = \frac{a_0}{s} + \frac{a_1}{s} + \frac{a_2}{s}$$

$$E(s) = \frac{R(s)}{1+G(s)H(s)} = \frac{\frac{a_0}{s} + \frac{a_1}{s^2} + \frac{a_2}{s^3}}{1 + \frac{10}{s(0.1s+1)}}$$

$$= \frac{\frac{a_0}{s} + \frac{a_1}{s^2} + \frac{a_2}{s^3}}{\frac{s(0.1s+1)+10}{s(0.1s+1)}}$$

$$= \frac{a_0}{s}\left[\frac{s(0.1s+1)}{s(0.1s+1)+10}\right] + \frac{a_1}{s^2}\left[\frac{s(0.1s+1)}{(0.1s+1)+10}\right] + \frac{a_2}{s^3}\left[\frac{s(0.1s+1)}{s(0.1s+1)+10}\right]$$

The steady state error, e_{ss}, can be obtained from final value theorem,

Steady state error, $e_{ss} = \underset{t\to\infty}{Lt}\, e(t) = \underset{s\to 0}{Lt}\, sE(s)$

$$e_{ss} = \underset{s\to 0}{Lt}\; s\left\{\frac{a_0}{s}\left[\frac{s(0.1s+1)}{s(0.1s+1)+10}\right] + \frac{a_1}{s}\left[\frac{s(0.1s+1)}{s(0.1s+1)+10}\right]\right.$$

$$\left. + \frac{a_2}{s^3}\left[\frac{s(0.1s+1)}{s(0.1s+1)+10}\right]\right\}$$

$$= \underset{s\to 0}{Lt}\left\{\frac{a_0 s(0.1s+1)}{s(0.1s+1)+10} + \frac{a_1(0.1s+1)}{s(0.1s+1)+10} + \frac{a_2(0.1s+1)}{s[s(0.1s+1)+10]}\right\}$$

$e_{ss} = 0 + a_1/10 + \infty = \infty$ $\quad e_{ss} = \infty$

Problem 3 :

The open loop T.F of a servo system with unity feedback is $G(s) = \frac{10}{s(0.1s+1)}$. Evaluate the static error constants of the system. Obtain the steady state error of the system, when subjected to an input given by the polynomial, $r(t) = a_0 + a_1 t + \frac{a_2}{2} t^2$.

Solution :

To find Static error constant.

For unity feedback system, $H(s) = 1$

$\therefore$ Loop T.F, $G(s)\,H(s) = G(s)$

The static error constants are, K_p, K_v and K_a

position error constant, $K_p = \underset{s\to 0}{Lt}\; G(s)$

$$= \underset{s\to 0}{Lt}\; \frac{10}{s(0.1s+1)} = \infty$$

Velocity error constant, $K_v = \underset{s\to 0}{Lt}\; sG(s)$

$$= \underset{s\to 0}{Lt}\; s\,\frac{10}{s(0.1s+1)} = 10$$

Acceleration error constant, $K_a = \underset{s\to 0}{Lt}\; s^2 G(s)$

$$= \underset{s\to 0}{Lt}\; s^2\,\frac{10}{s(0.1s+1)}$$

$$= 0.$$

The steady state error with <u>unit step input,</u>

$$e_{ss} = \frac{1}{1+K_p}$$

position error constant, $K_p = \underset{s\to 0}{Lt}\ G(s)\,H(s) = \underset{s\to 0}{Lt}\ G(s)$

$$= \underset{s\to 0}{Lt}\ \frac{10}{(s+2)(s+3)}$$

$$= \frac{10}{2\times 3} = \frac{5}{3}$$

Steady state error, $e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+5/3}$

$$= \frac{3}{3+5} = \frac{3}{8}$$

$$e_{ss} = 0.375$$

c) . $G(s) = \frac{10}{s^2(s+1)(s+2)}$

Let us assume unity feedback system, $H(s) = 1$

The open loop system has two poles at origin. Hence it is a type-2 system. In systems with type no. 2 the acceleration (parabolic) input will give a constant steady state error.

The steady state error with <u>unit acceleration input,</u>

$$e_{ss} = \frac{1}{K_a}$$

Acceleration error constant, $K_a = \underset{s\to 0}{Lt}\ s^2\,G(s)\,H(s)$

$$= \underset{s\to 0}{Lt}\ s^2\,G(s)$$

$$= \underset{s\to 0}{Lt}\ s^2\,\frac{10}{s^2(s+1)(s+2)} = \frac{10}{1\times 2}$$

$$= 5$$

Steady state error, $e_{ss} = \frac{1}{K_a} = \frac{1}{5} = 0.2$.

a) $G(s) = \frac{20(s+2)}{s(s+1)(s+3)}$ b) $G(s) = \frac{10}{(s+2)(s+3)}$ c) $G(s) = \frac{10}{s^2(s+1)(s+2)}$

Sol:

a) $G(s) = \frac{20(s+2)}{s(s+1)(s+3)}$

Let us assume unity feedback system, $H(s) = 1$

The open loop system has a pole at origin. Hence it is a type-1 system. In systems with type no.-1, the velocity (ramp) input will give a constant steady state error.

The steady state error with unit velocity input, $e_{ss} = \frac{1}{K_v}$

Velocity error constant, $K_v = \lim_{s \to 0} s\,G(s)\,H(s)$

$= \lim_{s \to 0} s\,G(s)$

$= \lim_{s \to 0} s \frac{20(s+2)}{s(s+1)(s+3)}$

$= \frac{20 \times 2}{1 \times 3} = \frac{40}{3}$

Steady state error, $e_{ss} = \frac{1}{K_v} = \frac{3}{40} = 0.075$

b) $G(s) = \frac{10}{(s+2)(s+3)}$

Let us assume unity feedback system, $H(s) = 1$

The open loop system has no pole at origin. Hence it is a type-0 system. In system with type number-0, the step I/P will give a constant steady state error.

$$= \frac{3}{s}\left[\frac{s^2(s+1)}{s^2(s+1)+10(s+2)}\right] - \frac{2}{s^2}\left[\frac{s^2(s+1)}{s^2(s+1)+10(s+2)}\right] + \frac{1}{3s^3}\left[\frac{s^2(s+1)}{s^2(s+1)+10(s+2)}\right]$$

The steady state error e_{ss} can be obtained from final value theorem.

Steady State Error, $e_{ss} = \underset{t \to \infty}{Lt}\ e(t) = \underset{s \to 0}{Lt}\ s\,E(s)$

$$e_{ss} = \underset{s \to 0}{Lt}\ s\left\{\frac{3}{s}\left[\frac{s^2(s+1)}{s^2(s+1)+10(s+2)}\right] - \frac{2}{s^2}\left[\frac{s^2(s+1)}{s^2(s+1)+10(s+2)}\right] + \frac{1}{3s^3}\left[\frac{s^2(s+1)}{s^2(s+1)+10(s+2)}\right]\right\}$$

$$= \underset{s \to 0}{Lt}\left\{\frac{3s^2(s+1)}{s^2(s+1)+10(s+2)} - \frac{2s(s+1)}{s^2(s+1)+10(s+2)} + \frac{(s+1)}{3s^2(s+1)+30(s+2)}\right\}$$

$$= 0 - 0 + \frac{1}{60}$$

$$e_{ss} = \frac{1}{60}$$

Result :

a) Position error constant, $K_p = \infty$

b) Velocity error constant, $K_v = \infty$

c) Acceleration error constant, $K_a = 20$

b.) When, $R(s) = \frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}$,

steady state error, $e_{ss} = \frac{1}{60}$

Problem 2 :

For servomechanisms with open loop T.F given below explain what type of input signal give rise to a constant steady state error and calculate their values.

Problem 1:

For a unity feedback control system the open loop T.F

$G(s) = \frac{10(s+2)}{s^2(s+1)}$. Find

a) the position, velocity and acceleration error constants.

b) the steady state error when the input is $R(s)$,

where $R(s) = \frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}$

Sol:

a) To find static error constants

For a unity feedback system, $H(s) = 1$

position error constant, $K_p = \underset{s\to 0}{Lt}\, G(s)H(s) = \underset{s\to 0}{Lt}\, G(s)$

$$= \underset{s\to 0}{Lt}\, \frac{10(s+2)}{s^2(s+1)} = \infty$$

Velocity error constant, $K_v = \underset{s\to 0}{Lt}\, s\, G(s)H(s) = \underset{s\to 0}{Lt}\, s\, G(s)$

$$= \underset{s\to 0}{Lt}\, s\, \frac{10(s+2)}{s^2(s+1)} = \infty$$

Acceleration error constant, $K_a = \underset{s\to 0}{Lt}\, s^2 G(s)H(s) = \underset{s\to 0}{Lt}\, s^2 G(s)$

$$= \underset{s\to 0}{Lt}\, s^2 \frac{10(s+2)}{s^2(s+1)} = \frac{10 \times 2}{1} = 20$$

b) To find steady state error.

The error signal in s-domain, $E(s) = \frac{R(s)}{1 + G(s)H(s)}$

Given that $R(s) = \frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}$; $G(s) = \frac{10(s+2)}{s^2(s+1)}$

$H(s) = 1$

$$E(s) = \frac{\frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}}{1 + \frac{10(s+2)}{s^2(s+1)}} = \frac{\frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}}{\frac{s^2(s+1) + 10(s+2)}{s^2(s+1)}}$$

Evaluation of Generalized Error coefficient :

$$C_n = (-1)^n \int_0^t T^n f(T)\, dT \quad , \quad F(s) = \frac{1}{1+G(s)H(s)}$$

$$L\{f(T)\} = F(s)$$

$$F(s) = \int_0^t f(T)\, e^{-sT}\, dT$$

$\underset{s\to 0}{Lt}$ on both side.

$$\underset{s\to 0}{Lt}\, F(s) = \underset{s\to 0}{Lt} \int_0^t f(T)\, e^{-sT}\, dT$$

$$= \int_0^t f(T)\, \underset{s\to 0}{Lt}\, e^{-sT}\, dT = \int_0^t f(T)\, dT = C_0$$

$$C_0 = \underset{s\to 0}{Lt}\, F(s)$$

$$C_1 = \underset{s\to 0}{Lt} \frac{d}{ds} F(s)$$

$$C_2 = \underset{s\to 0}{Lt} \frac{d^2}{ds^2} F(s)$$

$$C_n = \underset{s\to 0}{Lt} \frac{d^n}{ds^n} F(s)$$

Correlation between Static and Dynamic Error coefficients.

The values of dynamic error coefficients can be used to calculate static error coefficients the following expressions shown the relationship between them.

$$C_0 = \frac{1}{1+K_p}$$

$$C_1 = \frac{1}{K_v}$$

$$C_2 = \frac{1}{K_a}$$

Proof :

$$C_0 = \underset{s\to 0}{Lt}\, F(s) = \underset{s\to 0}{Lt} \frac{1}{1+G(s)H(s)} = \frac{1}{1+\underset{s\to 0}{Lt}\, G(s)H(s)} = \frac{1}{1+K_p}$$

Static Error constant for Various Type Number of Systems

Error Constant	Type Number of System			
	0	1	2	3
K_p	constant	∞	∞	∞
K_v	0	constant	∞	∞
K_a	0	0	Constant	∞

Steady state Error for Various Types of Inputs.

Input signal	Type Number of system			
	0	1	2	3
Unit Step	$\frac{1}{1+K_p}$	0	0	0
Unit Ramp	∞	$\frac{1}{K_v}$	0	0
Unit parabolic	∞	∞	$\frac{1}{K_a}$	0

GENERALIZED ERROR COEFFICIENTS :

The error signal in s-domain $E(s) = \dfrac{R(s)}{1+G(s)H(s)}$

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)\,H(s)}$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)} = C_0 + C_1 s + \frac{C_2}{2!}s^2 + \frac{C_3}{3!}s^3 + \dots$$

$$E(s) = C_0 R(s) + C_1 s R(s) + \frac{C_2}{2!}s^2 R(s) + \frac{C_3}{3!}s^3 R(s) + \dots$$

Taking L.T of eq. above,

$$e(t) = C_0\, r(t) + C_1 s\, r(t) + \frac{C_2}{2!}s^2 r(t) + \frac{C_3}{3!}s^3 r(t) + \dots$$

This method will be useful to find generalized error coefficients without using diff. but using L.T.

STEADY STATE ERROR WHEN THE INPUT IS UNIT PARABOLIC SIGNAL :

$$e_{ss} = \underset{s\to 0}{Lt} \frac{s R(s)}{1+G(s)H(s)} \qquad i/p \quad R(s) = \frac{1}{s^3}$$

$$e_{ss} = \underset{s\to 0}{Lt} \frac{s \frac{1}{s^3}}{1+G(s)H(s)} = \underset{s\to 0}{Lt} \frac{1}{s^2 + s^2 G(s)H(s)}$$

$$= \frac{1}{\underset{s\to 0}{Lt}\, s^2 G(s) H(s)} = \frac{1}{K_a} \qquad K_a = \underset{s\to 0}{Lt}\, s^2 G(s)H(s)$$

acceleration error constant

Type 0 system :

$$K_a = \underset{s\to 0}{Lt}\, s^2 G(s) H(s)$$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$$

Type 1 system :

$$K_a = \underset{s\to 0}{Lt}\, s^2 G(s) H(s) = 0$$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$$

Type 2 system :

$$K_a = \underset{s\to 0}{Lt}\, s^2 G(s) H(s) = \text{constant}$$

$$e_{ss} = \frac{1}{K_a} = \text{constant}.$$

Type 3 system :

$$K_a = \underset{s\to 0}{Lt}\, s^2 G(s) H(s) = \infty$$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{\infty} = 0$$

Type no. 3 and above for unit parabolic i/p

$K_a = \infty$ and $e_{ss} = 0$.

STEADY STATE ERROR WHEN THE INPUT IS UNIT RAMP SIGNAL.

$$e_{ss} = \underset{s \to 0}{Lt} \frac{s R(s)}{1 + G(s) H(s)} \quad , \quad R(s) = \frac{1}{s^2}$$

$$e_{ss} = \underset{s \to 0}{Lt} \frac{s \, {}^{1}/_{s^2}}{1 + G(s) H(s)} = \underset{s \to 0}{Lt} \frac{1}{s + s G(s) H(s)}$$

$$= \frac{1}{\underset{s \to 0}{Lt} s G(s) H(s)} = \frac{1}{K_V} \qquad K_V = \underset{s \to 0}{Lt} s G(s) H(s)$$

Velocity error constant.

Type 0 System :

$$K_V = \underset{s \to 0}{Lt} s \, G(s) H(s) = 0$$

$$e_{ss} = \frac{1}{K_V} = \frac{1}{0} = \infty$$

Type 1 system :

$$K_V = \underset{s \to 0}{Lt} s \, G(s) H(s) = \text{Constant. } = K_V$$

$$e_{ss} = \frac{1}{K_V} = \text{Constant.}$$

Type 2 system :

$$K_V = \underset{s \to 0}{Lt} s \, G(s) H(s) = \infty$$

$$e_{ss} = \frac{1}{K_V} = \frac{1}{\infty} = 0$$

In system with type number 2 and above, for unit ramp input, the value of K_V is infinity so, the steady state error is zero.

For the three cases mentioned above the steady state error is associated with one of the constants defined as follows.

positional error constant, $K_p = \lim_{s \to 0} G(s)\,H(s)$

Velocity error constant, $K_v = \lim_{s \to 0} s\,G(s)\,H(s)$

Acceleration error constant, $K_a = \lim_{s \to 0} s^2 \cdot G(s)\,H(s)$

K_p, K_v and K_a are in general called static error constants.

STEADY STATE ERROR WHEN THE INPUT IS UNIT STEP SIGNAL :

$$e_{ss} = \lim_{s \to 0} \frac{s\,R(s)}{1+G(s)\,H(s)} \qquad R(s) = \frac{1}{s}$$

$$e_{ss} = \lim_{s \to 0} \frac{s\,\frac{1}{s}}{1+G(s)\,H(s)} = \frac{1}{1+\lim_{s \to 0} G(s)\,H(s)} = \frac{1}{1+K_p}$$

$$K_p = \lim_{s \to 0} G(s)\,H(s)$$

K_p is positional error constant.

Type 0 system :

$$K_p = \lim_{s \to 0} G(s)\,H(s) \text{ is ... constant}$$

$$e_{ss} = \frac{1}{1+K_p}$$

Type 1 system :

$$K_p = \lim_{s \to 0} G(s)\,H(s) = \infty$$

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$$

For type 1 and above for unit step input the value of K_p is ∞, so steady state error is 0. $e_{ss} = 0$

$e(t)$ = error signal in time domain.

$$e(t) = L^{-1}\{E(s)\} = L^{-1}\left\{\frac{R(s)}{1+G(s)H(s)}\right\}$$

e_{ss} = steady state error.

The steady state error is defined as the value of $e(t)$ when t tends to infinity.

$$e_{ss} = \underset{t\to\infty}{Lt}\ e(t)$$

The final value theorem of L.T states that,

If, $F(s) = L\{f(t)\}$ then, $\underset{t\to\infty}{Lt}\ f(t) = \underset{s\to 0}{Lt}\ F(s)$

Using final value theorem,

The steady state error,

$$e_{ss} = \underset{t\to\infty}{Lt}\ e(t) = \underset{s\to 0}{Lt}\ sE(s)$$

$$= \underset{s\to 0}{Lt}\left[\frac{sR(s)}{1+G(s)H(s)}\right]$$

STATIC ERROR CONSTANTS :

When a control system is excited with standard input signal, the steady state error may be zero, constant or infinity. The value of steady state error depends on the type and the input signal. Type-0 system will have a constant steady state error when the input is step signal.

Type-1 system will have constant steady state error when input is ramp signal or velocity signal.

Type-2 system will have constant steady state error when input is parabolic signal or acceleration signal.

Time constant, $T = \frac{1}{\zeta \omega_n} = \frac{1}{0.5 \times 4} = 0.5$ sec.

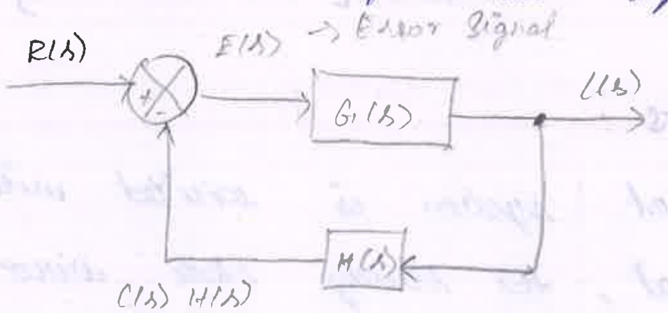
For 5% error, Settling Time, $t_s = 3T = 3 \times 0.5 = 1.5$ sec.

For 2% error, Settling time, $t_s = 4T = 4 \times 0.5 = 2$ sec.

STEADY STATE ERROR :

The steady state error is the value of error signal $e(t)$, when t tends to infinity. The steady state error is a measure of system accuracy. These errors arise from the nature of inputs, type of system and from non linearity of system components. The steady state performance of a stable control system is generally judged by its steady state error to step, ramp and parabolic inputs.

If N is no. of poles at the origin then the type no. is N.



order = s^{n} → order

Type no. of system

$$G(s)H(s) = K \frac{P(s)}{Q(s)}$$

$$= K \frac{(s+z_1)(s+z_2)\ldots}{s^n (s+p_1)(s+p_2)\ldots}$$

$N = 0$, $T = 0$, $s^0 = 1$

No. poles = Type

$$E(s) = R(s) - C(s)H(s)$$

$$C(s) = E(s)\,G(s).$$

Sub $C(s)$ in $E(s)$

$$E(s) = R(s) - [E(s)\,G(s)]\,H(s)$$

$$E(s) + E(s)\,G(s)\,H(s) = R(s)$$

$$E(s)\,[1 + G(s)\,H(s)] = R(s)$$

$$E(s) = \frac{R(s)}{1 + G(s)\,H(s)}$$

On equating the coefficient of s^2 we get,

$$0 = A + B \quad \therefore \; B = -A = -1$$

On equating the coefficient of s we get

$$0 = 4A + C \quad \therefore \; C = -4A = -4$$

$$C(s) = \frac{1}{s} + \frac{-s-4}{s^2+4s+16} = \frac{1}{s} - \frac{s+4}{s^2+4s+4+12}$$

$$= \frac{1}{s} - \frac{s+2+2}{(s+2)^2+12} = \frac{1}{s} - \frac{s+2}{(s+2)^2+12} - \frac{2}{\sqrt{12}} \frac{\sqrt{12}}{(s+2)^2+12}$$

The ~~time~~ domain response is obtained by taking inverse L.T of $C(s)$.

the response in time domain,

$$c(t) = \mathcal{L}^{-1}\{C(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{s+2}{(s+2)^2+12} - \frac{2}{\sqrt{12}} \frac{\sqrt{12}}{(s+2)^2+12}\right\}$$

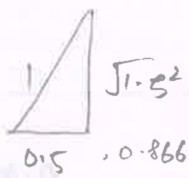
$$= 1 - e^{-2t} \cos \sqrt{12}\, t - \frac{2}{2\sqrt{3}} e^{-2t} \sin \sqrt{12}\, t$$

$$\boxed{c(t) = 1 - e^{-2t}\left[\frac{1}{\sqrt{3}} \sin(\sqrt{12}\, t) + \cos(\sqrt{12}\, t)\right]}$$

Damped frequency of oscillation, $\omega_d = \omega_n \sqrt{1-\zeta^2}$

$$= 4\sqrt{1-0.5^2}$$

$$= 3.464 \text{ rad/sec.}$$



$, 0.866$

Rise time, $t_r = \frac{\pi - \theta}{\omega_d} = \frac{\pi - 1.047}{3.464}$

$$= 0.6046 \text{ sec.}$$

$\cos 0.5 = \frac{\sqrt{3}}{2}$

$\sin \theta = 0.866 = \frac{\sqrt{3}}{2}$

$\tan \theta = 1.732$

$\theta = 60°$

$= 1.047$ rad

Peak time, $t_p = \frac{\pi}{\omega_d} = \frac{\pi}{3.464} = 0.907$ sec.

% Max. overshoot } $\% M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100 = e^{\frac{-0.5\pi}{\sqrt{1-0.5^2}}} \times 100$

$$= 0.163 \times 100 = 16.3\%$$

$$\frac{C(s)}{R(s)} = \frac{16}{s^2 + (0.8 + 16 \times 0.2)s + 16} = \frac{16}{s^2 + 4s + 16}$$

Given that damping ratio, $\zeta = 0.5$. Hence the system is underdamped and so the response of the system will have damped oscillations. The roots of char. polynomial will be complex conjugate.

The response in s-domain, $C(s) = R(s) \frac{16}{s^2 + 4s + 16}$

For unit step input, $R(s) = \frac{1}{s}$

$$C(s) = \frac{1}{s} \frac{16}{s^2 + 4s + 16} = \frac{16}{s(s^2 + 4s + 16)}$$

By partial fraction expansion we can write,

$$C(s) = \frac{16}{s(s^2 + 4s + 16)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 4s + 16}$$

The residue A is obtained by mult. $C(s)$ by s and letting $s = 0$.

$$A = C(s) \times s \Big|_{s=0} = \frac{16}{s^2 + 4s + 16}\Big|_{s=0} = \frac{16}{16} = 1$$

The residues B and C are evaluated by cross multiplying the following eq. and equating the coefficients of like powers of s.

$$\frac{16}{s(s^2 + 4s + 16)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 4s + 16}$$

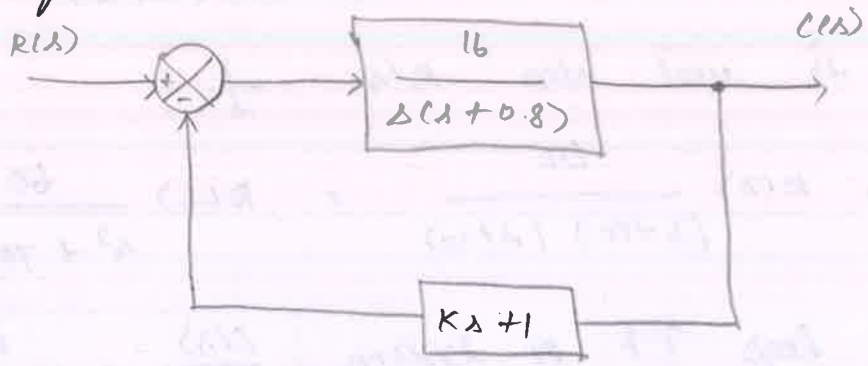
On cross multiplication we get,

$$16 = A(s^2 + 4s + 16) + (Bs + C)s$$

$$16 = As^2 + 4As + 16A + Bs^2 + Cs.$$

Calculate rise time, peak time, maximum overshoot and settling time.

Sol:



Sol:

The closed loop T.F, $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$

Given that $G(s) = \frac{16}{s(s+0.8)}$ and $H(s) = Ks+1$

$$\frac{C(s)}{R(s)} = \frac{16/s(s+0.8)}{1+\frac{16}{s(s+0.8)}(Ks+1)} = \frac{16}{s(s+0.8)+16(Ks+1)}$$

$$= \frac{16}{s^2+0.8s+16Ks+16} = \frac{16}{s^2+(0.8+16K)s+16}$$

The values of K and ω_n are obtained by comparing the system T.F with std. form of 2nd order T.F.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{16}{s^2+(0.8+16K)s+16}$$

On comp. we get,

$\omega_n^2 = 16$

$\omega_n = 4$ rad/sec.

$0.8+16K = 2\zeta\omega_n$

$K = \frac{2\zeta\omega_n - 0.8}{16}$

$= \frac{2 \times 0.5 \times 4 - 0.8}{16}$

$= 0.2$

$$= \frac{600}{s(s+60)(s+10)} = \frac{1}{s}\frac{600}{(s+60)(s+10)}$$

Since input is unit step $R(s) = \frac{1}{s}$

$$C(s) = R(s)\frac{60}{(s+60)(s+10)} = R(s)\frac{60}{s^2+70s+600}$$

The closed loop T.F of system, $\boxed{\frac{C(s)}{R(s)} = \frac{600}{s^2+70s+600}}$

The damping ratio and natural frequency of oscillation can be estimated by comparing the system T.F with std. form of 2nd order T.F.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{600}{s^2+70s+600}$$

on comparing, we get,

$$\omega_n^2 = 600$$

Natural frequency of oscillation

$$\boxed{\omega_n = \sqrt{600} = 24.49 \text{ rad/sec.}}$$

$$2\zeta\omega_n = 70$$

Damping ratio. ζ

$$\zeta = \frac{70}{2\omega_n} = \frac{70}{2 \times 24.49} \quad \boxed{\zeta = 1.43}$$

$$\boxed{\omega_n = 24.49 \text{ rad/sec} \; ; \; \zeta = 1.43}$$

Problem 3:

A positional control system with velocity feedback is shown in fig. what is the response c(t) to the unit step input. Given that $\zeta = 0.5$. Also

The time domain response c(t) is obtained by taking inverse L.T of C(s)

Response in time domain, $c(t) = L^{-1}\{C(s)\}$

$$c(t) = L^{-1}\left\{\frac{1}{s} - \frac{4}{3}\frac{1}{s+1} + \frac{1}{3}\frac{1}{s+4}\right\}$$

$$= 1 - \frac{4}{3}e^{-t} + \frac{1}{3}e^{-4t}$$

$$= 1 - \frac{1}{3}\left[4e^{-t} - e^{-4t}\right]$$

Result:

Response of unit feedback system,

$$c(t) = 1 - \frac{1}{3}\left[4e^{-t} - e^{-4t}\right]$$

Problem 2:

The response of a servomechanism is $c(t) = 1 + 0.2\,e^{-60t} - 1.2\,e^{-10t}$ when subject to a unit step input. Obtain an expression for closed loop transfer function. Determine the undamped natural frequency ω_n and damping ratio (ζ).

Sol:

Given that, $c(t) = 1 + 0.2\,e^{-60t} - 1.2\,e^{-10t}$

On taking L.T of c(t) or $L\{c(t)\}$

$$C(s) = \frac{1}{s} + 0.2\frac{1}{s+60} - 1.2\frac{1}{s+10}$$

$$= \frac{(s+60)(s+10) + 0.2\,s\,(s+10) - 1.2\,s\,(s+60)}{s\,(s+60)\,(s+10)}$$

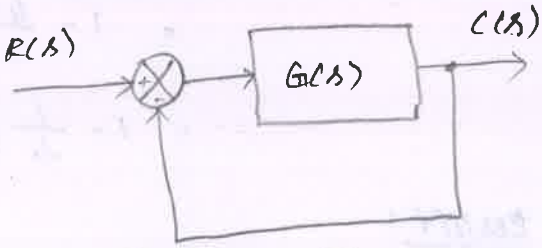
$$= \frac{\cancel{s^2} + 70s + 600 + \cancel{0.2s^2} + 2s - \cancel{1.2s^2} - 72s}{s\,(s+60)\,(s+10)}$$

problem 1:

Obtain the response of unity feedback system whose open loop transfer function is $G(s) = \frac{4}{s(s+5)}$ and when the input is unit step.

Sol:

The closed loop system is shown



closed loop T.F, $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$

$$\frac{C(s)}{R(s)} = \frac{\frac{4}{s(s+5)}}{1+\frac{4}{s(s+5)}} = \frac{4/\cancel{s(s+5)}}{s(s+5)+4/\cancel{s(s+5)}}$$

$$\frac{C(s)}{R(s)} = \frac{4}{s(s+5)+4} = \frac{4}{s^2+5s+4} = \frac{4}{(s+4)(s+1)}$$

The response in s-domain, $C(s) = R(s)\frac{4}{(s+4)(s+1)}$

Since input is unit step, $R(s) = \frac{1}{s}$

$$C(s) = \frac{1}{s}\frac{4}{(s+4)(s+1)}$$

By partial fraction expansion, we can write,

$$C(s) = \frac{4}{s(s+4)(s+1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+4}$$

$$A = C(s) \times s\Big|_{s=0} = \frac{4}{(s+1)(s+4)}\Big|_{s=0} = \frac{4}{1\times 4} = 1$$

$$B = C(s) \times (s+1)\Big|_{s=-1} = \frac{4}{s(s+4)}\Big|_{s=-1} = \frac{4}{-1(-1+4)} = \frac{-4}{3}$$

$$C = C(s) \times (s+4)\Big|_{s=-4} = \frac{4}{s(s+1)}\Big|_{s=-4} = \frac{4}{-4(-4+1)} = \frac{1}{3}$$

Expressions for Time Domain Specifications :

1. Rise time (t_r) :

$$t_r = \frac{\pi - \theta}{\omega_d} \qquad \theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$t_r = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_n\sqrt{1-\xi^2}} \text{ in sec.}$$

2. Peak time (t_p) :

$$t_p = \frac{\pi}{\omega_d}$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

3. Peak overshoot (M_p)

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \theta)$$

$$\% M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100$$

→ percentage

$$\% M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100$$

4. Settling time (t_s)

For second order system, time constant,

$$T = 1/\xi\omega_n$$

$$t_s = \frac{4}{\xi\omega_n} = 4T \quad (\text{for } 2\% \text{ error})$$

$$t_s = \frac{3}{\xi\omega_n} = 3T \quad (\text{for } 5\% \text{ error})$$

$$t_s = \frac{\ln(\% \text{ error})}{\xi\omega_n} = \frac{\ln(\% \text{ error})}{T}$$

2. RISE TIME (t_r) : It is the time taken for response to raise from 0 to 100% for the very first time. For underdamped system, the raise time is calculated from 0 to 100%. But for overdamped system it is the time taken by the response to raise from 10% to 90%. For critically damped system, it is the time taken for response to raise from 5% to 95%

3. PEAK TIME (t_p) : It is the time taken for the response to reach the peak value the very first time. (or) It is the time taken for the response to reach the peak overshoot, M_p.

4. PEAK OVERSHOOT (M_p) : It is defined as the ratio of the maximum peak value to the final value, where, the maximum peak value is measured from final value.

Let, $c(\infty)$ = Final value of $c(t)$

$c(t_p)$ = Maximum value of $c(t)$.

Now, peak overshoot, $M_p = \dfrac{c(t_p) - c(\infty)}{c(\infty)}$

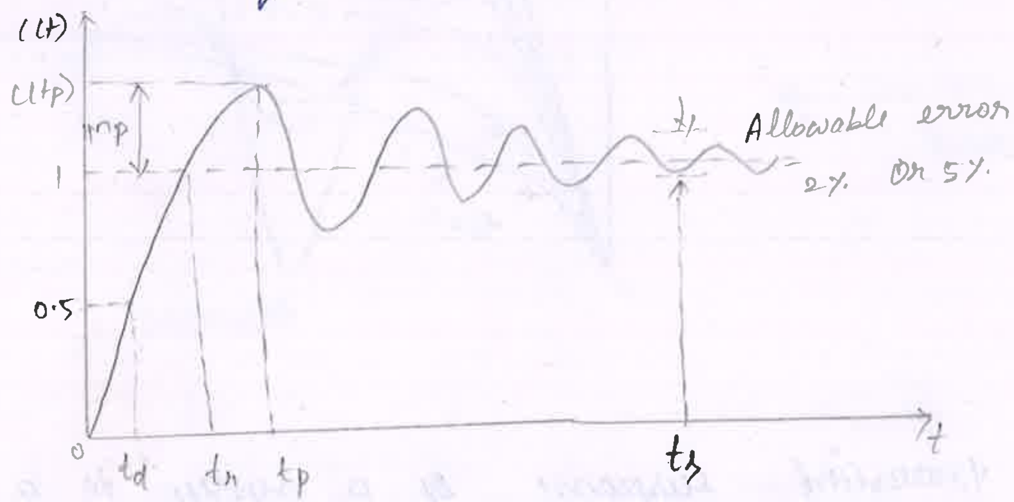
% peak overshoot, $\% M_p = \dfrac{c(t_p) - c(\infty)}{c(\infty)} \times 100$

5. SETTLING TIME (t_s) : It is defined as the time by the response to reach and stay within a specified error. It is usually expressed as % of final value. The usual tolerable error is 2% or 5% of the final value.

The most practical standard is to start with the system at rest and so output and all time derivatives before $t=0$ will be zero. The transient response of a practical control system often exhibits damped oscillation before reaching steady state. A typical damped oscillatory response of a system is shown in fig.

The transient response char. of a control system to a unit step input is specified in terms of following time domain specifications.

1. Delay time, t_d.
2. Rise time, t_r.
3. Peak time, t_p
4. Maximum overshoot, M_p
5. Settling time, t_s.



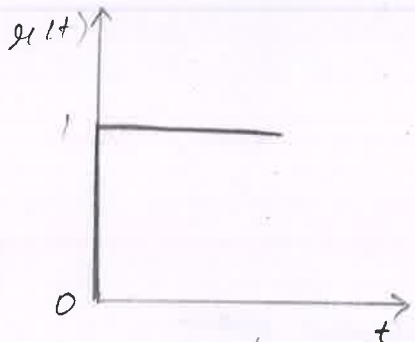
Time domain specifications are defined as follows.

1. Delay Time (t_d) : It is the time taken for response to reach 50% of the final value, for the very first time.

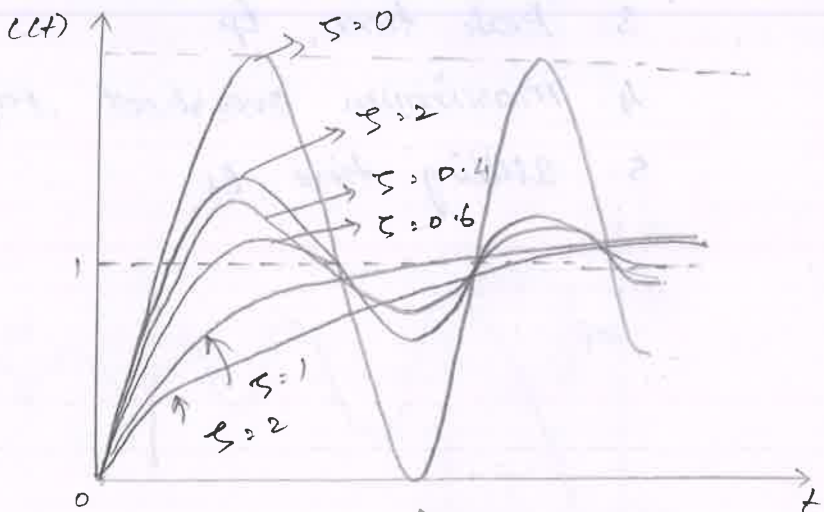
TIME DOMAIN SPECIFICATIONS :

The desired performance characteristics of control system are specified in terms of time domain specifications. Systems with energy storage elements cannot respond instantaneously and will exhibit transient responses, whenever they are subjected to inputs or disturbances.

The desired performance char. of a system of any order may be specified in terms of the transient response to a unit step input signal. The response of a second order system for unit-step input with various values of damping ratio is shown in fig.



Input



Response

The transient response of a system to a unit step input depends on the initial conditions. Therefore to compare the time response of various systems it is necessary to start with standard initial conditions.

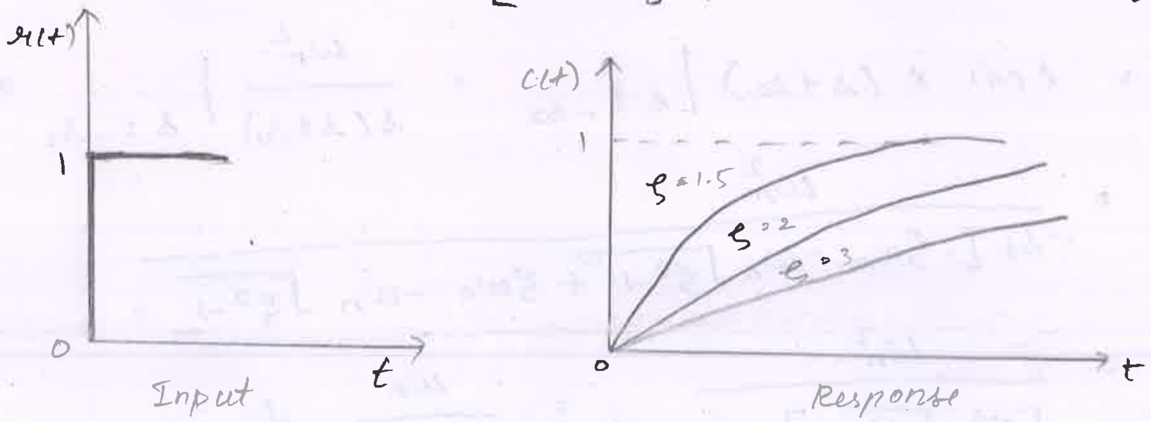
The above eq. is the response of overdamped closed loop system for unit step input. For step input of value A, the above eq. is multiplied by A.

For closed loop over damped second order system

$$\text{Unit step response} = 1 - \frac{\omega_n}{2\sqrt{\xi^2-1}} \frac{1}{s_1}\left(\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2}\right)$$

where, $s_1 = \xi\omega_n - \omega_n\sqrt{\xi^2-1}$

$$\text{Step response} = A\left[1 - \frac{\omega_n}{2\sqrt{\xi^2-1}} \frac{1}{s_1}\left(\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2}\right)\right]$$



Response of over damped second order system for unit step input

Using above eq., the response of overdamped second order system is sketched as shown in fig. and observed that the response has no oscillation but it takes longer time for the response to reach the final steady value.

$$= \frac{\omega_n^2}{\xi^2\omega_n^2 - \omega_n^2(\xi^2-1)} = \frac{\omega_n^2}{\omega_n^2} = 1$$

$$B = (s+s_1) \times C(s)\Big|_{s=-s_1} = \frac{\omega_n^2}{s(s+s_2)}\Big|_{s=-s_1} = \frac{\omega_n^2}{-s_1(-s_1+s_2)}$$

$$= \frac{-\omega_n^2}{s_1[-\xi\omega_n + \omega_n\sqrt{\xi^2-1} + \xi\omega_n + \omega_n\sqrt{\xi^2-1}]}$$

$$= \frac{-\omega_n^2}{[2\omega_n\sqrt{\xi^2-1}]s_1} = \frac{-\omega_n}{2\sqrt{\xi^2-1}} \frac{1}{s_1}$$

$$C = C(s) \times (s+s_2)\Big|_{s=-s_2} = \frac{\omega_n^2}{s(s+s_1)}\Big|_{s=-s_2} = \frac{\omega_n^2}{-s_2(-s_2+s_1)}$$

$$= \frac{\omega_n^2}{-s_2[-\xi\omega_n - \omega_n\sqrt{\xi^2-1} + \xi\omega_n - \omega_n\sqrt{\xi^2-1}]}$$

$$= \frac{\omega_n^2}{[2\omega_n\sqrt{\xi^2-1}]s_2} = \frac{\omega_n}{2\sqrt{\xi^2-1}} \frac{1}{s_2}$$

The response in time domain $c(t)$ is given by,

$$c(t) = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{\omega_n}{2\sqrt{\xi^2-1}}\frac{1}{s_1}\frac{1}{(s+s_1)} + \frac{\omega_n}{2\sqrt{\xi^2-1}}\frac{1}{s_2}\frac{1}{(s+s_2)}\right\}$$

$$c(t) = 1 - \frac{\omega_n}{2\sqrt{\xi^2-1}}\frac{1}{s_1}e^{-s_1 t} + \frac{\omega_n}{2\sqrt{\xi^2-1}}\frac{1}{s_2}e^{-s_2 t}$$

$$\boxed{c(t) = 1 - \frac{\omega_n}{2\sqrt{\xi^2-1}}\left(\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2}\right)}$$

where, $s_1 = \xi\omega_n - \omega_n\sqrt{\xi^2-1}$

$s_2 = \xi\omega_n + \omega_n\sqrt{\xi^2-1}$

RESPONSE OF OVER DAMPED SECOND ORDER SYSTEM FOR UNIT STEP INPUT

The standard form of closed loop transfer function of second order system is,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

For overdamped system $\zeta > 1$. the roots of the denominator of transfer function are real and distinct. Let the roots of the denominators be s_a, s_b.

$$s_a, s_b = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} = -\left[\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1}\right]$$

Let $s_1 = -s_a$ and $s_2 = -s_b$ $\therefore$ $s_1 = \zeta\omega_n - \omega_n\sqrt{\zeta^2-1}$

$$s_2 = \zeta\omega_n + \omega_n\sqrt{\zeta^2-1}$$

The closed loop transfer function can be written in terms of s_1 and s_2 as shown below.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s+s_1)(s+s_2)}$$

For unit step input $r(t) = 1$ and $R(s) = 1/s$

$$C(s) = R(s)\frac{\omega_n^2}{(s+s_1)(s+s_2)} = \frac{\omega_n^2}{s(s+s_1)(s+s_2)}$$

By partial fraction expansion we can write,

$$C(s) = \frac{\omega_n^2}{s(s+s_1)(s+s_2)} = \frac{A}{s} + \frac{B}{s+s_1} + \frac{C}{s+s_2}$$

$$A = s \times C(s)\Big|_{s=0} = s \times \frac{\omega_n^2}{s(s+s_1)(s+s_2)}\Bigg|_{s=0} = \frac{\omega_n^2}{s_1 s_2}$$

$$= \frac{\omega_n^2}{\left[\zeta\omega_n - \omega_n\sqrt{\zeta^2-1}\right]\left[\zeta\omega_n + \omega_n\sqrt{\zeta^2-1}\right]}$$

$$B = (s+\omega_n)^2 \times C(s)\Big|_{s=-\omega_n} = \frac{\omega_n^2}{s}\Big|_{s=-\omega_n} = -\omega_n$$

$$C = \frac{d}{ds}\left[(s+\omega_n)^2 \times C(s)\right]\Big|_{s=-\omega_n} = \frac{d}{ds}\left(\frac{\omega_n^2}{s}\right)\Big|_{s=-\omega_n} = \frac{-\omega_n^2}{s^2}\Big|_{s=-\omega_n} = -1$$

$$C(s) = \frac{A}{s} + \frac{B}{(s+\omega_n)^2} + \frac{C}{(s+\omega_n)} = \frac{1}{s} - \frac{\omega_n}{(s+\omega_n)^2} - \frac{1}{s+\omega_n}$$

The response in time domain

$$c(t) = L^{-1}C(s) = L^{-1}\left\{\frac{1}{s} - \frac{\omega_n}{(s+\omega_n)^2} - \frac{1}{s-\omega_n}\right\} \qquad L\{1\} = \frac{1}{s}$$

$$c(t) = 1 - \omega_n t\, e^{-\omega_n t} - e^{-\omega_n t} \qquad L\{t e^{-at}\} = \frac{1}{(s+a)^2}$$

$$\boxed{c(t) = 1 - e^{-\omega_n t}(1+\omega_n t)} \qquad L\{e^{-at}\} = \frac{1}{s+a}$$

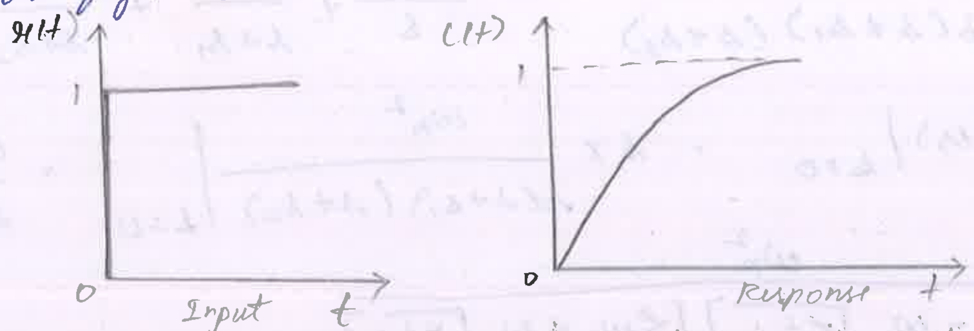
The above eq. is the response of critically damped closed loop second order system for unit step input. For step input of step value A, the above eq. should be multiplied by A.

For closed loop critically damped second order system,

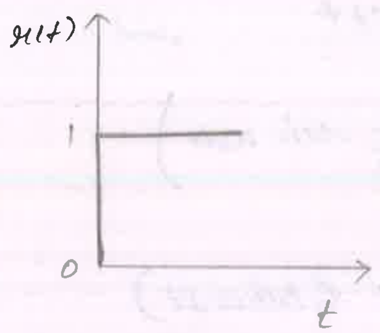
$$\text{Unit step response} = 1 - e^{-\omega_n t}(1+\omega_n t)$$

$$\text{Step response} = A\left[1 - e^{-\omega_n t}(1+\omega_n t)\right]$$

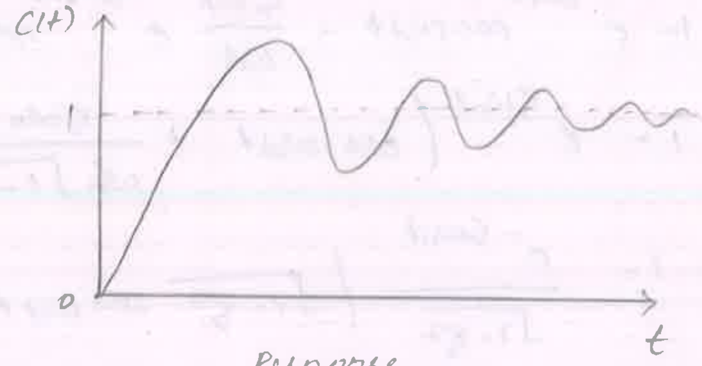
Using above eq., the response of critically damped second order system is sketched as shown in fig. and observed that the response has no oscillations.



Response of critically damped second order system for unit step input



Input



Response

response of under damped second order system for unit step input.

RESPONSE OF CRITICALLY DAMPED SECOND ORDER SYSTEM FOR UNIT STEP INPUT :

The standard form of closed loop transfer function of second order system is,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

For critical damping $\xi = 1$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s+\omega_n)^2}$$

when input is unit step, $r(t) = 1$ and $R(s) = 1/s$

The response in s-domain.

$$C(s) = R(s)\frac{\omega_n^2}{(s+\omega_n)^2} = \frac{1}{s}\frac{\omega_n^2}{(s+\omega_n)^2} = \frac{\omega_n^2}{s(s+\omega_n)^2}$$

By partial fraction expansion, we can write.

$$C(s) = \frac{\omega_n^2}{s(s+\omega_n)^2} = \frac{A}{s} + \frac{B}{(s+\omega_n)^2} + \frac{C}{s+\omega_n}$$

$$A = s \times C(s)\Big|_{s=0} = \frac{\omega_n^2}{(s+\omega_n)^2}\Big|_{s=0} = \frac{\omega_n^2}{\omega_n^2} = 1$$

$$= 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi\omega_n}{\omega_d} e^{-\xi\omega_n t}\sin\omega_d t \qquad \omega_d = \omega_n\sqrt{1-\xi^2}$$

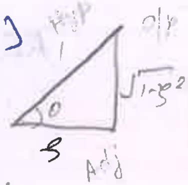
$$= 1 - e^{-\xi\omega_n t}\left(\cos\omega_d t + \frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}}\sin\omega_d t\right)$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left(\sqrt{1-\xi^2}\cos\omega_d t + \xi\sin\omega_d t\right)$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left(\sin\omega_d t \times \xi + \cos\omega_d t \times \sqrt{1-\xi^2}\right)$$

Let us express $c(t)$ in a std. form as shown below.

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left(\sin\omega_d t \times \cos\theta + \cos\omega_d t \times \sin\theta\right)$$



$\sin\theta = \sqrt{1-\xi^2}$

$\cos\theta = \xi$

$\tan\theta = \sqrt{1-\xi^2}/\xi$

$$\boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \theta)}$$

where, $\left(\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)$

The above eq. is the response of under damped closed loop second order system for unit step input. For step input of step value A, the above eq. should be multiplied by A.

For closed loop under damped second order system,

$$\text{Unit step response} = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \theta) \; ; \; \theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

$$\text{Step response} = A\left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \theta) \; ; \; \theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right]$$

Using above eq. the response of underdamped second order system for unit step input is sketched and observed that the response oscillates before settling to a final value. The oscillations depends on the value of damping ratio.

On cross mult. eq after sub $A=1$, we get,

$\omega_n^2 = s^2 + 2\zeta\omega_n s + \omega_n^2 + (Bs+C)s$

$\omega_n^2 = s^2 + 2\zeta\omega_n s + \omega_n^2 + Bs^2 + Cs.$

Equating coefficients of s^2 we get, $0 = 1+B$ $\therefore B = -1$

Eq. coefficients of s we get, $0 = 2\zeta\omega_n + C$ $\therefore C = -2\zeta\omega_n.$

$$C(s) = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Let us add and subtract $\zeta^2\omega_n^2$ to the denominator of second term in eq. above.

$$C(s) = \frac{1}{s} - \frac{s+2\zeta\omega_n}{s^2 + 2\zeta\omega_n s + \omega_n^2 + \zeta^2\omega_n^2 - \zeta^2\omega_n^2}$$

$$= \frac{1}{s} - \frac{s+2\zeta\omega_n}{(s^2 + 2\zeta\omega_n s + \zeta^2\omega_n^2) + (\omega_n^2 - \zeta^2\omega_n^2)}$$

$$= \frac{1}{s} - \frac{s+2\zeta\omega_n}{(s+\zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)} = \frac{1}{s} - \frac{s+2\zeta\omega_n}{(s+\zeta\omega_n)^2 + \omega_d^2}$$

$$C(s) = \frac{1}{s} - \frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2 + \omega_d^2} - \frac{\zeta\omega_n}{(s+\zeta\omega_n)^2 + \omega_d^2}$$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

Let us multiply and divide by ω_d in third term of eq. above.

$$C(s) = \frac{1}{s} - \frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2 + \omega_d^2} - \frac{\zeta\omega_n}{\omega_d}\,\frac{\omega_d}{(s+\zeta\omega_n)^2 + \omega_d^2}$$

The response in time domain is given by

$$c(t) = \mathcal{L}^{-1}\{C(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2 + \omega_d^2} - \frac{\zeta\omega_n}{\omega_d}\,\frac{\omega_d}{(s+\zeta\omega_n)^2 + \omega_d^2}\right\}$$

$\mathcal{L}\{1\} = \frac{1}{s}$, $\mathcal{L}\{e^{-at}\sin\omega t\} = \frac{\omega}{(s+a)^2 + \omega^2}$

$$\mathcal{L}\{e^{-at}\cos\omega t\} = \frac{s+a}{(s+a)^2 + \omega^2}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

For underdamped system, $0 < \zeta < 1$ and roots of the denominator (char. eq.) are complex conjugate.

The root of the denominator are,

$$s = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

since, $\zeta < 1$, ζ^2 is also less than 1, and so, $1 - \zeta^2$ is always positive.

$$s = -\zeta\omega_n \pm \omega_n\sqrt{(-1)(1-\zeta^2)} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

The damped frequency of oscillation, $\omega_d = \omega_n\sqrt{1-\zeta^2}$

$$s = -\zeta\omega_n \pm j\omega_d$$

The response in s-domain, $C(s) = R(s)\dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

For unit step input, $r(t) = 1$ and $R(s) = 1/s$

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

By partial fraction expansion,

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

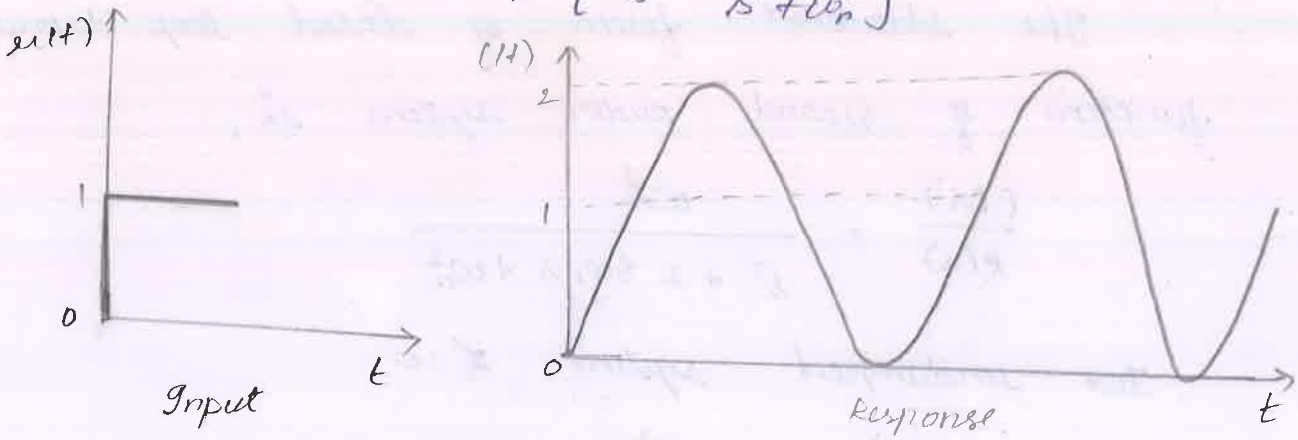
A is obtained by multiplying $C(s)$ by s and letting $s = 0$.

$$A = s \times C(s)\big|_{s=0} = s \times \left\{\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}\right\}\Bigg|_{s=0} = \frac{\omega_n^2}{\omega_n^2} = 1$$

To solve for B and C, cross multiply above, and equate like power of s.

Time domain response, $c(t) = L^{-1}\{C(s)\}$

$$= L^{-1}\left\{\frac{1}{s} - \frac{s}{s^2+\omega_n^2}\right\} = 1 - \cos\omega_n t.$$



Response of undamped second order system for unit step input.

Using above eq., the response of undamped second order system for unit step input is sketched in fig. and observed that the response is completely oscillatory.

The above eq. is the response of undamped closed loop second order system for unit step input. For step input of step value A, the eq above should be multiplied by A.

For closed loop undamped second order system,

$$\boxed{\begin{aligned} \text{Unit step response} &= 1 - \cos\omega_n t \\ \text{Step response} &= A(1 - \cos\omega_n t) \end{aligned}}$$

RESPONSE OF UNDERDAMPED SECOND ORDER SYSTEM FOR UNIT STEP INPUT

The standard form of closed loop transfer function of second order system is,

RESPONSE OF UNDAMPED SECOND ORDER SYSTEM FOR UNIT STEP INPUT

The standard form of closed loop transfer function of second order system is,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

For undamped system, $\zeta = 0$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + \omega_n^2}$$

when the input is unit step, $r(t) = 1$ and $R(s) = \frac{1}{s}$

$\therefore$ The response in s-domain, $C(s) = R(s) \frac{\omega_n^2}{s^2 + \omega_n^2}$

$$= \frac{1}{s} \frac{\omega_n^2}{s^2 + \omega_n^2}$$

By partial fraction expansion,

$$C(s) = \frac{\omega_n^2}{s(s^2 + \omega_n^2)} = \frac{A}{s} + \frac{B}{s^2 + \omega_n^2}$$

A is obtained by multiplying C(s) by s and letting s = 0.

$$A = C(s) \times s \Big|_{s=0} = \frac{\omega_n^2}{s(s^2 + \omega_n^2)} \times s \Big|_{s=0}$$

$$= \frac{\omega_n^2}{s^2 + \omega_n^2} \Big|_{s=0} = \frac{\omega_n^2}{\omega_n^2} = 1$$

B is obtained by multiplying C(s) by $(s^2 + \omega_n^2)$ and letting $s^2 = -\omega_n^2$ or $s = j\omega_n$.

$$B = C(s) \times (s^2 + \omega_n^2) \Big|_{s=j\omega} = \frac{\omega_n^2}{s(s^2 + \omega_n^2)} \times (s^2 + \omega_n^2) \Big|_{s=j\omega}$$

$$= \frac{\omega_n^2}{s} \Big|_{s=j\omega_n} = \frac{\omega_n^2}{j\omega_n} = -j\omega_n = -s$$

$L\{1\} = \frac{1}{s}$

$$C(s) = \frac{A}{s} + \frac{B}{s^2 + \omega_n^2} = \frac{1}{s} - \frac{s}{s^2 + \omega_n^2}$$

$L\{\cos \omega t\} = \frac{s}{s^2 + \omega^2}$

It is a quadratic eq. and the roots of this eq. is given by,

$$s_1, s_2 = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= \frac{-2\zeta\omega_n \pm \sqrt{4\omega_n^2(\zeta^2-1)}}{2}$$

$$= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1}$$

when, $\zeta = 0$, $s_1, s_2 = \pm j\omega_n$; $\left\{\begin{array}{l}\text{roots are purely imaginary}\\ \text{and the system is undamped.}\end{array}\right.$

when, $\zeta = 1$, $s_1, s_2 = -\omega_n$; $\left\{\begin{array}{l}\text{roots are real and equal and}\\ \text{the system is critically damped.}\end{array}\right.$

when, $\zeta > 1$, $s_1, s_2 = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1}$

$\left\{\begin{array}{l}\text{roots are real and unequal}\\ \text{and they system is overdamped.}\end{array}\right.$

when $0 < \zeta < 1$, $s_1, s_2 = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1}$

$$= -\zeta\omega_n \pm \omega_n\sqrt{(-1)(1-\zeta^2)}$$

$$= -\zeta\omega_n \pm \omega_n\sqrt{-1}\sqrt{1-\zeta^2}$$

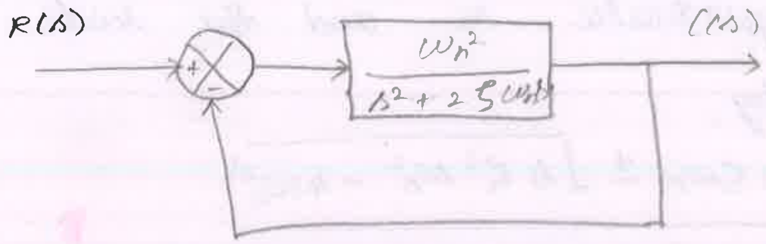
$$= -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

$$= -\zeta\omega_n \pm j\omega_d \ ;$$

$\left\{\begin{array}{l}\text{roots are complex conjugate}\\ \text{the system is underdamped.}\end{array}\right.$

where, $\omega_d = \omega_n\sqrt{1-\zeta^2}$

Here ω_d is called damped frequency of oscillation of the system and its unit is rad / sec.



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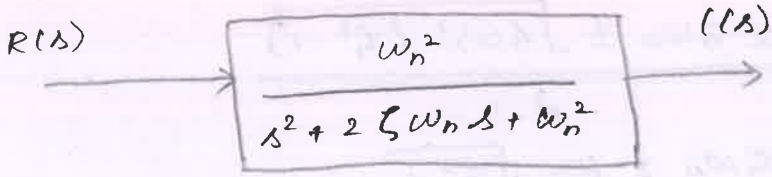


closed loop second order system.

The standard form of closed loop transfer function of second order system is given by,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where, ω_n = Undamped natural frequency, rad/sec.

ζ = Damping ratio.

The damping ratio is defined as the ratio of the actual damping to the critical damping. The response c(t) of second order system depends on the value of damping ratio. Depending on the value of ζ, the system can be classifeid into the following four cases,

case 1 : Undamped system, $\zeta = 0$

case 2 : Under damped system, $0 < \zeta < 1$

case 3 : Critically damped system, $\zeta = 1$

case 4 : Over damped system, $\zeta > 1$

The characteristics eq. of 2nd order system is,

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

For closed loop first order system,

unit step response $= 1 - e^{-t/T}$

Step response $= A\left(1 - e^{-t/T}\right)$

when, $t = 0$, $c(t) = 1 - e^{0} = 1$

$t = 1T$, $c(t) = 1 - e^{-1} = 0.632$

$t = 2T$, $c(t) = 1 - e^{-2} = 0.865$

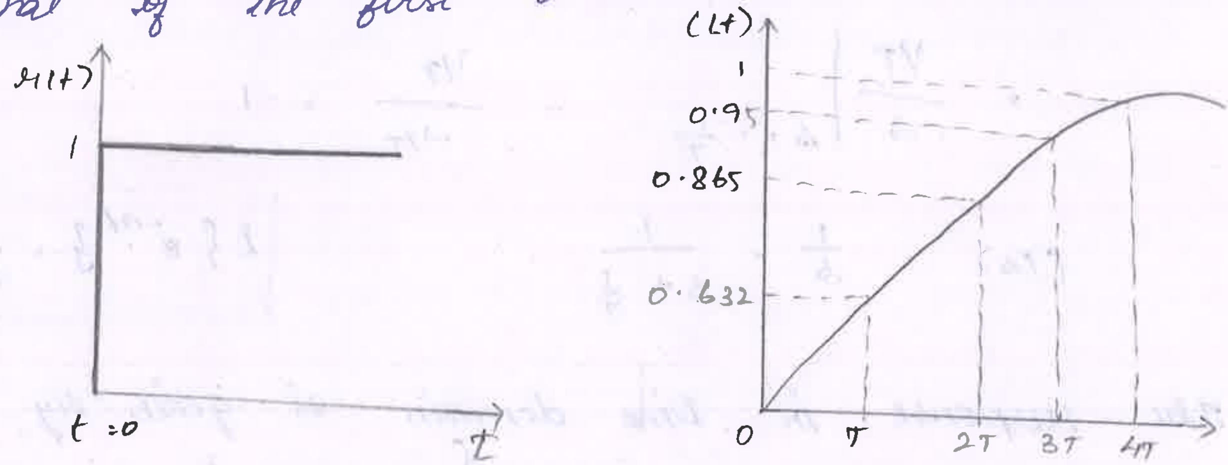
$t = 3T$, $c(t) = 1 - e^{-3} = 0.95$

$t = 4T$, $c(t) = 1 - e^{-4} = 0.9817$

$t = 5T$, $c(t) = 1 - e^{-5} = 0.993$

$t = \infty$, $c(t) = 1 - e^{-\infty} = 1$

" Here T is called Time constant of the system. In a time of 5T, the system is assumed to have attained steady state. The input and output signal of the first order system is shown in fig.



Unit step input   Response of unit step i/p

Response of first order system to unit step input.

SECOND ORDER SYSTEM:

The closed loop. ~~second~~ order system is shown in fig.

By partial fraction expansion,

$$C(s) = \frac{1/T}{s(s+\frac{1}{T})} = \frac{A}{s} + \frac{B}{(s+\frac{1}{T})}$$

A is obtained by multiplying $C(s)$ by s and letting $s=0$

$$A = C(s) \times s \Big|_{s=0} = \frac{1/T}{s(s+1/T)} \times s \Big|_{s=0}$$

$$= \frac{1/T}{s+1/T}\Big|_{s=0} = \frac{1/T}{1/T} = 1$$

B is obtained by multiplying $C(s)$ by $(s+1/T)$ and letting $s=-1/T$

$$B = C(s) \times (s+\frac{1}{T})\Big|_{s=-\frac{1}{T}} = \frac{1/T}{s(s+1/T)} \times (s+1/T)\Big|_{s=-\frac{1}{T}}$$

$$= \frac{1/T}{s}\Big|_{s=-\frac{1}{T}} = \frac{1/T}{-1/T} = -1$$

$$C(s) = \frac{1}{s} - \frac{1}{s+\frac{1}{T}}$$

$$\boxed{L\{e^{-at}\} = \frac{1}{s+a}}$$

The response in time domain is given by,

$$c(t) = L^{-1}\{C(s)\} = L^{-1}\left\{\frac{1}{s} - \frac{1}{s+\frac{1}{T}}\right\} = 1 - e^{-t/T} \quad \rightarrow (1)$$

The eq. (1) is the response of the closed loop first order system for unit step input. For step input of step value, A, the eq. (1) is multiplied by A.

$$T(s) = \frac{P(s)}{Q(s)} = \frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)} \rightarrow ③$$

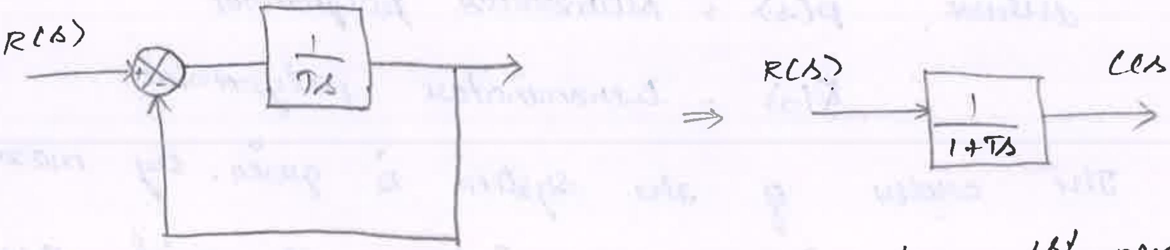
where, z_1 z_2 ... z_m are zeros of the system

p_1 p_2 .... p_n are poles of the system.

Now, the value of n gives the number of poles in the transfer function. Hence the order is also given by the number of poles of the transfer function.

RESPONSE OF FIRST ORDER SYSTEM FOR UNIT STEP INPUT:

The closed loop order system with unity feedback is shown.



closed loop system for 1^{st} order system

The closed loop transfer function of first order system, $\frac{C(s)}{R(s)} = \frac{1}{1+Ts}$

If the input is unit step then,

$r(t) = 1$ and $R(s) = \frac{1}{s}$

$\therefore$ The response in s-domain, $C(s) = R(s)\frac{1}{1+Ts}$

$$= \frac{1}{s}\,\frac{1}{1+Ts}$$

$$= \frac{1}{sT\left(\frac{1}{T}+s\right)} = \frac{\frac{1}{T}}{s\left(s+\frac{1}{T}\right)}$$

The order of the system is given by the order of the differential eq. governing the system. If the system is governed by n^{th} order diff. eq., then the system is called n^{th} order system.

Alternatively, the order can be determined from the transfer function of the system. The transfer function of the system can be obtained by taking L.T of diff. eq. governing the system and rearranging them as a ratio of two polynomials as in s, as shown in eq.

$$T.F, T(s) = \frac{P(s)}{Q(s)} = \frac{b_0 s^m + b_1 s^{m-1} + b_2 s^{m-2} + \dots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1} s + a_n} \rightarrow ②$$

where, $P(s)$ = Numerator polynomial

$Q(s)$ = Denominator polynomial.

The order of the system is given by maximum power of s in the denominator polynomial, $Q(s)$.

Here, $Q(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1} s + a_n$.

Now, n is order of system

when $n = 0$, the system is zero order system

$n = 1$, " " First order system

$n = 2$, " " second order system and so on.

∴ Order can be specified for both open loop and closed loop system.

The num. and denom. polynomial of eq. ② can be expressed in the factorized form as shown in eq. ③

IMPULSE RESPONSE :

The response of the system, with input as impulse signal is called weighing function or impulse response of the system. It is also given by the inverse L.T of a system transfer function, denoted by $m(t)$.

Impulse response, $m(t) = L^{-1}\{R(s)\, M(s)\}$

$= L^{-1}\{M(s)\}$

where, $M(s) = \dfrac{G(s)}{1+G(s)\,H(s)}$ $\boxed{R(s) = 1, \text{ for impulse.}}$

Since impulse response (or weighting function) is obtained from the transfer function of the system, it shows the characteristics of the system. Also the response for any input can be obtained by convolution of input with impulse response.

Order of the system :

The input and output relationship of a control system can be expressed by n^{th} order differential eq. shown in eq.

$$a_0 \frac{d^n}{dt^n} p(t) + a_1 \frac{d^{n-1}}{dt^{n-1}} p(t) + a_2 \frac{d^{n-2}}{dt^{n-2}} p(t) + \cdots a_{n-1} \frac{d}{dt} p(t) +$$

$$a_n\, p(t) = b_0 \frac{d^m}{dt^m} q(t) + b_1 \frac{d^{m-1}}{dt^{m-1}} q(t) + b_2 \frac{d^{m-2}}{dt^{m-2}} q(t) + \cdots +$$

$$b_{m-1} \frac{d}{dt} q(t) + b_m\, q(t) \longrightarrow (1)$$

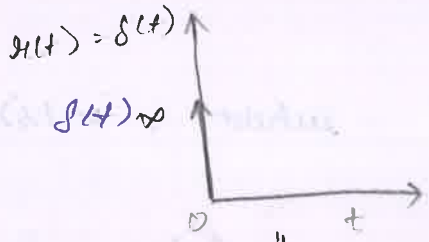
where, $p(t)$ = output / Response

$q(t)$ = Input / Excitation.

Integral of step signal is ramp signal. Integral of ramp signal of parabolic signal.

Impulse signal :

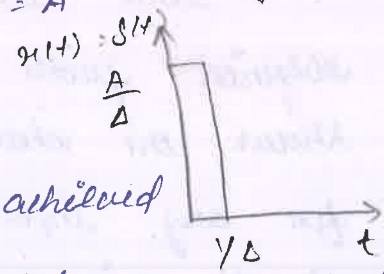
A signal of very large magnitude which is available for very short duration is called impulse signal. Ideal impulse signal is a signal with infinite magnitude and zero duration but with an area of A. The unit impulse signal is a special case in which A is unity.



The impulse signal is denoted by δ(t) and math. it is expressed as

$\delta(t) = \infty \; ; \; t = 0$ and $\int_{-\infty}^{+\infty} \delta(t)\, dt = A$

$= 0 \; ; \; t \neq 0$



Since a perfect impulse cannot be achieved in practice it is usually approximated by a pulse of small width but with area, A. Mathematically an impulse signal is the derivative of a step signal.

L.T of impulse function is unity.

Standard Test signals.

Name of signal	Time domain eq. of signal x(t)	L.T of the signal, R(s)
Step	A	A/s
Unit step	1	$1/s$
Ramp	At	A/s^2
Unit ramp	t	$1/s^2$
Parabolic	$At^2/2$	A/s^3
Unit parabolic	$t^2/2$	$1/s^3$
Impulse	$\delta(t)$	1

A special case of step signal is unit step in which A is unity.

The mathematical representation of step signal is

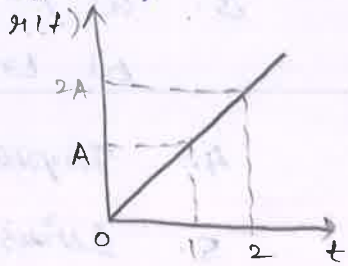
$$u(t) = 1 \quad ; \; t \geq 0$$
$$= 0 \quad ; \; t < 0$$

Ramp Signal:

The ramp signal is a signal whose value increases linearly with time from a initial value of zero at $t=0$. The ramp signal resembles a constant velocity input to a system. A special case of ramp signal is unit ramp signal in which the value of A is unity.

The mathematical representation of ramp signal is

$$r(t) = At \quad ; \; t \geq 0$$
$$= 0 \quad ; \; t < 0$$

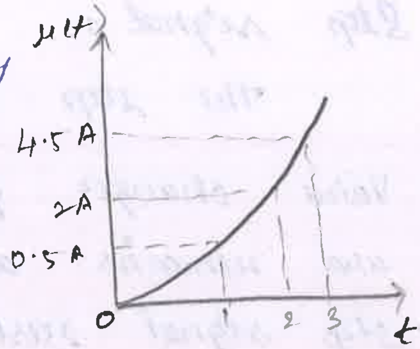


Parabolic signal

In parabolic signal, the instantaneous value varies as square of the time from an initial value of zero at $t=0$. The sketch of the signal with respect to time resembles a parabola. The parabolic signal resembles a constant acceleration input to the system. A specific case of parabolic signal is unit parabolic signal in which A is unity.

The mathematical representation of the parabolic signal is

$$p(t) = \frac{At^2}{2} \quad ; \; t \geq 0$$
$$= 0 \quad ; \; t < 0$$



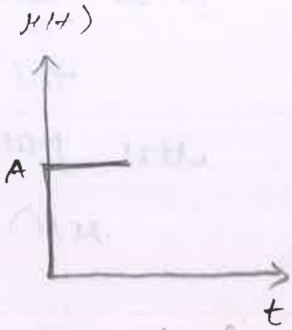
The characteristics of actual input signals are a sudden shock, a sudden change, a constant velocity, and a constant acceleration. Hence test signals which resembles these characteristics are used as input signals to predict the performance of the system. The commonly used test input signals are as follows. The standard test signals are.

1. a) Step signal
   b) Unit step signal
2. a) Ramp signal
   b) Unit ramp signal
3. a) parabolic signal
   b) Unit parabolic signal
4. Impulse signal
5. Sinusoidal signal.

The use of test signals can be justified because of a correlation existing between the response characteristics of a system to a test input signal and capability of a system to cope with actual input signals.

Step signal:

The step signal is a signal whose value changes from zero to A at $t=0$ and remains constant at A for $t>0$. The step signal resembles an actual steady input to a system.



Step signal

Response in time domain, $c(t) = L^{-1}\{C(s)\}$

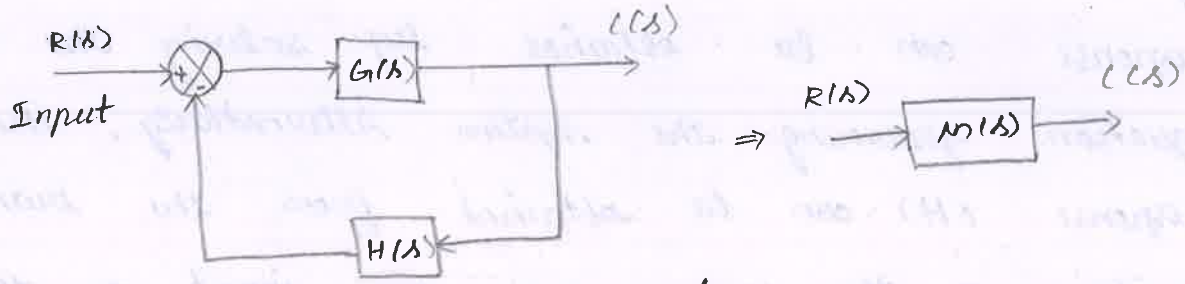
$= L^{-1}\{R(s) \times M(s)\}$

where, $M(s) = \dfrac{G(s)}{1+G(s)H(s)}$

The time response of a control system consists of two parts:

* Transient response
* Steady state response.

The transient response is the response of system when the input changes from one state to another. The steady state response is the response as time, t approaches infinity.



closed loop system

$$M(s) = \frac{G(s)}{1+G(s)\ H(s)}$$

Test signals:

The knowledge of input signal is required to predict the response of a system. In most of the systems, the input signals are not known ahead of time and also it is difficult to express the input signals mathematically by simple equations.

UNIT - II TIME RESPONSE ANALYSIS

SYLLABUS :

Transient response - steady state response - Measures of performance of the standard first order and second order system - effect on an additional zero and an additional pole - steady error constant and system - type number - PID control - Analytical design for PD, PI, PID control systems.

TIME RESPONSE :

The time response of the system is the output of the closed loop system as a function of time. It is denoted by $c(t)$. The time response can be obtained by solving the differential equation governing the system. Alternatively, the response $c(t)$ can be obtained from the transfer function of the system and the input to the system.

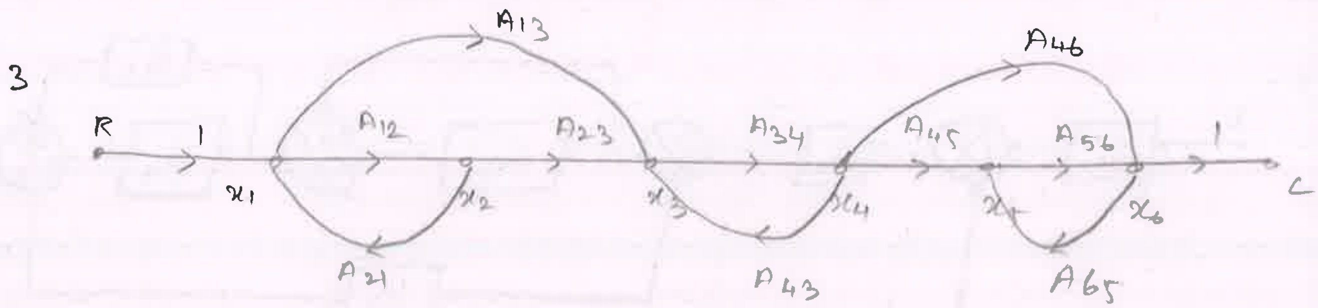
The closed loop transfer function,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = M(s)$$

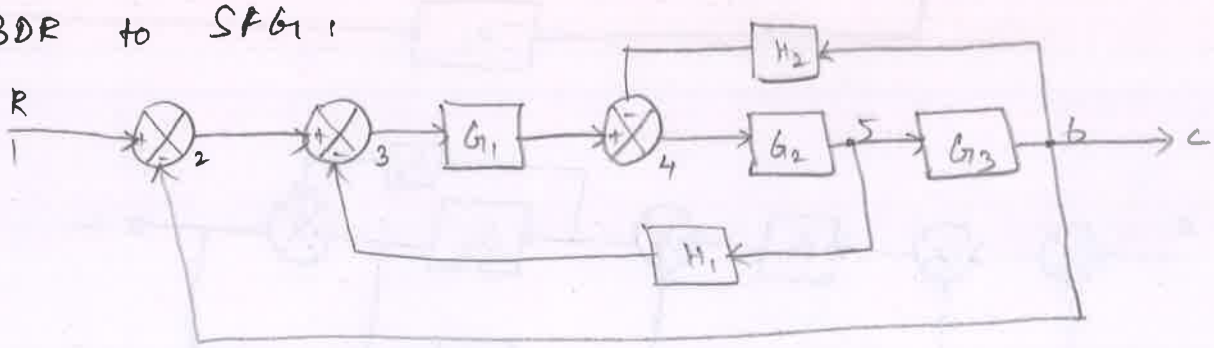
The output or Response in s-domain, $C(s)$ is given by the product of the transfer function and the input, $R(s)$, on taking L.T of this product the time domain response, $c(t)$ can be obtained.

Response in s-domain, $C(s) = R(s)\,M(s)$

3



4. BDR to SFG :

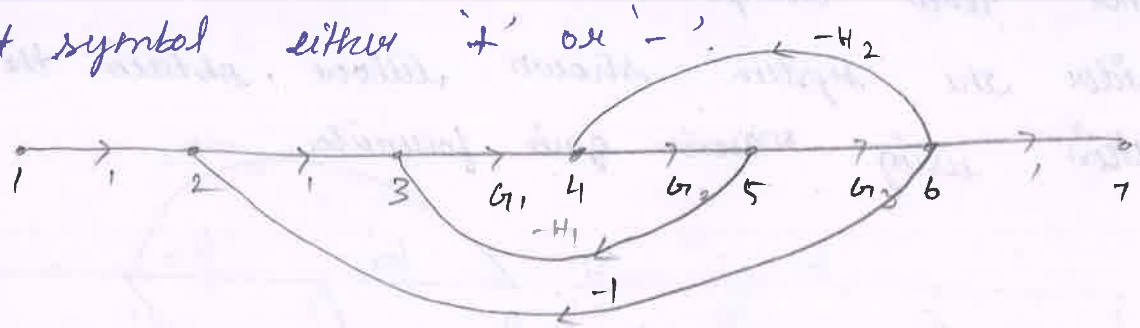


i) consider the node point on all summing points, branch point and R & C

2.) connect all points as per block diagram flow with the block value.

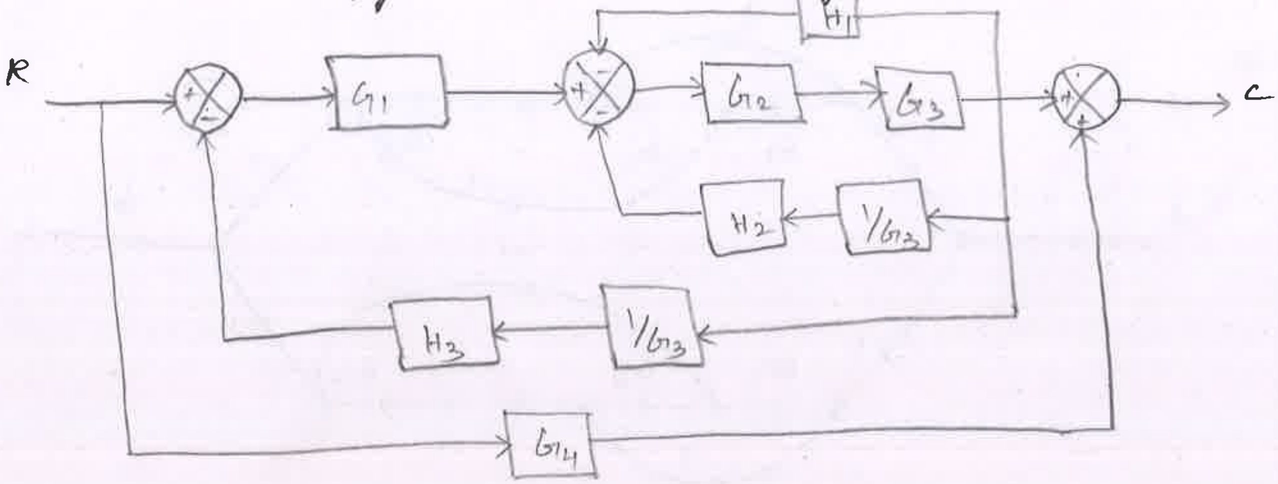
3) If there is no block consider it as 1

4.) Give the value for the feedback loop with the summing point symbol either '+' or '-'.

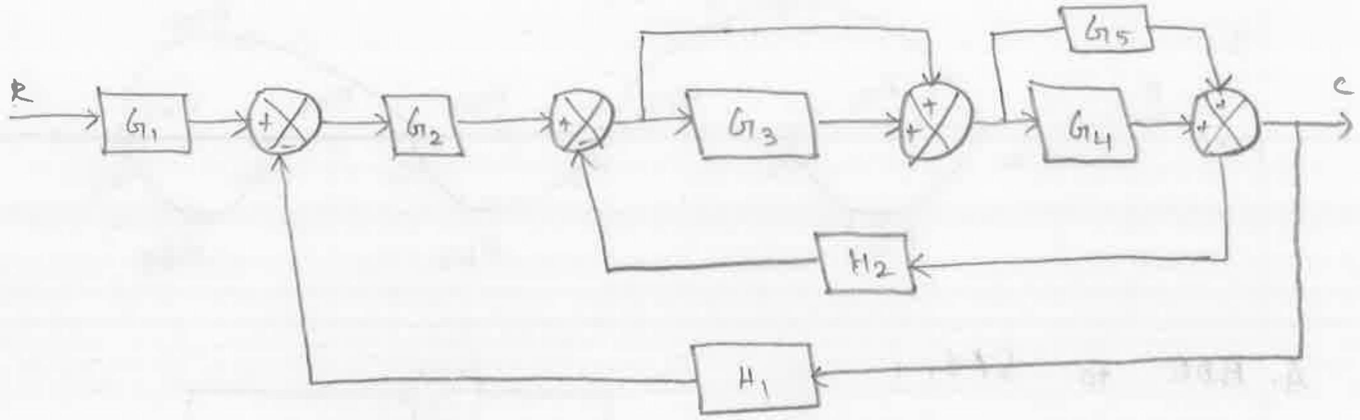


Tutorial on BDR to SFG :

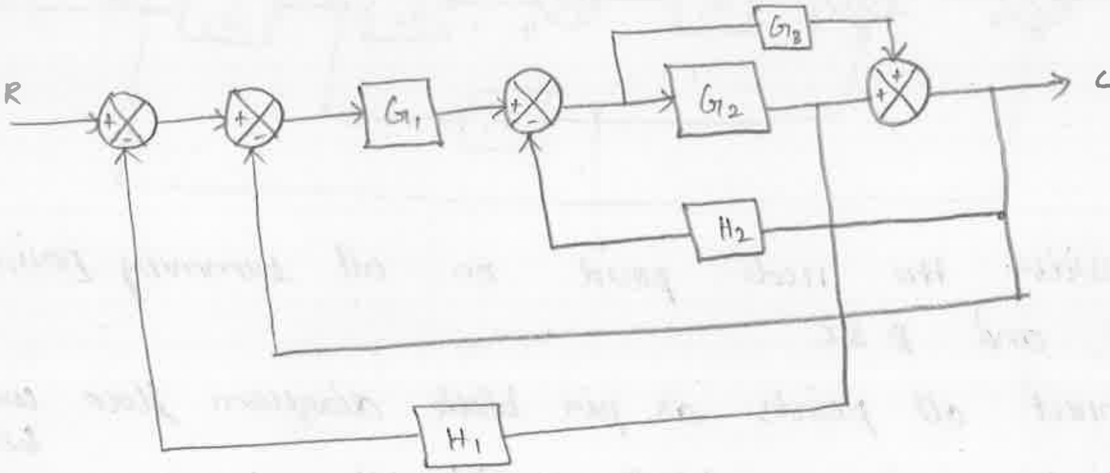
Find the transfer function of the block diagram using BDR and also verify with SFG by converting it.



2.



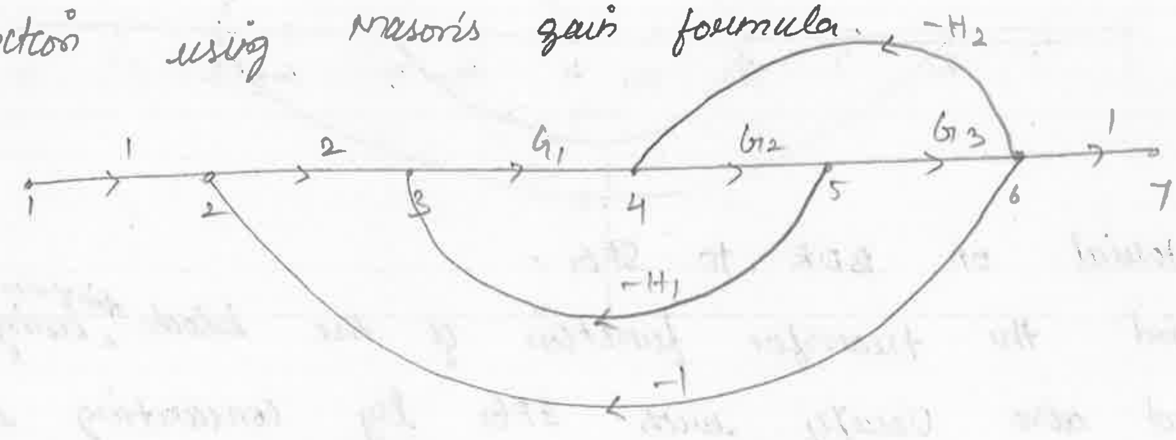
3.



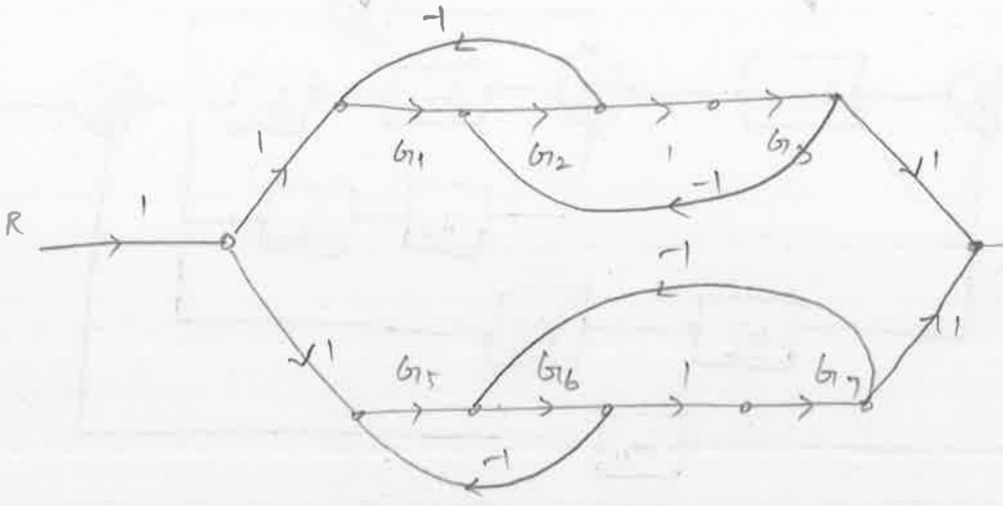
3. Signal Flow Graph:

consider the system shown below, obtain the transfer function using Mason's gain formula.

1.



2.

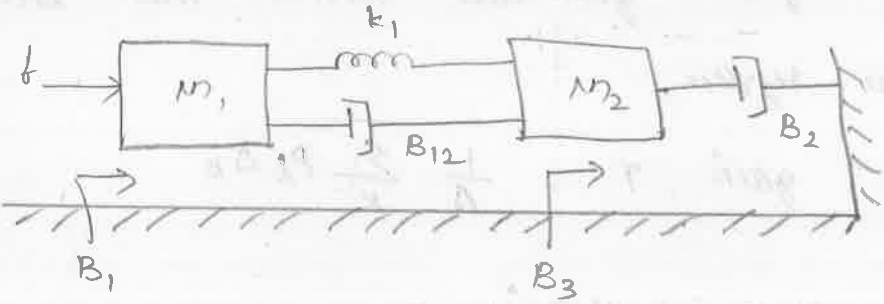


16 MARKS :

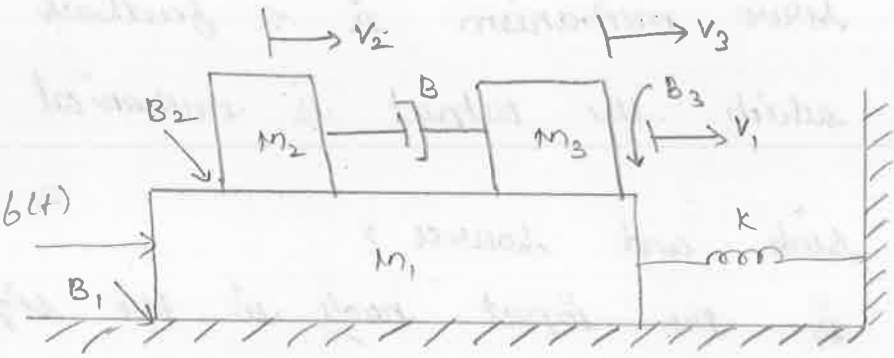
1. MTS and MRS

Find the differential equations and output transfer function for the following MTS and MRS. And also draw in corresponding F-V, F-I and T-V, T-I analogous circuit.

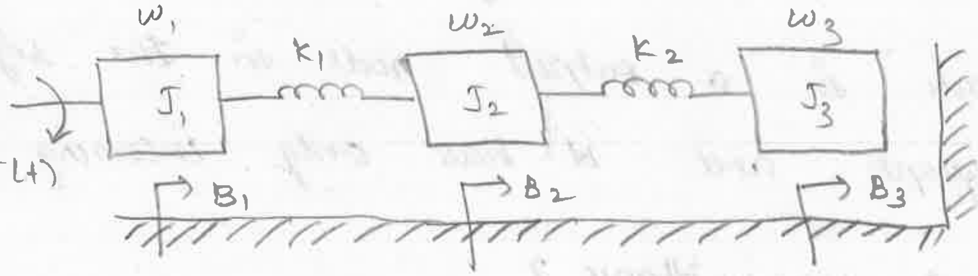
1.



2.



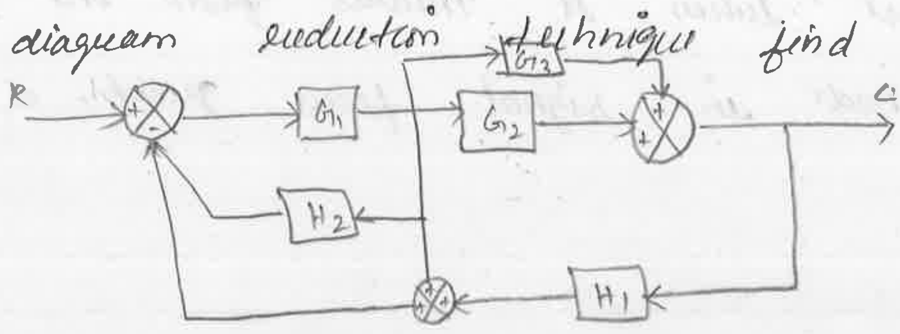
3.



2. BDR :

consider the block diagram shown below, using block diagram reduction technique find C/R

1)



21. what is signal flow graph?

A signal flow graph is a diagram that represents a set of simultaneous linear algebric equations.

22. write the mason's gain formula.

Mason's gain formula states that the overall gain of the system.

Overall gain, $T = \frac{1}{\Delta} \sum_k P_k \Delta_k$

23. What is servo mechanism?

The servo mechanism is a feedback control system in which the output is mechanical position.

24. what is sink and source?

Source is the input node in the signal flow graph and it has only outgoing branches.

Sink is a output node in the signal flow graph and it has only incoming branches.

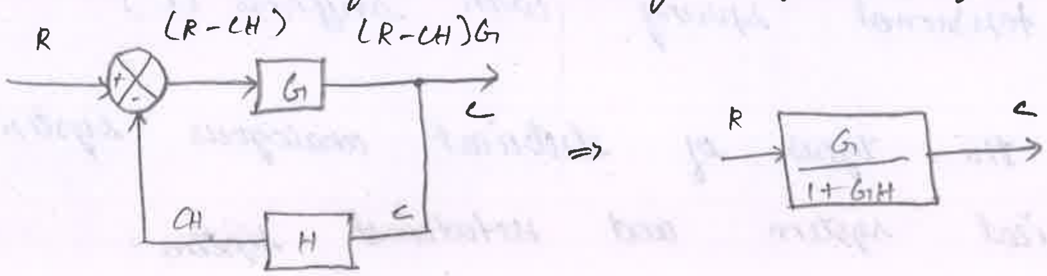
25. what is transmittance?

The transmittance is the gain acquired by the signal when it travels from one node to another node in signal flow graph.

18. What is the basis for framing the rules of block diagram reduction technique?

The rules for block diagram reduction technique are framed such that any modification made on the diagram does not alter the input output relation.

19. Write the rule for eliminating negative feedback loop.



$$(R - CH)G = C$$

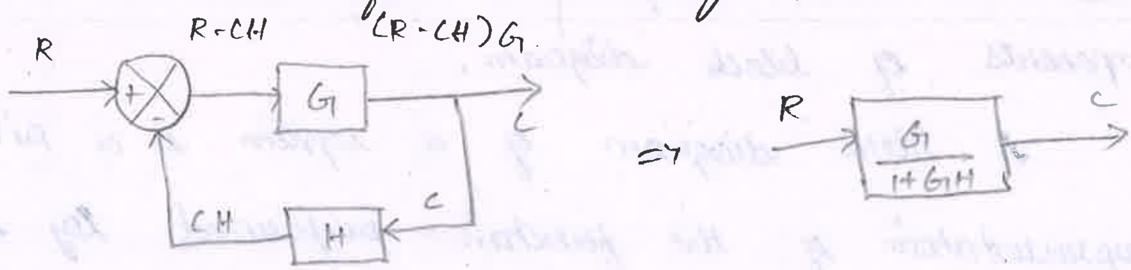
$$GR - CGH = C$$

$$GR = C + CGH$$

$$GR = C(1 + GH)$$

$$\boxed{\frac{C}{R} = \frac{G}{1+GH}}$$

20. Write the rule for eliminating positive feedback loop.



$$(R + CH)G = C$$

$$GR + CGH = C$$

$$GR = C - CGH$$

$$GR = C(1 - GH)$$

$$\boxed{\frac{C}{R} = \frac{G}{1-GH}}$$

15. What are the basic elements used for modelling mechanical rotational system?

The model of mechanical rotational system can be obtained using three basic elements

* mass with moment of Inertia (J)
* Dash pot with rotational frictional coefficient (B)
* torsional spring with stiffness (K)

16. Name the types of electrical analogous system for mechanical system and rotational system.

In mechanical translational system

* Force Voltage analogous system
* Force current analogous system

In mechanical rotational system

* Torque Voltage analogous system
* Torque current analogous system.

17. What is block diagram? What are the basic components of block diagram.

A block diagram of a system is a pictorial representation of the function performed by each component of the system and shows the flow of signals. The basic elements of block diagram are

* block
* branch point
* Summing point.

12. State the principle of superposition / homogenity.

The principle of superposition and homogenity states that if the system has responses $c_1(t)$ and $c_2(t)$ for the inputs $r_1(t)$ and $r_2(t)$ respectively then the system response to the linear combination of these input $a_1 r_1(t) + a_2 r_2(t)$ is given by linear combination of the individual outputs $a_1 c_1(t) + a_2 c_2(t)$, where a_1 and a_2 are constants.

13. Define Transfer function.

The Transfer function of a system is defined as the ratio of Laplace transform of output to Laplace transform of input with zero initial conditions.

$$T.F = \left. \frac{\text{Laplace Transform of output}}{\text{Laplace Transform of input}} \right| \text{with zero initial conditions.}$$

14. What are the basic elements used for modeling mechanical translational system.

The model of mechanical translational system can be obtained by using three basic elements

* mass
* spring
* Dashpot.

It also has low sensitivity to parameter variations. Hence negative feedback is preferred in closed loop system.

9. What are the characteristics of negative feedback?
   i) accuracy in tracking steady state value.
   ii) rejection of disturbance signals.
   iii) low sensitivity to parameter variations.
   iv) reduction in gain at the expense of better stability.

10. What is the effect of positive feedback on stability?
The positive feedback increases the error signal and drives the output to instability. But sometimes the positive feedback is used in minor loops in control systems to amplify certain internal signals or parameters.

11. Distinguish b/w open loop and closed loop system

open loop	closed loop
1. Inaccurate & unreliable	1. Accurate & reliable
2. Simple and economical	2. complex and costly
3. Changes in o/p due to external disturbances are not corrected automatically.	3. Changes in output due to external disturbances are corrected automatically.
4. They are generally stable	4. Great efforts are needed to design a stable system.

$$= \frac{ks+b}{s^2+(a-k)s} = \frac{ks+b}{s[s+(a-k)]}$$

Velocity error constant, $K_v = \underset{s\to 0}{Lt}\ s\,G(s)\,H(s)$

$$= \underset{s\to 0}{Lt}\ s\,G(s)$$

$$= \underset{s\to 0}{Lt}\ s\,\frac{ks+b}{s[s+(a-k)]}$$

$$K_v = \frac{b}{a-k}$$

with velocity input,

steady state error, $e_{ss} = \frac{1}{K_v} = \frac{a-k}{b}$

Problem 5:

A unity feedback system has the forward T.F

$G(s) = \frac{K_1(2s+1)}{s(5s+1)(1+s^2)}$. when the input $r(t) = 1+6t$

determine the minimum value of K_1 so that the steady state error is less than 0.1.

Sol:

Given that, input, $r(t) = 1+6t$

on taking L.T of $r(t)$ → $R(s)$

$$R(s) = L\{r(t)\} = L\{1+6t\} = \frac{1}{s}+\frac{6}{s^2}$$

The error signal in s-domain $E(s)$ is given by

$$E(s) = \frac{R(s)}{1+G(s)H(s)} = \frac{\frac{1}{s}+\frac{6}{s^2}}{1+\frac{k_1(2s+1)}{s(5s+1)(1+s)^2}}$$

$$= \frac{\frac{1}{s} + \frac{6}{s^2}}{\dfrac{s(5s+1)(1+s)^2 + K_1(2s+1)}{s(5s+1)(1+s)^2}}$$

$$= \frac{1}{s}\left[\frac{s(5s+1)(1+s)^2}{s(5s+1)(1+s)^2 + K_1(2s+1)}\right] + \frac{6}{s^2}\left[\frac{s(5s+1)(1+s)^2}{s(5s+1)(1+s)^2 + K_1(2s+1)}\right]$$

The steady state error, ess can be obtained from final value theorem.

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s\,E(s)$$

$$= \lim_{s \to 0} s\left\{\frac{1}{s}\left[\frac{s(5s+1)(1+s)^2}{s(5s+1)(1+s)^2 + K_1(2s+1)}\right] + \frac{6}{s^2}\left[\frac{s(5s+1)(1+s)^2}{s(5s+1)(1+s)^2 + K_1(2s+1)}\right]\right\}$$

$$= \lim_{s \to 0}\left\{\frac{s(5s+1)(1+s)^2}{s(5s+1)(1+s)^2 + K_1(2s+1)} + \frac{6(5s+1)(1+s)^2}{s(5s+1)(1+s)^2 + K_1(2s+1)}\right\}$$

$$= 0 + \frac{6}{K_1} = \frac{6}{K_1}$$

Given that, $e_{ss} < 0.1$,

$$0.1 = \frac{6}{K_1}$$

or

$$K_1 = \frac{6}{0.1}$$

$$K_1 = 60$$

Result:

For steady state error, $e_{ss} < 0.1$, the value of K_1 should be greater than 60.

CONTROLLERS :

A controller is a device introduced in the system to modify the error signal and to produce a control signal. The manner in which the controller produces the control signal is called control action. The controller modifies the transient response of the system. The electronic controllers using operational amplifiers are presented in this section.

The following six basic control actions are very common among industrial analog controllers.

1. Two position or ON - OFF control action
2. Proportional control action
3. Integral control action
4. Proportional - plus - integral control action
5. Proportional - plus - derivative control action
6. Proportional - plus integral plus derivative control action.

Depending on the control action provided the controllers can be classified by same as follows.

1. Two position or ON - OFF controllers
2. proportional controller action
3. Integral controller action
4. Proportional - plus - integral controller action
5. proportional - plus - derivative controller
6. proportional - plus integral plus derivative controller.

RESPONSE WITH P, PI, PD AND PID CONTROLLERS :

In feedback control system a controller may be introduced to modify the error signal and to achieve better control action. The introduction of controllers will modify the transient response and the steady state error of the system. The effects due to introduction of P, PI, PD and PID controllers are discussed in this section.

P - Controller : (PROPORTIONAL CONTROLLER).

P - controller is a device that produces a control signal, $u(t)$ proportional to the i/p error signal, $e(t)$

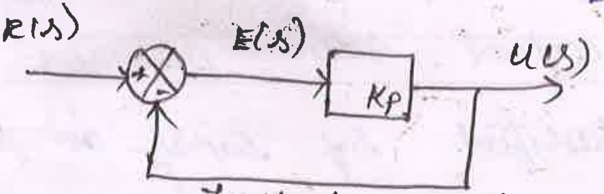
$$u(t) \propto e(t)$$

$$u(t) = K_p\, e(t)$$

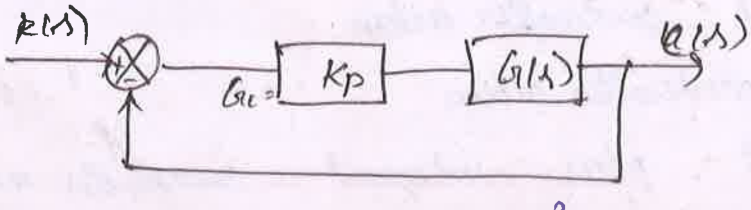
where K_p - proportional gain or constant.

L.T $\quad U(s) = K_p E(s)$

T.F of P- controller, $\dfrac{U(s)}{E(s)} = K_p$



Feedback signal.



OLTF. $G(s) = \dfrac{\omega_n^2}{s(s+2\zeta\omega_n)}$

$$LTF = G_c(s)\, G(s) H(s) = G_c(s)\, G(s)$$

$$= K_p \times \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$$

$$G(s) = \frac{K_p \omega_n^2}{s(s + 2\zeta\omega_n)}$$

$$H(s) = 1$$

$$\frac{C(s)}{R(s)} = \frac{\dfrac{\omega_n^2 K_p}{s(s + 2\zeta\omega_n)}}{1 + \dfrac{\omega_n^2 K_p}{s(s+2\zeta\omega_n)}} \qquad \frac{\dfrac{\omega_n^2 K_p}{s(s+2\zeta\omega_n)}}{\dfrac{s(s+2\zeta\omega_n) + \omega_n^2 K_p}{s(s+2\zeta\omega_n)}}$$

$$= \frac{\omega_n^2 K_p}{s^2 + 2\zeta\omega_n s + \omega_n^2 K_p}$$

Advantages :

* Improves steady state accuracy ↑
* Disturbance signal rejection
* Relative stability of system
* Increases the open loop gain ↑ of system but it reduces the ↓ sensitivity of the system.

Disadvantages :

* It produces constant steady state error.

PI controller (Proportional Integral controller) :

PI controller produces an o/p signal consisting of two terms : one proportional to error signal & other proportional to integral of error signal

$$u(t) \propto \left[e(t) + \int e(t)\, dt \right]$$

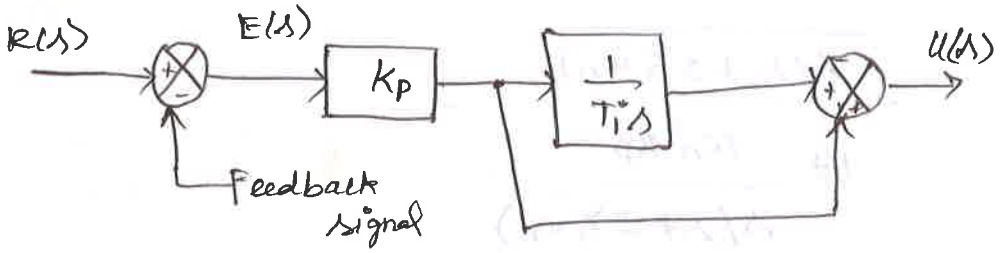
$$u(t) = K_p\, e(t) + \frac{K_p}{T_i} \int e(t)\, dt.$$

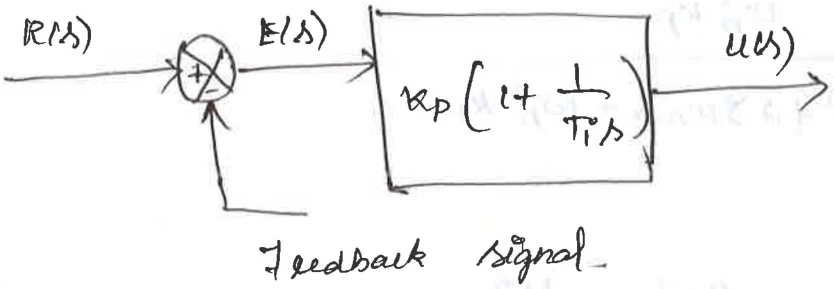
L.T $\quad U(s) = K_p E(s) + \frac{K_p}{T_i} \frac{E(s)}{s}$

K_p = proportional gain

T_i - Integral time

T.F of PI controller, $\frac{U(s)}{E(s)} = K_p + \frac{K_p}{T_i s} = K_p\left[1 + \frac{1}{sT_i}\right]$

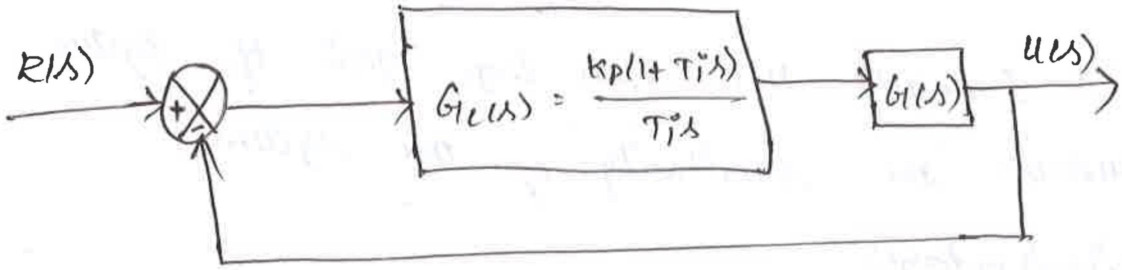




$e(t) + \int e(t)\, dt$

$K_p e(t) + \frac{K_p}{T_i}$

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s}\right) = K_p\left(\frac{T_i s + 1}{T_i s}\right)$$



$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s}$$

$$LTF = G_c(s)\, G(s)$$

$$= K_p\left(\frac{T_i s + 1}{T_i s}\right)\left(\frac{\omega_n^2}{s(s + 2\zeta\omega_n)}\right)$$

$$= \frac{K_p \omega_n^2 (1 + T_i s)}{s^2 T_i (s + 2\zeta\omega_n)}$$

CLTF, $$\frac{C(s)}{R(s)} = \frac{G_c(s)\, G(s)}{1 + G_c(s)\, G(s)}$$

$$= \frac{K_p \omega_n^2 (1+T_i s) \Big/ s^2 T_i (s + 2\zeta\omega_n)}{1 + \dfrac{K_p \omega_n^2 (1+T_i s)}{s^2 T_i (s+2\zeta\omega_n)}}$$

$$= \frac{K_p \omega_n^2 (1+T_i s)}{s^2 T_i (s + 2\zeta\omega_n) + K_p \omega_n^2 (1+T_i s)}$$

$$= \frac{K_p \omega_n^2 (1+T_i s)}{T_i s^2 + 2\zeta\omega_n T_i s^2 + K_p \omega_n^2 T_i s + K_p \omega_n^2}$$

$$= \frac{(K_p / T_i)\, \omega_n^2 (1+T_i s)}{s^2 + 2\zeta\omega_n s^2 + K_p \omega_n^2 + \dfrac{K_p}{T_i}\omega_n^2}$$

$$= \frac{K_i \omega_n^2 (1+T_i s)}{s^2 + 2\zeta\omega_n s^2 + K_p \omega_n^2 s + K_i \omega_n^2}$$

$$\boxed{K_i = \frac{K_p}{T_i}}$$

Advantages:

* Reduces steady state error
* Increases order and types of system

Disadvantages:

* Less stable than the original.

PD controller : (Proportional Derivative controller) :

PD controller produces an o/p signal consisting of two terms : 1) $\propto$ to error signal 2) $\propto$ to derivative of error signal.

$$u(t) \propto \left[e(t) + \frac{d}{dt} e(t) \right]$$

$$u(t) = K_p\, e(t) + K_p T_d \frac{d}{dt} e(t)$$

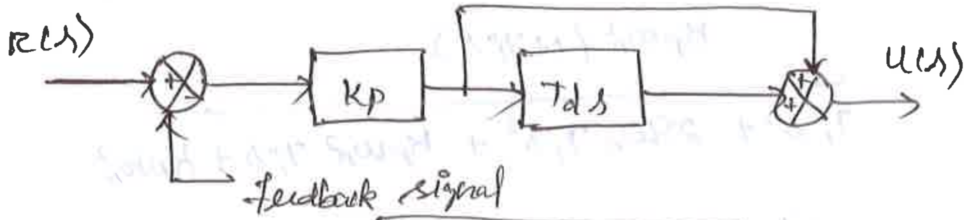
K_p : proportional gain

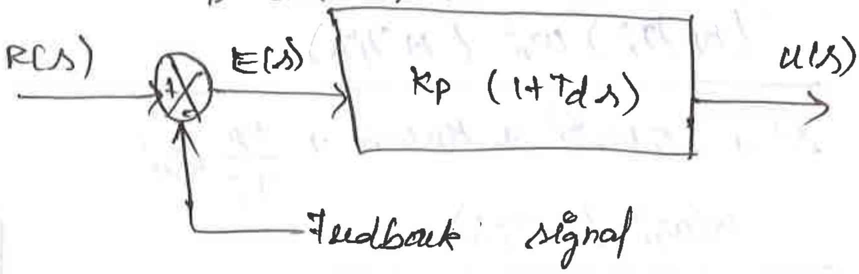
T_d : Derivative time.

L.T, $U(s) = K_p E(s) + K_p T_d s E(s)$

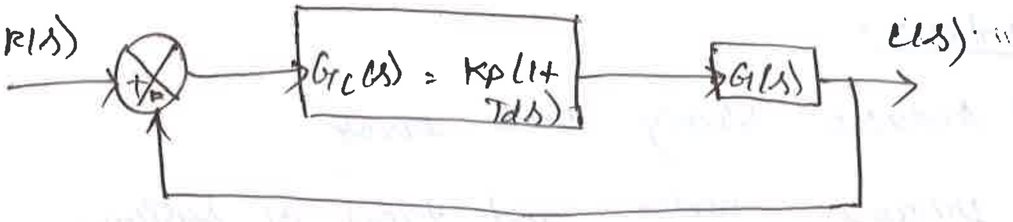
T.F, $\frac{U(s)}{E(s)} = K_p + K_p T_d s$

$$= K_p (1 + T_d s)$$





$G_c(s) = K_p (1 + T_d s)$



OLTF, $G(s) = \frac{\omega_n^2}{s(s + 2\xi\omega_n)}$

LTF, $= G_c(s)\, G(s)\, H(s) = G_c(s)\, G(s)$

$$= K_p(1+T_d s) \times \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

$$= \frac{K_p\omega_n^2(1+T_d s)}{s(s+2\xi\omega_n)}$$

CLT.F , $\frac{C(s)}{R(s)} = \frac{G_c(s)\,G(s)}{1+G_c(s)\,G(s)}$

$$= \frac{\frac{K_p\omega_n^2(1+T_d s)}{s(s+2\xi\omega_n)}}{1+\frac{K_p\omega_n^2(1+T_d s)}{s(s+2\xi\omega_n)}}$$

$$= \frac{K_p\omega_n^2(1+T_d s)}{s(s+2\xi\omega_n)+K_p\omega_n^2(1+T_d s)}$$

$$= \frac{K_p\omega_n^2(1+T_d s)}{s^2+2\xi\omega_n s+K_p\omega_n^2\cancel{T_d})+K_p\omega_n^2 T_d s}$$

$$= \frac{K_p\omega_n^2(1+T_d s)}{s^2+(2\xi\omega_n+K_p\omega_n^2 T_d)s+K_p\omega_n^2}$$

$$= \frac{\omega_n^2(K_p+K_d s)}{s^2+(2\xi\omega_n+K_d\omega_n^2)s+K_p\omega_n^2} \qquad \boxed{K_d = K_p T_d}$$

Advantages :

* Derivative control results in the addition of a zero,↑ so reduction of rise time↓

* Increase of damping ratio↑ so maximum overshoot↓ is reduced.

Disadvantages :

* Steady state error is not affected by derivative control

PID Controller : (Proportional Integral Derivative controller)

PID controller produces an o/p signal consisting of three terms, 1.) $\propto$ to error signal, 2.) $\propto$ to integral of error signal, 3.) $\propto$ to derivative of error signal.

$$u(t) \propto \left[e(t) + \int e(t)dt + \frac{d}{dt} e(t)\right]$$

$$u(t) = K_p e(t) + \frac{K_p}{T_i}\int e(t)\, dt + K_p T_d \frac{d}{dt} e(t)$$

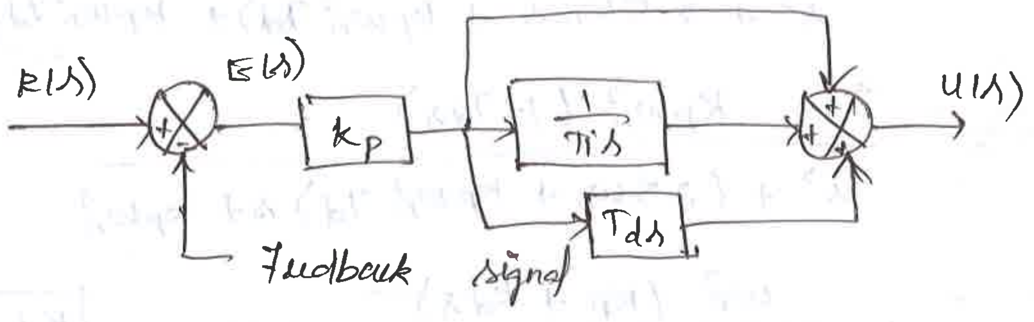
L.T

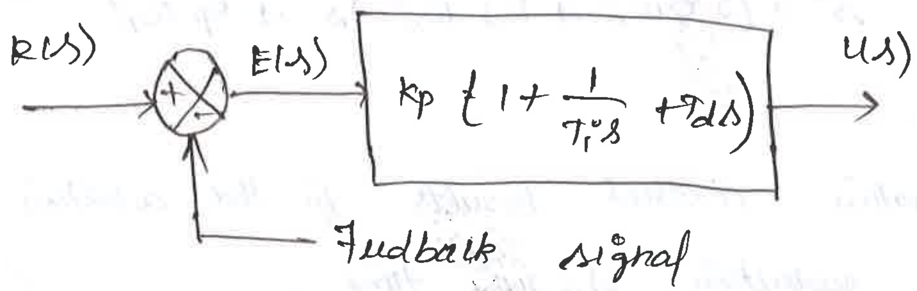
$$U(s) = K_p E(s) + \frac{K_p}{T_i}\frac{E(s)}{s} + K_p T_d\, s E(s)$$

T.F,

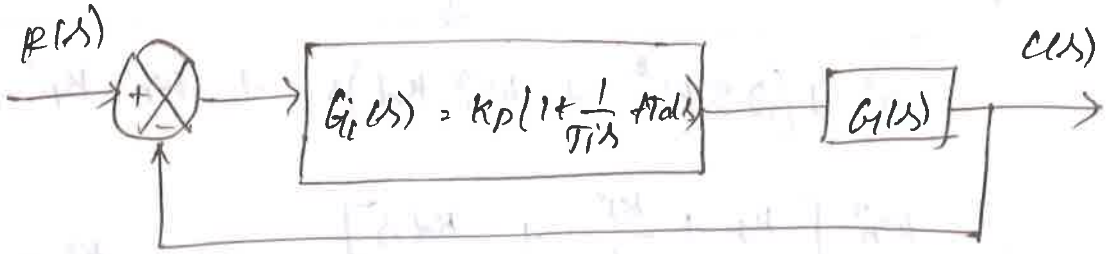
$$\frac{U(s)}{E(s)} = K_p + \frac{K_p}{T_i s} + K_p T_d s$$

$$\frac{U(s)}{E(s)} = K_p\left[1 + \frac{1}{T_i s} + T_d s\right]$$





$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$



OLTF, $G(s) = \dfrac{\omega_n^2}{s(s+2\zeta\omega_n)}$

Loop T.F, $= G_c(s)\,G(s)\,H(s) = G_c(s)\,G(s)$

$$= K_p\left(1+\frac{1}{T_i s}+T_d s\right) \times \frac{\omega_n^2}{s(s+\omega_n^2 2\zeta)}$$

$$= \frac{\omega_n^2 K_p\left(1+\frac{1}{T_i s}+T_d s\right)}{s(s+2\zeta\omega_n^2)}$$

CL T.F, $\dfrac{C(s)}{R(s)} = \dfrac{G_c(s)\,G(s)}{1+G_c(s)\,G(s)}$

$$= \frac{\dfrac{\omega_n^2 K_p\left(1+\frac{1}{T_i s}+T_d s\right)}{s(s+2\zeta\omega_n^2)}}{1+\dfrac{\omega_n^2 K_p\left(1+\frac{1}{T_i s}+T_d s\right)}{s(s+2\zeta\omega_n^2)}}$$

$$= \frac{\omega_n^2 K_p\left(1+\frac{1}{T_i s}+T_d s\right)}{s(s+2\zeta\omega_n^2)+\omega_n^2 K_p\left(1+\frac{1}{T_i s}+T_d s\right)}$$

$$= \frac{\omega_n^2 K_p + \frac{\omega_n^2 K_p}{T_i s} + \omega_n^2 K_p T_d s}{s^2+2\zeta\omega_n^2 s+\omega_n^2 K_p+\frac{\omega_n^2 K_p}{T_i s}+\omega_n^2 K_p T_d s}$$

$$= \frac{\omega_n^2 K_p + K_i \frac{\omega_n^2}{s} + \omega_n^2 K_d s}{s^2 + (2\zeta\omega_n + \omega_n^2 K_d)s + \omega_n^2 K_p + \frac{\omega_n^2 K_i}{s}}$$

$$= \frac{\omega_n^2 \left[K_p + \frac{K_i}{s} + K_d s\right]}{s^2 + \left\{\left[\frac{2\zeta}{\omega_n} + K_d\right]\omega_n^2 s + K_p \omega_n^2 + \frac{K_i}{s}\right\}\omega_n^2}$$

$K_i = \frac{K_p}{T_i}$

$K_d = K_p T_d$

$$= \frac{\omega_n^2 \left[K_p + \frac{K_i}{s} + K_d s\right]}{s^2 + (2\zeta\omega_n + \omega_n^2 K_d)s + \omega_n^2 K_p + \omega_n^2 \frac{K_i}{s}}$$

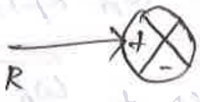
$$\frac{C(s)}{R(s)} = \frac{\omega_n^2 \left[K_p + \frac{K_i}{s} + K_d s\right]}{s^2 + \left[2\zeta\omega_n + \omega_n^2 K_d\right]s + \omega_n^2 K_p + \omega_n^2 \frac{K_i}{s}}$$

Advantages :

* Stability in gain
* Reduced steady state error ↓ and maximum overshoot ↑ of the system.

Problem 1 :

The following diagram shows unity feedback with the derivative control. By using this derivative control the damping ratio is to be made 0.5. Determine the value of T_d.



2 marks :

1. what is time response ?

The time response is the output of closed loop system as a function of time. It is denoted by c(t). It is given by inverse L.T of the product of input and transfer function of the system.

2. what is transient and steady state response ?

Transeint response is the response of the system when the input changes from one state to another.

The response of the system as ~~time~~ tends to infinity is called steady state response.

3. Name the test signal used in control system.

The commonly used test input signal in control system are,

1. Impulse
2. Step
3. Ramp
4. Parabolic
5. Sinusoidal.

14. Define peak time.

It is the time taken for the response to reach the peak value, the way first time (or). It is the time taken for the response to reach peak overshoot, Mp.

15. Define peak overshoot.

It is defined as the ratio of maximum peak value to final value, where maximum peak value is measured from final value.

16. Define settling time.

It is defined as the time taken by the response to reach and stay within a specified error and the error is usually specified as % of final value. The usual tolerable error is 2% or 5% of the final value.

17. What is type number of a system? What its significance.

The type number is given by number of poles of loop transfer function at the origin. The type number of the system decides the steady state error.

18. Distinguish between type and order of a system.

* Type number is specified for loop transfer function but order can be specified for any transfer function.

* The type number is given by no. of poles of loop transfer function lying at origin of s-plane but the order is given by the number of poles of Transfer function.

19. what is steady state error?

The steady state error is the value of error signal $e(t)$ when 't' tends to infinity. The steady state is a measure of system accuracy.

20. what are static error constant?

The K_p, K_v and K_a are called static error constants. These constants are associated with steady state error in a particular type of system and for a standard input.

21. Define positional error constant.

The positional error constant $K_p = \lim_{s \to 0} G(s) H(s)$

The steady state error in type 0 system when the input is unit step is given by $\frac{1}{1+K_p}$

22. Define velocity error constant.

The velocity error constant $K_v = \lim_{s \to 0} s \cdot G(s) H(s)$

the steady state error in type 1 system for unit ramp input is given by $\frac{1}{K_v}$

23. Define acceleration error constant.

The acceleration error constant $K_a = \lim_{s \to 0} s^2 G(s) H(s)$.

The steady state error in type 2 system, when the input is unit parabolic is given by $\frac{1}{K_a}$.

24. what are the generalized error coefficients? Give the relation between generalized and static error coefficients.

They are the coefficients of generalized error ~~static~~ series. The generalized error series is given by,

$$e(t) = C_0 r(t) + C_1 \dot{r}(t) + \frac{C_2}{2!} \ddot{r}(t) + \frac{C_3}{3!} \dddot{r}(t) + \dots + \frac{C_n}{n!} r^{n}(t).$$

The coefficients $C_0, C_1, C_2 \dots C_n$ are called generalized error coefficients or dynamic error coefficients.

The n^{th} coefficients, $C_n = \lim_{s \to 0} \frac{d^n}{ds^n} F(s)$

where, $F(s) = \frac{1}{1 + G(s) H(s)}$

* The following expressions shows the relation between generalized and static error coefficient.

$$C_0 = \frac{1}{1 + K_P} \; ; \quad C_1 = \frac{1}{K_v} \; ; \quad C_2 = \frac{1}{K_a}$$

25. what is automatic controller? what is its need and types?

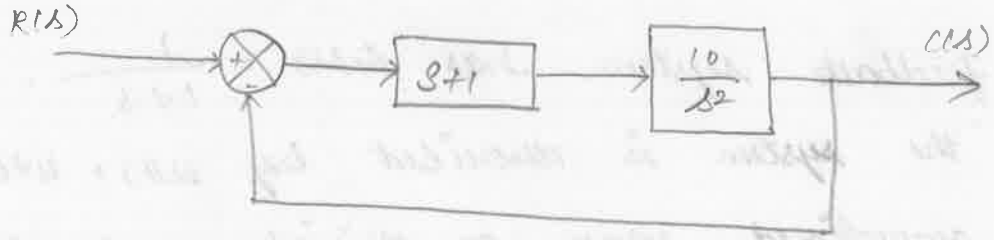
The combined unit of error detector, amplifier and controller is called automatic controller.

The controller is provided to modify the error signal for better control action.

The different types of controller used in control system are P, PI, PD and PID controllers.

16 Marks :

1. What is the unit step response of the system shown in fig.



2. The open loop transfer function of an unity feedback control system is given by $G(s) = \frac{100}{s(s+2)(s+5)}$. For unit step input, find the time response of the closed loop system and determine % overshoot and the rise time.

3. For a second order system whose open loop T.F $G(s) = \frac{4}{s(s+2)}$, Determine the maximum overshoot, the time to reach the maximum overshoot when a step displacement of 18' is given to the system. Find the rise time, ~~time~~ 1st constant and the settling time for an error of 7%.

4. Consider the unity feedback closed loop system where the forward transfer function is $G(s) = \frac{25}{s(s+5)}$. Obtain the rise time, peak time, maximum overshoot and the settling time when the system is subjected to a unit - step input.

5. For a system whose $G(s) = \frac{10}{s(s+1)(s+2)}$. Find the steady state error when it is subjected to the input, $r(t) = 1+2t+1.5t^2$

6. A unity feedback system has $G(s) = \frac{1}{1+s}$. The input to the system is described by $r(t) = 4+6t+2t^3$. Find the generalized error co-efficients and steady state error.

7. For unity feedback system having open loop transfer function as $G(s) = \frac{K(s+2)}{s^2(s^2+7s+12)}$. Determine
   i) type of system
   ii) error constants K_p, K_v and K_a
   iii) Steady state error for parabolic input.

8. The open loop transfer function of a unity feedback control system is $G(s) = \frac{9}{s+1}$, using the generalized error series determine the error signal and steady state error of the system, when the system is excited by (i) $r(t) = 1+2t+3t^2/2$ (ii) $r(t) = \frac{3t^2}{2}$

9. Derive the first order response of the unit step and unit ramp input.

10. Derive the second order response of the unit step input for all the four cases.
    i) Undamped
    ii) Under damped
    iii) critically damped
    iv) over damped.

UNIT - III FREQUENCY RESPONSE AND SYSTEM ANALYSIS

SYLLABUS :

Closed loop frequency response - Performance specification in frequency domain - Frequency response of standard second order system - Bode plot - Polar Plot - Nyquist plots - Design of compensators using Bode plots - cascade lead compensation - cascade lag compensation - cascade lag - lead compensation.

SINUSOIDAL TRASFER FUNCTION :

The response of a system for the sinusoidal input is called sinusoidal response. The ratio of sinusoidal response and sinusoidal input is called Sinusoidal transfer function of the system and in general, it is denoted by $T(j\omega)$. The sinusoidal transfer function is the frequency domain representation of the system, and so it is also called frequency domain transfer function.

The sinusoidal T.F ($T(j\omega)$ can be obtained as shown below.

1. construct a physical model of a system using basic elements / parameters.

2. Determine the diff. eq. governing the system from the physical model of the system.

3. Take L.T of differential equations in order to convert them to s-domain equations.

4. Determine s-domain transfer function, $T(s)$, which is ratio of s-domain output and input.

5. Determine the frequency domain transfer function, $T(j\omega)$ by replacing s by $j\omega$ in s-domain transfer function, $T(s)$.

$$\boxed{T(s) \xrightarrow{s = j\omega} T(j\omega)}$$

Consider a linear time invariant system with frequency domain transfer function, $T(j\omega)$ shown in fig. Let the system be excited by a sinusoidal signal frequency ω, amplitude A, and phase θ. Now the response or output will also be a sinusoidal signal of same frequency ω, but the amplitude and phase of response will be modified by amplitude and phase of the transfer function respectively.

Now, the amplitude of the response is given by the product of the amplitude of the input and transfer function. The phase of the response is given by the sum of the phase of the input and transfer function.

Let, $T(j\omega) = |T(j\omega)| \angle T(j\omega)$

where $|T(j\omega)|$ = Magnitude of $T(j\omega)$

$\angle T(j\omega)$ = phase of $T(j\omega)$

Let Input, $r(t) = A \sin(\omega t + \theta) = A \angle \theta$.

where A = Amplitude of input

ω = ~~frr~~ Frequency of input.

θ = phase of input.

Now, Response, $c(t) = r(t) \times T(j\omega)$

$$= A\angle\theta \times |T(j\omega)| \angle T(j\omega)$$

$$= A \times |T(j\omega)| \angle(\theta + \angle T(j\omega))$$

$$= B\angle\phi.$$

where, $B = A \times |T(j\omega)|$ = Magnitude of response

$\phi = \theta + \angle T(j\omega)$ = phase of response.

$r(t) = A \sin(\omega t + \theta) = A\angle\theta.$ $\quad c(t) = B\angle\phi$

$$r(t) \longrightarrow \boxed{T(j\omega) = |T(j\omega)| \angle T(j\omega)} \xrightarrow{c(t)}$$

where, $B = A \times |T(j\omega)|$

$\phi = \theta + \angle T(j\omega)$

System with sinusoidal T.F $T(j\omega)$

FREQUENCY RESPONSE :

The frequency domain transfer function $T(j\omega)$ is a complex function of ω. Hence it can be separated into magnitude function and phase function. Now, the magnitude and phase functions will be real functions of ω, and they are called frequency response.

The frequency response can be evaluated for open loop system and closed loop system. The frequency domain transfer function of open loop and closed loop systems can be obtained from the s-domain transfer function by replacing s by $j\omega$ shown below.

<u>open loop T.F</u> : $G(s) \xrightarrow{s = j\omega} G(j\omega) = |G(j\omega)| \angle G(j\omega)$

<u>Loop T.F</u> : $G(s)H(s) \xrightarrow{s = j\omega} G(j\omega)H(j\omega) = |G(j\omega)H(j\omega)| \angle G(j\omega)H(j\omega)$

closed loop T.F $M(s) \xrightarrow{s = j\omega} M(j\omega) = |M(j\omega)| \angle M(j\omega)$

where, $|G(j\omega)|$, $|H(j\omega)|$, $|G(j\omega)H(j\omega)|$ are magnitude fn.

$\angle G(j\omega)$, $\angle H(j\omega)$, $\angle G(j\omega)H(j\omega)$ are phase functions.

Unity feedback system, $H(s) = 1$, open loop & loop T.F are same.

Advantages :

1. The absolute and relative stability of the closed loop system can be estimated from the knowledge of their open loop frequency response.

2. The practical testing of systems can be easily carried with available sinusoidal signal generators and precise measurement equipments.

3. The T.F of complicated systems can be determined experimentally by frequency response tests.

4. The design and parameters adjustment of the open loop T.F of a system for specified closed loop performance is carried out more easily in frequency domain.

5. When the system is designed by use of the frequency response analysis, the effects of noise disturbance and parameters variations are relatively easy to visualize and incorporate corrective measures.

6. The frequency response analysis and designs can be extended to certain non-linear control systems.

FREQUENCY DOMAIN SPECIFICATIONS :

The performance and characteristics of a system in frequency domain are measured in terms of frequency domain specifications. the requirements of a system to be designed are usually specified in terms of these specifications.

The frequency domain specifications are,

1. Resonant peak, M_r.
2. Resonant Frequency, ω_r
3. Bandwidth, ω_b
4. cut-off rate.
5. Gain margin, K_g.
6. Phase margin, γ.

Resonant peak (M_r): $M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$

The maximum value of magnitude of closed loop T.F is called the resonant peak, M_r. A large resonant peak corresponds to a large overshoot in transient response.

Resonant Frequency (ω_r) $\omega_r = \omega_n\sqrt{1-2\xi^2}$

The frequency at which the resonant frequency peak occurs is called resonant frequency, ω_r. This is ~~called~~ related to frequency of oscillation in the step response and thus it is indicative of the speed of transient response.

Bandwidth (ω_b) : $\omega_b = \omega_n \left[1 - 2\zeta^2 + \sqrt{2 - 4\zeta^2 + 4\zeta^4}\right]^{1/2}$

The Bandwidth is the range of frequencies for which normalized gain of the system is more than -3 db. The frequency at which the gain is -3 db. is called cut-off frequencies. Bandwidth is usually defined for closed loop system and it transmits the signals whose frequencies are less than the cut-off frequency. The Bandwidth is a measure of the ability of a feedback system to reproduce the input signal, noise rejection characteristics and rise time. A large bandwidth corresponds to a small rise time or fast response.

Cut-off Rate :

The slope of the log-magnitude curve near the cut off frequency is called cut-off rate. The cut-off rate indicates the ability of the system to distinguish the signal from noise.

Gain margin, Kg : $K_g = \infty$

The gain margin, Kg is defined as the value of gain, to be added to system, in order to bring the system to the verge of instability.

The Gain margin, Kg is given by reciprocal of the magnitude of open loop T.F at phase cross over frequency. The frequency at which the phase of open loop T.F is 180° is called the phase cross-over frequency, ω_{pc}.

GCOF ⇒ Frq. Mag. of OLT.F is unity = 1

PCOF ⇒ Frq. phase of open L.T.F is $\underline{180^\circ}$

Gain margin, $K_g = \dfrac{1}{|G(j\omega_{pc})|}$

The gain margin in db can be expressed as

K_g in db $= 20 \log K_g = 20 \log \dfrac{1}{|G(j\omega)|}$

$|G(j\omega_{pc})|$ is magnitude of $G(j\omega)$ at $\omega = \omega_{pc}$.

The gain margin in db is given by the negative of the db magnitude of $G(j\omega)$ at phase cross-over frequency. The gain margin indicates the additional gain that can be provided to system without affecting the stability of system.

Phase margin (γ): $\gamma = 90 - \tan^{-1}\left[\dfrac{\left[-2\zeta^2 + \sqrt{4\zeta^4+1}\right]^{1/2}}{2\zeta}\right]$

The phase margin γ, is defined as the additional phase lag to be added at the gain cross over frequency in order to bring the system to the verge of instability. The gain cross over frequency ω_{gc} is the frequency at which the magnitude of open loop T.F is unity (or it is the frequency at which the db magnitude is zero).

The phase margin γ, is obtained by adding 180° to the phase angle ϕ of the open loop transfer function at the gain cross over frequency.

phase margin, $\gamma = 180^\circ + \phi_{gc}$.

where $\phi_{gc} = \angle G(j\omega_{gc})$

$\angle G(j\omega_{gc})$ is phase angle of $G(j\omega)$ at $\omega = \omega_{gc}$.

The phase margin indicates the additional phase lag that can be provided to the system without affecting stability.

FREQUENCY RESPONSE PLOTS :

Frequency response analysis of control systems can be carried either analytically or graphically. The various graphical techniques available for frequency response analysis are,

OL & CL

1. Bode plot
2. Polar plot (or Nyquist plot)
3. Nichols plot
4. M and N circles
5. Nichols chart.

Bode plot, polar plot and Nichols plot are usually drawn for open loop systems. From the open loop response plot, the performance and stability of closed loop system are estimated.

The M and N circles and Nichols chart are used to graphically determine the frequency response of unity feedback closed loop system from the knowledge of open loop response.

The frequency response plots are used to determine the frequency domain specifications, to study the stability of the systems and to adjust the gain of the system to satisfy the desired specifications.

BODE PLOT :

The Bode plot is a frequency response plot of the sinusoidal transfer function of a system. A Bode plot consists of two graphs. One is a plot of magnitude of a sinusoidal T.F versus log ω. the other is a plot of the phase angle of a sinusoidal T.F versus log ω.

the Bode plot can be drawn for both open loop and closed loop system. Usually the bode plot is drawn for open loop system. The standard representation of the logarithmic magnitude of open loop T.F of $G(j\omega)$ is $20 \log |G(j\omega)|$ where the base of the log is 10. The unit used in this representation of the magnitude is the decibel, usually abbreviated as db. the curves are drawn on semilog paper, using the log scale (abscissa) for frequency and the linear scale (ordinate) for either magnitude (in decibels) or phase angle (in degrees).

The main adv. of the bode plot is that multiplication of magnitudes can be converted into addition. Also a simple method for sketching an approximate log-magnitude curve is available.

Consider the open loop T.F, $G(s) = \dfrac{K(1+sT_1)}{s(1+sT_2)(1+sT_3)}$

$$G(j\omega) = \frac{K(1+j\omega T_1)}{j\omega(1+j\omega T_2)(1+j\omega T_3)}$$

$$= \frac{K\angle 0^\circ \sqrt{1+\omega^2 T_1^2}\ \angle \tan^{-1}\omega T_1}{\omega\angle 90^\circ \sqrt{1+\omega^2 T_2^2}\ \angle \tan^{-1}\omega T_2 \sqrt{1+\omega^2 T_3^2}\ \angle \tan^{-1}\omega T_3}$$

the magnitude of $G(j\omega) = |G(j\omega)| = \dfrac{K\sqrt{1+\omega^2 T_1^2}}{\omega\sqrt{1+\omega^2 T_2^2}\sqrt{1+\omega^2 T_3^2}}$

the phase angle of the $G(j\omega) = \angle G(j\omega)$

$$= \tan^{-1}\omega T_1 - 90^\circ - \tan^{-1}\omega T_2 - \tan^{-1}\omega T_3$$

the magnitude of $G(j\omega)$ can be expressed in decibels as shown below.

$$|G(j\omega)| \text{ in db} = 20\log|G(j\omega)|$$

$$= 20\log\left[\frac{K\sqrt{1+\omega^2 T_1^2}}{\omega\sqrt{1+\omega^2 T_2^2}\ \sqrt{1+\omega^2 T_3^2}}\right]$$

$$= 20\log\left[\frac{K}{\omega} \times \sqrt{1+\omega^2 T_1^2} \times \frac{1}{\sqrt{1+\omega^2 T_2^2}} \times \frac{1}{\sqrt{1+\omega^2 T_3^2}}\right]$$

$$= 20\log\frac{K}{\omega} + 20\log\sqrt{1+\omega^2 T_1^2} + 20\log\frac{1}{\sqrt{1+\omega^2 T_2^2}} + 20\log\frac{1}{\sqrt{1+\omega^2 T_3^2}}$$

$$= 20\log\frac{K}{\omega} + 20\log\sqrt{1+\omega^2 T_1^2} - 20\log\sqrt{1+\omega^2 T_2^2} - 20\log\sqrt{1+\omega^2 T_3^2}$$

From the above eq., it is clear that, when the magnitude is expressed in db, the multiplication is converted to addition. Hence in magnitude plot, the db magnitudes of individuals factor of $G(j\omega)$ can be added.

Problem 1 :

Sketch Bode plot for the following Transfer Function and determine the system gain K for the gain cross over frequency to be 5 rad/sec.

$$G(s) = \frac{Ks^2}{(1+0.2s)(1+0.02s)}$$

Sol :

The sinusoidal T-F $G(j\omega)$ is obtained by replacing s by $j\omega$ in the given s-domain T-F.

$$G(j\omega) = \frac{K(j\omega)^2}{(1+0.2j\omega)(1+0.02j\omega)}$$

Let, $K = 1$, $$G(j\omega) = \frac{(j\omega)^2}{(1+j0.2\omega)(1+j0.02\omega)}$$

Magnitude plot :

The corner frequencies are

$$\omega_{c1} = \frac{1}{0.2} = 5 \text{ rad/sec.}$$

$$\omega_{c2} = \frac{1}{0.02} = 50 \text{ rad/sec.}$$

The various terms of $G(j\omega)$ are listed in Table 1. in the increasing order of their corner frequency. Also the table shows the slope contributed by each term and the change in slope at the corner frequency.

Table - 1

Term	corner frequency rad / sec.	Slope db/ dec.	change in slope db / dec.
$(j\omega)^2$	-	+40	
$\frac{1}{1+j0.2\omega}$	$\omega_{c1} = \frac{1}{0.2} = 5$	-20	$40 - 20 = 20$
$\frac{1}{1+j0.02\omega}$	$\omega_{c2} = \frac{1}{0.02} = 50$	-20	$20 - 20 = 0$

Choose a low frequency ω_l such that $\omega_l < \omega_{c1}$ and choose a high frequency ω_h such that $\omega_h > \omega_{c2}$

Let, $\omega_l = 0.5$ rad / sec and $\omega_h = 100$ rad / sec.

Let, $A = |G(j\omega)|$ in db

Let us calculate A at ω_l, ω_{c1}, ω_{c2} and ω_h.

At $\omega = \omega_l$, $A = 20 \log |(j\omega)^2| = 20 \log (\omega)^2$

$= 20 \log (0.5)^2$

$= -12$ db a

At $\omega = \omega_{c1}$, $A = 20 \log |(j\omega)^2| = 20 \log (\omega)^2$

$= 20 \log (5)^2 = 28$ db. b

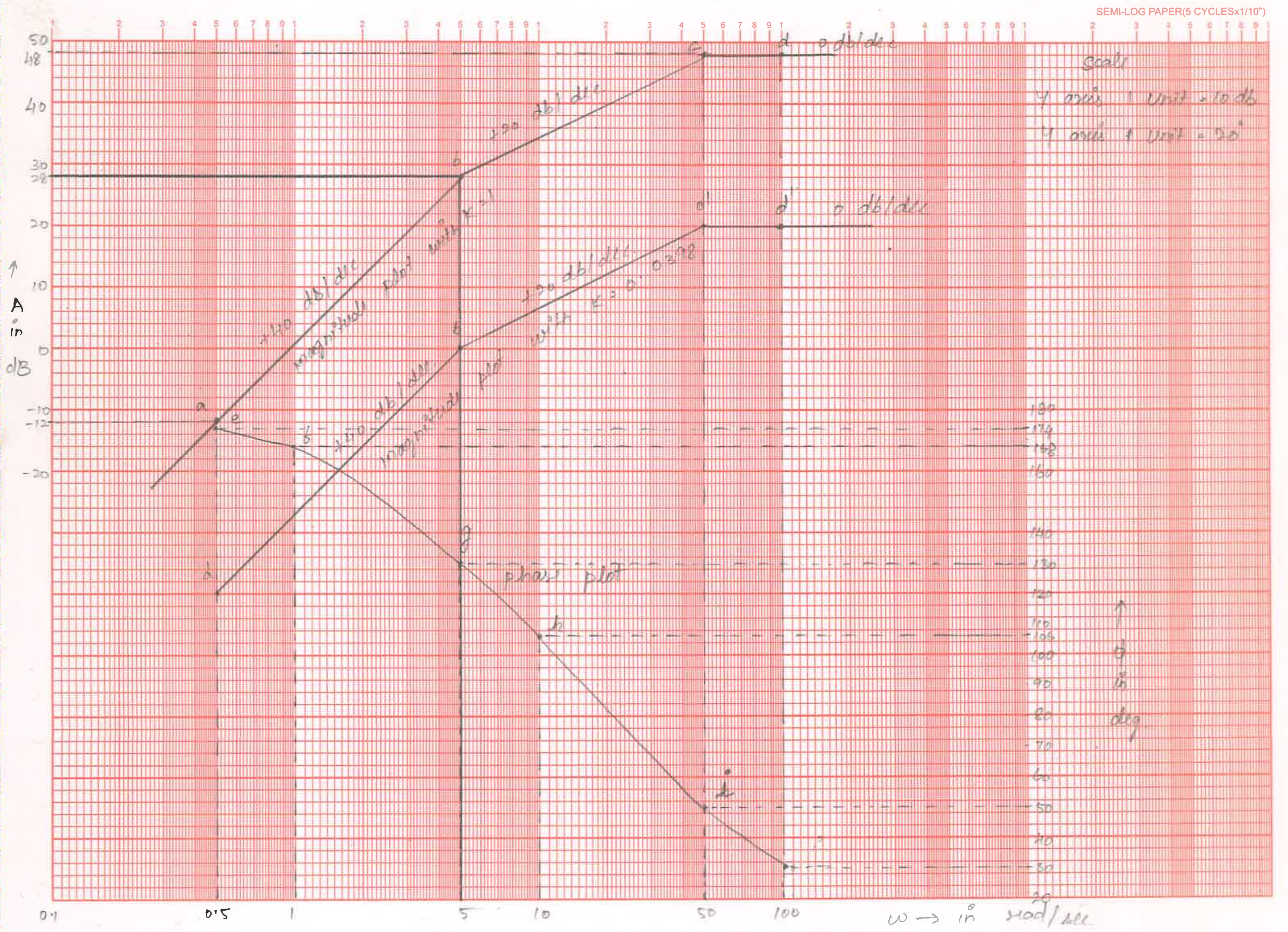
At, $\omega = \omega_{c2}$, $A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log \frac{\omega_{c2}}{\omega_{c1}}\right] + A_{(at\ \omega = \omega_{c1})}$

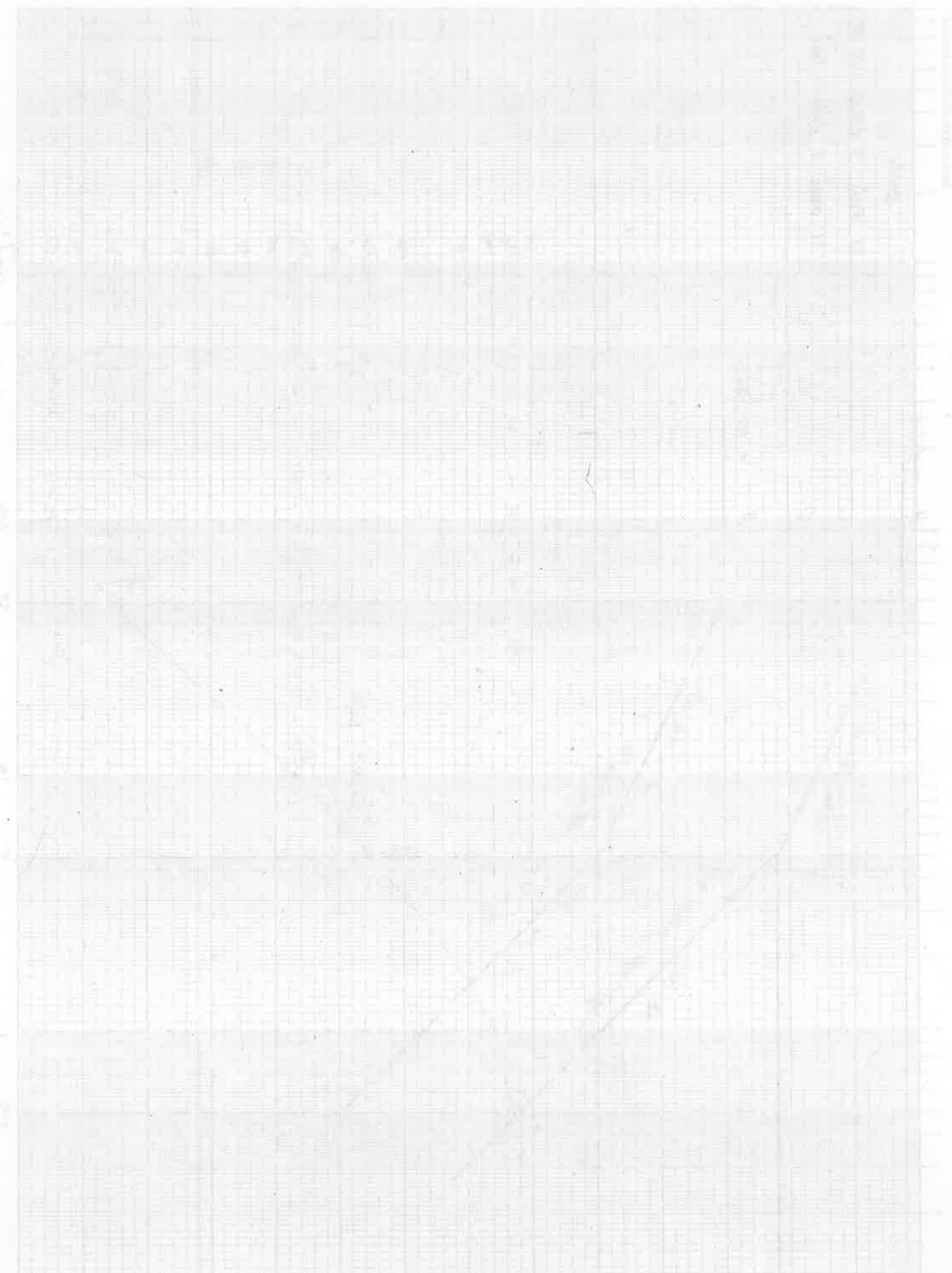
$= 20 \times \log \frac{50}{5} + 28 = 48$ db c

At, $\omega = \omega_h$, $A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log \frac{\omega_h}{\omega_{c2}}\right] + A_{(at\ \omega = \omega_{c2})}$

$= 0 \times \log \frac{100}{50} + 48 = 48$ db. d

1 Unit = 10 db on y-axis


SEMI-LOG PAPER(5 CYCLESx1/10")
Scale
Y axis 1 unit = 10 db
Y axis 1 unit = 20°
A in dB
φ in deg
ω → in rad/sec
+40 db/dec
+20 db/dec
0 db/dec
magnitude plot with K = 1
magnitude plot with K = 0.0398
phase plot
0.1
0.5
1
5
10
50
100
50
48
40
30
28
20
10
0
-10
-12
-20
180
174
168
160
140
130
120
110
106
100
90
80
70
60
50
40
30
20



Phase plot:

$$\phi = \angle G(j\omega) = 180^\circ - \tan^{-1} 0.2\omega - \tan^{-1} 0.02\omega$$

Table -2

ω rad/sec.	$\tan^{-1} 0.2\omega$ deg	$\tan^{-1} 0.02\omega$ deg	$\phi = \angle G(j\omega)$ deg	point in phase plot.
0.5	5.7	0.6	173.7 ≈ 174	e
1	11.3	1.1	167.6 ≈ 168	f
5	45	5.7	129.3 ≈ 130	g
10	63.4	11.3	105.3 ≈ 106	h
50	84.3	45	50.7 ≈ 50	i
100	87.1	63.4	29.5 ≈ 30	j

1 Unit = 20° on y-axis

Calculation of k

$$20 \log k = -28\,db$$

$$\log k = \frac{-28}{20}$$

$$k = 10^{-(28/20)}$$

$$k = 0.0398$$

the magnitude of plot with $k=1$ and 0.0398 and phase plot are shown.

Problem 2:

Sketch the bode plot for the following T.F and determine phase margin and gain margin

$$G(s) = \frac{75(1+0.2s)}{s(s^2+16s+100)}$$

Sol:

On comparing the quadratic factor in the denominator of G(s) with standard form of quadratic factor we can estimate ζ and ω_n.

$$s^2+16s+100 = s^2+2\zeta\omega_n s+\omega_n^2$$

on comparing we get,

$$\omega_n^2 = 100 \Rightarrow \omega_n = 10$$

$$2\zeta\omega_n = 16 \Rightarrow \zeta = \frac{16}{2\omega_n} = \frac{16}{2\times 10} = 0.8$$

Let us convert the given s-domain T.F into bode form or time constant form.

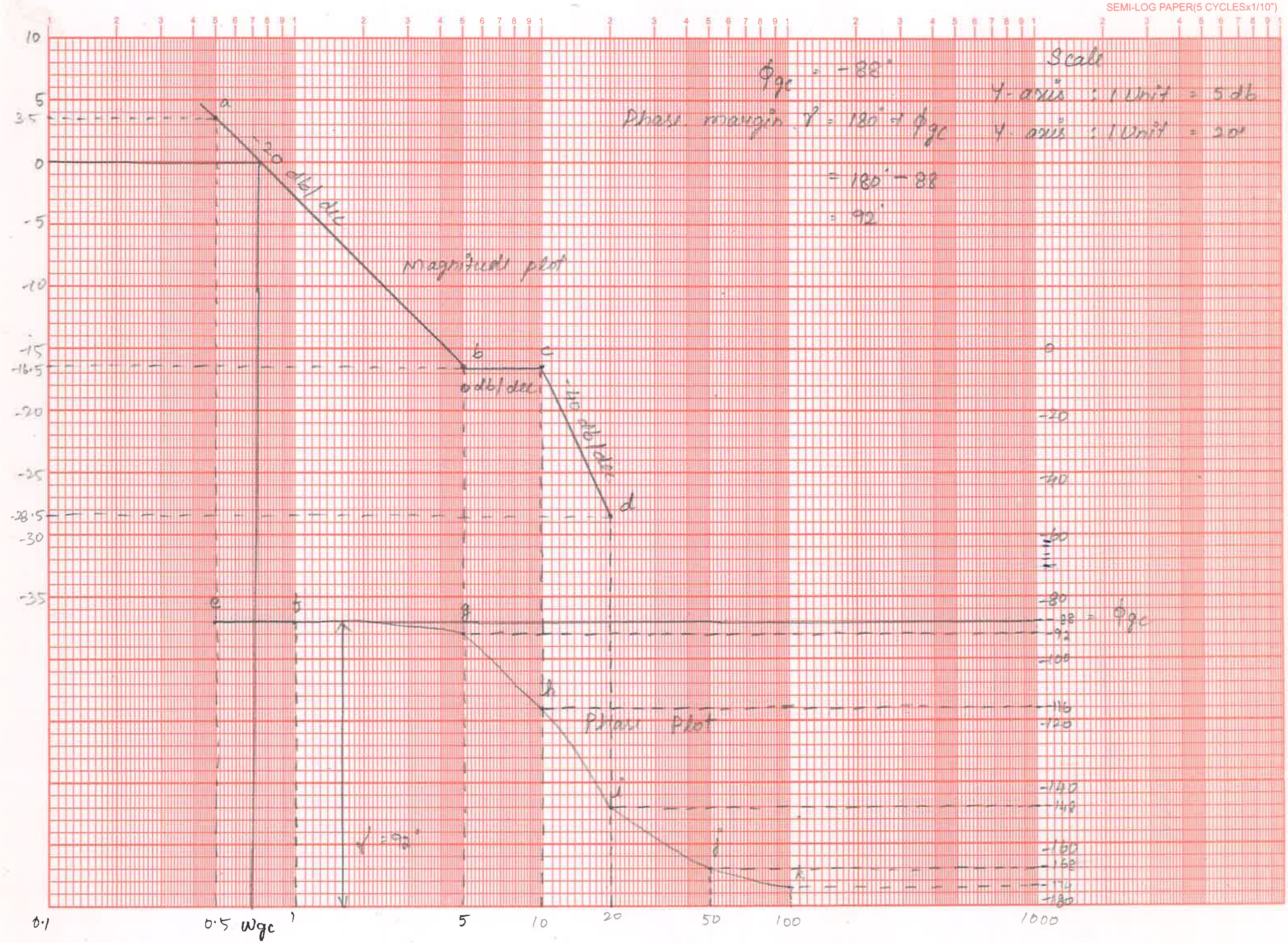
$$G(s) = \frac{75(1+0.2s)}{s(s^2+16s+100)} = \frac{75(1+0.2s)}{s\times 100\left(\frac{s^2}{100}+\frac{16s}{100}+1\right)}$$

$$= \frac{0.75(1+0.2s)}{s(1+0.01s^2+0.16s)}$$

The sinusoidal T.F $G(j\omega)$ is obtained by replacing s by $j\omega$ in G(s)

$$G(j\omega) = \frac{0.75(1+0.2j\omega)}{j\omega(1+0.01(j\omega)^2+0.16j\omega)}$$

$$= \frac{0.75(1+j0.2\omega)}{j\omega(1-0.01\omega^2+j0.16\omega)}$$


SEMI-LOG PAPER(5 CYCLESx1/10")
Scale
Y-axis : 1 Unit = 5 db
Y-axis : 1 Unit = 20°
φgc = −88°
Phase margin γ = 180° + φgc
= 180° − 88
= 92°
Magnitude plot
−20 db/dec
0 db/dec
−40 db/dec
Phase Plot
γ = 92°
φgc
0.1
0.5
Wgc
5
10
20
50
100
1000
10
5
3.5
0
−5
−10
−15
−16.5
−20
−25
−28.5
−30
−35
−20
−40
−60
−80
−88
−92
−100
−116
−120
−140
−148
−160
−168
−174
−180

magnitude plot :

for Quadratic factor the corner frequency is ω_n

The corner Frequency are

$$\omega_{c1} = \frac{1}{0.2} = 5 \text{ rad/sec.}$$

$$\omega_{c2} = \omega_n = 10 \text{ rad/sec}$$

The Various terms of $G(j\omega)$ are listed in table. in the increasing order of their corner frequencies. Also the table shows the slope contributed by each term and the change in slope at the corner frequency.

Table 1:

Term	corner frequency rad/sec.	Slope db/dec.	change in slope db/dec.
$\frac{0.75}{j\omega}$	—	-20	
$1+j0.2\omega$	$\omega_{c1} = \frac{1}{0.2} = 5$	20	$-20+20 = 0$
$\frac{1}{1+0.01\omega^2 + j0.16\omega}$	$\omega_{c2} = \omega_n = 10$	-40	$0-40 = -40$

choose a low frequency ω_l such that $\omega_l < \omega_{c1}$ and choose a high frequency ω_h such that $\omega_h > \omega_{c2}$

Let, $\omega_l = 0.5$ rad/sec ,

$\omega_h = 20$ rad/sec.

Let, $A = |G(j\omega)|$ in db.

Let us calulcte A at ω_1, ω_{c1}, ω_{c2} and ω_h

At $\omega = \omega_l$ (0.5), $A = 20 \log \left| \frac{0.75}{j\omega} \right| = 20 \log \frac{0.75}{0.5} = 3.5$ db   a

At, $\omega = \omega_{c1}$ (5), $A = 20 \log \left| \frac{0.75}{j\omega} \right| = 20 \log \frac{0.75}{5} = -16.5$ db   b

At, $\omega = \omega_{c2}$ (10), $A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log \frac{\omega_{c2}}{\omega_{c1}}\right] + A_{at\ (\omega = \omega_{c1})}$

$= 0 \times \log \frac{10}{5} + (-16.5) \Rightarrow -16.5$ db   c

At, $\omega = \omega_h$ (200), $A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log \frac{\omega_h}{\omega_{c2}}\right] + A_{at\ (\omega = \omega_{c2})}$

$= -40 \times \log \frac{20}{10} + (-16.5) = -28.5$ db   d.

In semi log sheet, 1 unit = 5 db on y-axis.

Frequencies decades from 0.1 to 100 rad/sec. on x-axis.

Phase plot:

the phase angle of $G(j\omega)$ as a function of ω is given by.

$\phi = \angle G(j\omega) = \tan^{-1} 0.2\omega - 90° - \tan^{-1} \frac{0.16\omega}{1 - 0.01\omega^2}$ for $\omega \leq \omega_n$

$\phi = \angle G(j\omega) = \tan^{-1} 0.2\omega - 90° - \left(\tan^{-1} \frac{0.16\omega}{1 - 0.01\omega^2} + 180°\right)$ for $\omega > \omega_n$

Table 2:

ω rad/sec	$\tan^{-1} 0.2\omega$ deg	$\tan^{-1} \frac{0.16\omega}{1 - 0.01\omega^2}$ deg	$\phi = \angle G(j\omega)$ deg	points in phase plot.
0.5	5.7	4.6	$-88.9 \approx -88$	e
1	11.3	9.2	$-87.9 \approx -88$	f
5	45	46.8	$-91.8 \approx -92$	g
10	63.4	90	$-116.6 \approx -116$	h
20	75.9	$-46.8 + 180 = 133.2$	$-147.3 \approx -148$	i
50	84.3	$-18.4 + 180 = 161.6$	$-167.3 \approx -168$	j
100	87.1	$-9.2 + 180 = 170.8$	$-173.7 \approx -174$	k.

1 Unit = 20° on y-axis.

Let ϕ_{gc} be the phase of $G(j\omega)$ at gain cross over frequency ω_{gc}

$\phi_{gc} = 88^\circ$

phase margin, $\gamma = 180^\circ + \phi_{gc} = 180^\circ - 88 = 92^\circ$

The phase plot crosses -180° only at infinity.

The $|G(j\omega)|$ at infinity is $-\infty$ db.

Hence gain margin is $-\infty$.

Problem 3:

Given, $G(s) = \dfrac{K e^{-0.2s}}{s(s+2)(s+8)}$. Find K so that the system is stable with, a) gain margin equal to 2 db

b) phase margin equal to 45°.

Sol:

Let us take $K = 1$

$$G(s) = \frac{e^{-0.2s}}{s(s+2)(s+8)} = \frac{e^{-0.2s}}{\{s \times 2(1+\frac{s}{2}) \times 8(1+\frac{s}{8})\}}$$

$$= \frac{0.0625\, e^{-0.2s}}{s(1+0.5s)(1+0.125s)}$$

The sinusoidal T.F $G(j\omega)$ is obtained by replacing s by $j\omega$ in $G(s)$.

$|0.0625\, e^{-j0.2\omega}| = 0.0625$

$\angle 0.0625\, e^{-j0.2\omega} = -0.2\omega$

$$G(j\omega) = \frac{0.0625\, e^{-j0.2\omega}}{j\omega(1+j0.5\omega)(1+j0.125\omega)}$$

Magnitude plot:

The corner frequencies are,

$\omega_{c1} = \dfrac{1}{0.5} = 2$ rad/sec.

$\omega_{c2} = \dfrac{1}{0.125} = 8$ rad/sec.

Table 1:

Term	corner frequency rad/sec	slope db/dec.	Change in slope db/dec.
$\frac{0.0625}{j\omega}$	–	-20	
$\frac{1}{1+j0.5\omega}$	$\omega_{c1} = \frac{1}{0.5} = 2$	-20	$-20-20 = -40$
$\frac{1}{1+j0.125\omega}$	$\omega_{c2} = \frac{1}{0.125} = 8$	-20	$-40-20 = -60$

Choose a low frequency ω_l such that $\omega_l < \omega_{c1}$ and choose a high frequency ω_h such that $\omega_h > \omega_{c2}$.

Let, $\omega_l = 0.5$ rad/sec.

$\omega_h = 50$ rad/sec

Let, $A = |G(j\omega)|$ in db

Let us calculate A at ω_l, ω_{c1}, ω_{c2} and ω_h

At $\omega = \omega_l$, $A = 20 \log \left| \frac{0.0625}{j\omega} \right| = 20 \log \frac{0.0625}{0.5} = -18$ db

At $\omega = \omega_{c1}$, $A = 20 \log \left| \frac{0.0625}{j\omega} \right| = 20 \log \frac{0.0625}{2} = -30$ db

At $\omega = \omega_{c2}$, $A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log \frac{\omega_{c2}}{\omega_{c1}} \right] + A_{(\text{at } \omega = \omega_{c1})}$

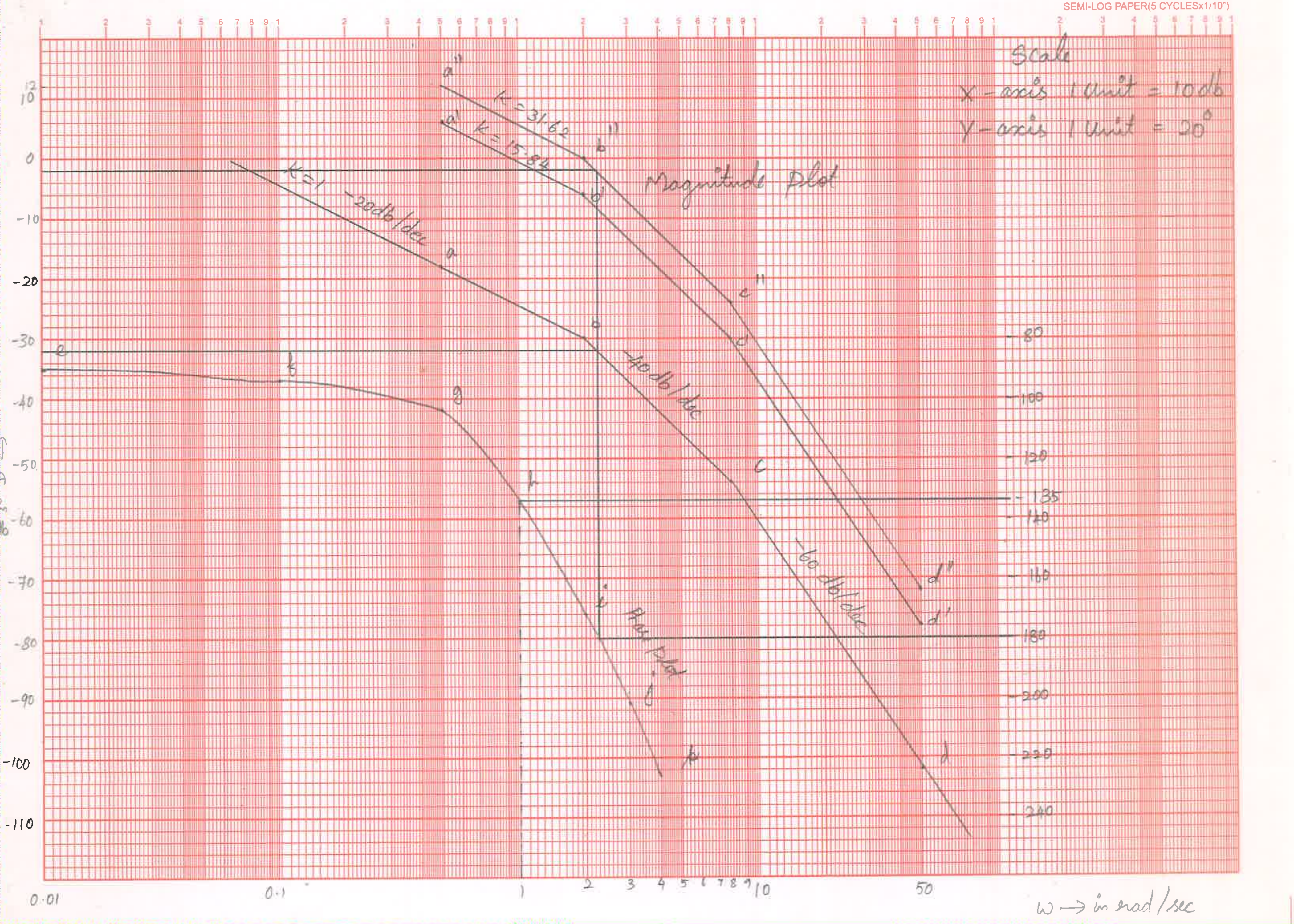
$= -40 \times \log \frac{8}{2} + (-30) = -54$ db

At $\omega = \omega_h$, $A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log \frac{\omega_h}{\omega_{c2}} \right] + A_{(\text{at } \omega = \omega_{c2})}$

$= -60 \times \log \frac{50}{8} + (-54) = -102$ db

1 unit = 10 db on y-axis

$\omega \rightarrow 0.01$ to 100 rad/sec $\Rightarrow$ x-axis.



SEMI-LOG PAPER(5 CYCLESx1/10")
Scale
X - axis 1 unit = 10 db
Y - axis 1 unit = 20°
Magnitude plot
K = 31.62
K = 15.84
K = 1
-20db/dec
-40db/dec
-60 db/dec
Phase plot
-80
-100
-120
-135
-140
-160
-180
-200
-220
-240
0.01
0.1
1
10
50
ω → in rad/sec

Phase plot:

$$\phi = -0.2\omega \times \frac{180^\circ}{\pi} - 90^\circ - \tan^{-1} 0.5\omega - \tan^{-1} 0.125\omega$$

Table 2:

ω rad/sec	$-0.2\omega(180^\circ/\pi)$ deg	$\tan^{-1} 0.5\omega$ deg	$\tan^{-1} 0.125\omega$ deg	$\phi = \angle G(j\omega)$ deg	point in phase plot
0.01	−0.1145	0.2864	0.0716	$-90.4 \approx -90$	e
0.1	−1.145	2.862	0.716	$-94.7 \approx -94$	f
0.5	−5.7	14	3.6	$-113.3 \approx -114$	g
1	−11.7	26	7.12	$-134.4 \approx -134$	h
2	−22.9	45	14	$-171.9 \approx -172$	i
3	−34.37	56.30	20.56	$-201.2 \approx -202$	j
4	−45.84	63.43	26.57	$-225.8 \approx -226$	k

1 unit = 20° on the y-axis.

Calculation of K:

phase margin, $\gamma = 180^\circ + \phi_{gc}$

where ϕ_{gc} is the phase of $G(j\omega)$ at $\omega = \omega_{gc}$

when $\gamma = 45^\circ$, $\phi_{gc} = \gamma - 180^\circ = 45^\circ - 180^\circ$
$= -135^\circ$

with $K = 1$, the db gain at $\phi = -135^\circ$ is -24 db

$20 \log K = 24$ $\quad$ 24 ↑.

$K = 10^{24/20}$

$$\boxed{K = 15.84}$$

with $K = 1$ Gain margin [illegible] = 30 db. $\quad$ 30 ↑

But required gain margin is 2 db or 30 db.

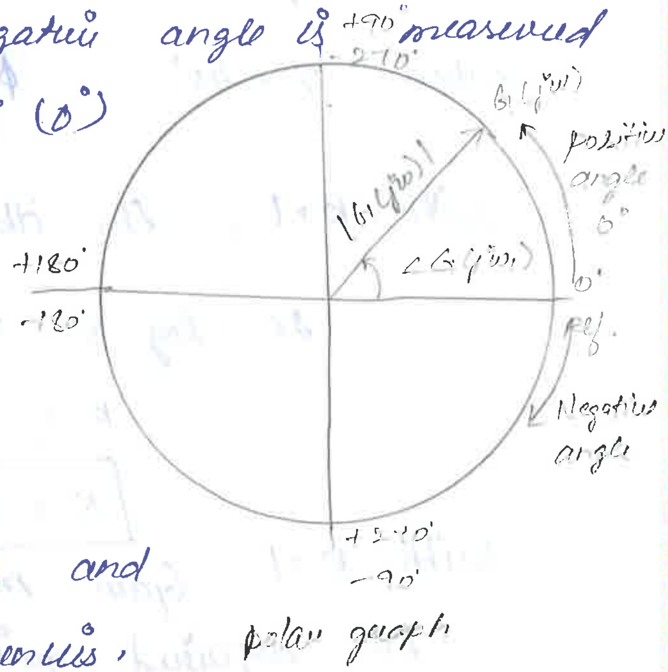
$20 \log K = 30$

$K = 10^{30/20}$ $\quad$ $\boxed{K = 31.62}$

POLAR PLOT: (NYQUIST PLOT).

The polar plot of a sinusoidal T.F $G(j\omega)$ is a plot of the magnitude of $G(j\omega)$ versus the phase angle of $G(j\omega)$ on polar co-ordinates as ω is varied from zero to infinity. Thus the polar plot is the locus of vectors $|G(j\omega)| \angle G(j\omega)$ as ω is varied from zero to infinity. The polar plot is also called Nyquist plot.

The polar plot is usually plotted on a polar graph sheet. The polar graph sheet has concentric circles and radial lines. The circles represent the magnitude and the radial lines represent the phase angles. Each point on the polar graph has a magnitude and phase angle. The magnitude of a point is given by the value of the circle passing through that point and the phase angle is given by the radial line passing through that point. In polar graph sheet a positive phase angle is measured in anticlockwise from the reference axis (0°) and a negative angle is measured clockwise from the ref. axis (0°)



In order to plot the polar plot, magnitude and phase of $G(j\omega)$ are computed for various values of ω and tabulated. Usually the choice of frequencies are corner frequencies and frequencies around corner frequencies. Choose proper scale for the magnitude circles.

Fix all the points on polar graph sheet and join the points by smooth curve. write the frequency corresponding to each point of the plot.

Alternatively, if $G(j\omega)$ can be expressed in rectangular co-ordinates as

$$G(j\omega) = G_R(j\omega) + jG_I(j\omega)$$

where, $G_R(j\omega)$ = Real part of $G(j\omega)$

$G_I(j\omega)$ = Imaginary part of $G(j\omega)$

then the polar plot can be plotted in ordinary graph sheet between $G_R(j\omega)$ and $G_I(j\omega)$ by varying ω from 0 to ∞. In order ~~the~~ to plot the polar plot on ordinary graph sheet, the magnitude and phase of $G(j\omega)$ are computed for various values of ω. Then convert the polar co-ordinates to rectangular co-ordinates using $P \rightarrow R$ conversion (polar to rectangular conversion) in the calculator. Sketch the polar plot using rectangular co-ordinates.

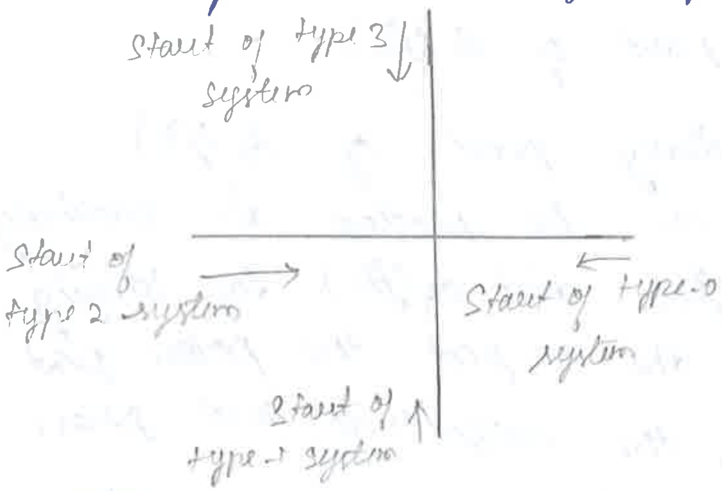
For minimum phase T.F with only poles, type no. of the system determine the quadrant at which the polar plot starts and the order of the system determine the quadrant at which the polar plot ends. The minimum phase systems are systems with all poles and zeros on left half of s-plane. The start and end of polar plot minimum phase system are shown below respectively.

Some typical sketches of polar plot are shown below.

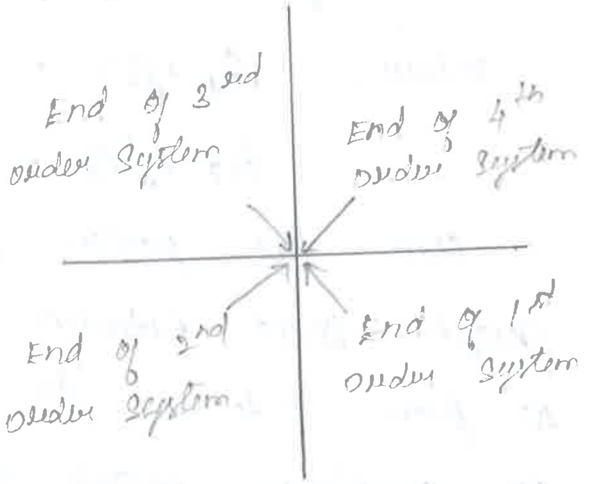
The change in shape of polar plot can be predicted due to addition of a pole or zero.

1. When a pole is added to a system, the polar plot end point will shift by $-90°$.

2. When a ~~pole~~ zero is added to a system, the polar plot end point will shift by $+90°$.



Start of polar plot of all pole minimum phase system.



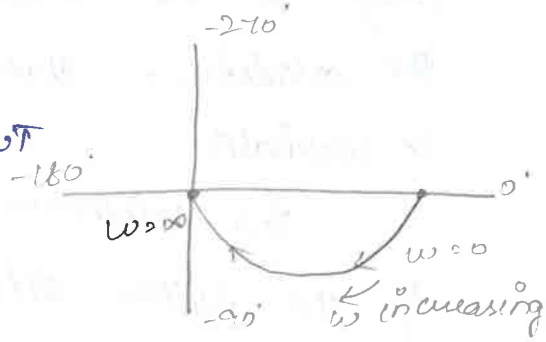
Start of polar plot of all pole minimum phase system

Type: 0, order: 1 $\quad G(s) = \dfrac{1}{1+sT}$

$$G(j\omega) = \frac{1}{1+j\omega T} = \frac{1}{\sqrt{1+\omega^2T^2}} \angle -\tan^{-1}\omega T$$

At $\omega \to 0 \quad G(j\omega) \to 1\angle 0°$

$\omega \to \infty \quad G(j\omega) \to 0\angle -90°$

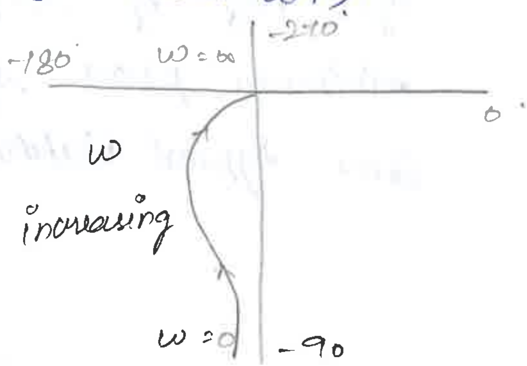


Type: 1, order: 2 $\quad G(s) = \dfrac{1}{s(1+sT)}$

$$G(j\omega) = \frac{1}{j\omega(1+j\omega T)} = \frac{1}{\omega\sqrt{1+\omega^2T^2}} \angle(-90° - \tan^{-1}\omega T)$$

At $\omega \to 0$, $G(j\omega) \to \infty\angle -90°$

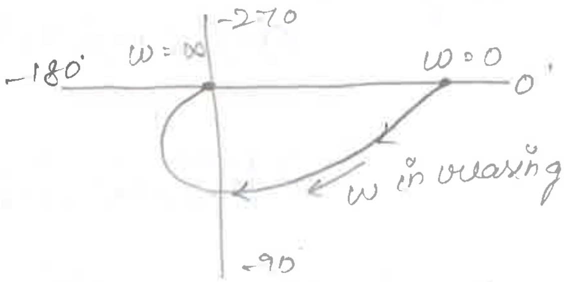
$\omega \to \infty$, $G(j\omega) \to 0\angle -180°$



Type : 0 , Order : 2 $\quad G(s) = \dfrac{1}{(1+sT_1)(1+sT_2)}$

$$G(j\omega) = \frac{1}{(1+j\omega T_1)(1+j\omega T_2)}$$

$$= \frac{1}{\sqrt{(1+\omega^2T_1^2)(1+\omega^2T_2^2)}} \angle(-\tan^{-1}\omega T_1 - \tan^{-1}\omega T_2)$$



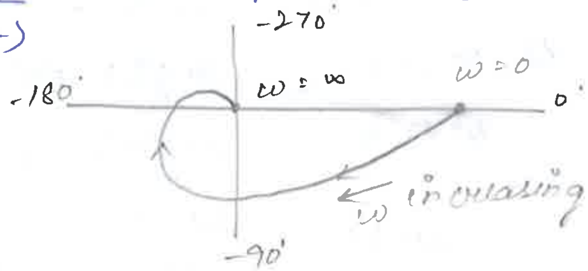
As $\omega \to 0$, $G(j\omega) \to 1\angle 0°$

$\omega \to \infty$ $\quad G(j\omega) \to 0\angle -180°$

Type : 0, Order : 3 $\quad G(s) = \dfrac{1}{(1+sT_1)(1+sT_2)(1+sT_3)}$

$$G(j\omega) = \frac{1}{(1+j\omega T_1)(1+j\omega T_2)(1+j\omega T_3)}$$

$$= \frac{1}{\sqrt{(1+\omega^2T_1^2)(1+\omega^2T_2^2)(1+\omega^2T_3^2)}} \angle(-\tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 - \tan^{-1}\omega T_3)$$



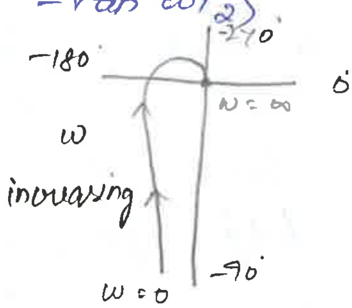
As $\omega \to 0$ $\quad G(j\omega) \to 1\angle 0°$

$\omega \to \infty$ $\quad G(j\omega) \to 0\angle -270°$

Type 1 : order 3. $\quad G(s) = \dfrac{1}{s(1+sT_1)(1+sT_2)}$

$$G(j\omega) = \frac{1}{j\omega(1+j\omega T_1)(1+j\omega T_2)}$$

$$= \frac{1}{\omega\sqrt{(1+\omega^2T_1^2)(1+\omega^2T_2^2)}} \angle(-90° - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2)$$



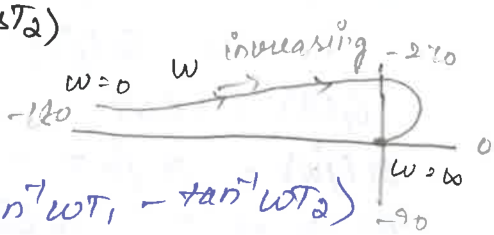
As $\omega \to 0$; $G(j\omega) \to \infty\angle -90°$

$\omega \to \infty$, $G(j\omega) \to 0\angle -270°$

Type 2 , order 4 : $\quad G(s) = \dfrac{1}{s^2(1+sT_1)(1+sT_2)}$

$$G(j\omega) = \frac{1}{(j\omega)^2(1+j\omega T_1)(1+j\omega T_2)}$$

$$= \frac{1}{\omega^2\sqrt{(1+\omega^2T_1^2)(1+\omega^2T_2^2)}} \angle(-180° - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2)$$



As $\omega \to 0$ $\quad G(j\omega) \to \infty \angle -180^\circ$

$\omega \to \infty$ $\quad G(j\omega) \to 0 \angle -360^\circ$

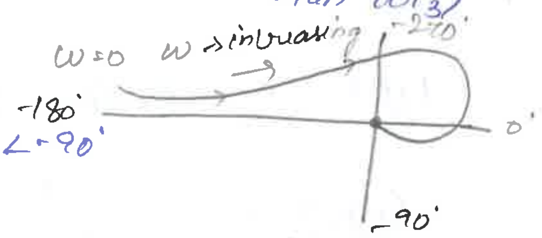
Type 2, order 5 : $G(s) = \dfrac{1}{s^2(1+sT_1)(1+sT_2)(1+sT_3)}$

$$G(j\omega) = \frac{1}{(j\omega)^2(1+j\omega T_1)(1+j\omega T_2)(1+j\omega T_3)}$$

$$= \frac{1}{\omega^2\sqrt{(1+\omega^2T_1^2)(1+\omega^2T_2^2)(1+\omega^2T_3^2)}} \angle(-180^\circ - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 - \tan^{-1}\omega T_3)$$

As $\omega \to 0$, $\quad G(j\omega) \to \infty \angle -180^\circ$

$\omega \to \infty$ $\quad G(j\omega) \to 0 \angle -450^\circ = 0 \angle -90^\circ$

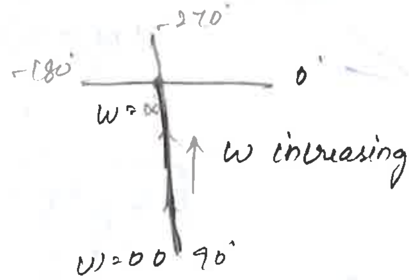


Type 1, Order : 1 $\quad G(s) = \dfrac{1}{s}$

$G(j\omega) = \dfrac{1}{j\omega} = \dfrac{1}{\omega\angle 90^\circ} = \dfrac{1}{\omega} \angle -90^\circ$

As $\omega \to 0$ $\quad G(j\omega) \to \infty \angle -90^\circ$

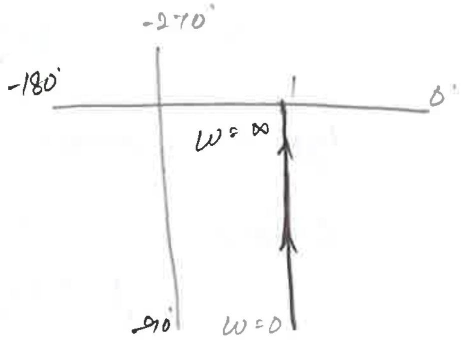
$\omega \to \infty$ $\quad G(j\omega) \to 0 \angle -90^\circ$



$G(s) = \dfrac{1+sT}{sT}$

$G(j\omega) = \dfrac{1+j\omega T}{j\omega T} = \dfrac{1}{j\omega T} + 1$

$= \dfrac{1}{\omega T \angle 90^\circ} + 1 = \dfrac{1}{\omega T} \angle -90^\circ + 1$



As $\omega \to 0$ $\quad G(j\omega) \to \infty \angle -90^\circ + 1$

$\omega \to \infty$ $\quad G(j\omega) \to 0 \angle -90^\circ + 1$

$G(s) = s$

$G(j\omega) = j\omega = \omega\angle 90^\circ$

As $\omega \to 0$ $\quad G(j\omega) \to 0 \angle 90^\circ$

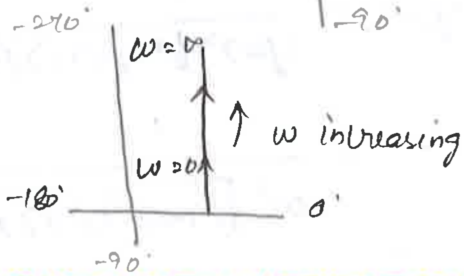
$\omega \to \infty$ $\quad G(j\omega) \to \infty \angle 90^\circ$

$G(s) = 1+sT$

$G(j\omega) = 1+j\omega T = 1+\omega T\angle 90^\circ$

As $\omega \to 0$, $\quad G(j\omega) \to 1 + 0\angle 90^\circ$

$\omega \to \infty$, $\quad G(j\omega) \to 1 + \infty \angle 90^\circ$



Determination of Gain Margin and Phase margin from Polar plot :

The Gain margin is defined as the inverse of the magnitude of $G(j\omega)$ at phase crossover frequency. The phase crossover frequency is the frequency at which the phase of $G(j\omega)$ is 180°

Let the polar plot cut the 180° axis at point B and the magnitude circle passing through the point B be G_B. Now the Gain margin $K_g = 1/G_B$. If the point B lies within unity circle, then the gain margin is positive otherwise negative. (If the polar plot is drawn in ordinary graph sheet using rectangular co-ordinates then the point B is the cutting point of $G(j\omega)$ locus with negative real axis and $K_g = 1/|G_B|$ where G_B is the magnitude corresponding to point B).

The phase margin is defined as, phase margin, $\gamma = 180^\circ + \phi_{gc}$ where ϕ_{gc} is the phase angle of $G(j\omega)$ at gain crossover frequency. The gain crossover frequency is the frequency at which the magnitude of $G(j\omega)$ is unity.

Let the polar plot cut the unity circle at point A as shown in figure. Now the phase margin, γ is given by $\angle AOP$. i.e, if $\angle AOP$ is below -180° axis then the phase margin is positive and if it is above -180° axis then the phase margin is negative.

Corner frequencies are

$$\omega_{c1} = \frac{1}{2} = 0.5 \text{ rad/sec.}$$

$$\omega_{c2} = \frac{1}{1} = 1 \text{ rad/sec.}$$

Magnitude and phase angle of $G(j\omega)$ are calculated for corner frequency and for frequencies around corner frequencies and tabulated in table 1.

Using polar to rectangular conversion, the polar co-ordinates listed in table-1 are converted to rectangular co-ordinates and tabulated in table 2.

The polar plot using polar co-ordinates is sketched on a polar graph sheet as shown.

The polar plot using rectangular co-ordinates is sketched on an ordinary graph sheet.

$$G(j\omega) = \frac{1}{(j\omega)(1+j\omega)(1+j2\omega)}$$

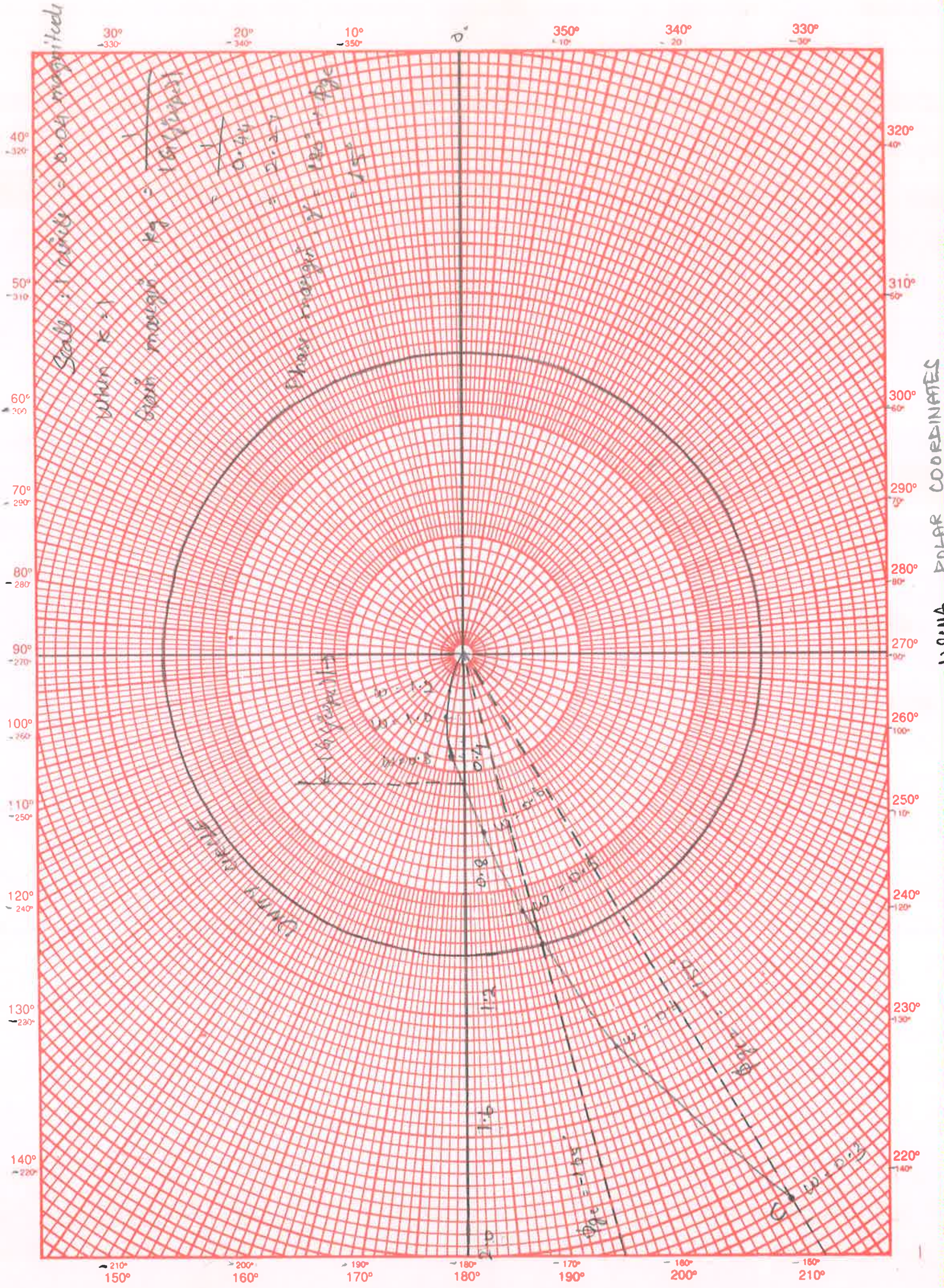
$$= \frac{1}{\omega\angle 90^\circ \sqrt{1+\omega^2}\angle \tan^{-1}\omega \sqrt{1+4\omega^2}\angle \tan^{-1} 2\omega}$$

$$= \frac{1}{\omega\sqrt{(1+\omega^2)(1+4\omega^2)}} \angle -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{(1+\omega^2)(1+4\omega^2)}} = \frac{1}{\omega\sqrt{1+4\omega^2+\omega^2+4\omega^4}}$$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{1+5\omega^2+4\omega^4}}$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$


Scale: 1 circle = 0.04 magnitude
When K = 1
Gain margin Kg = 1/|G(jω)H(jω)|
= 1/0.44
= 2.27
Phase margin γ = 180° + φgc
= 15°
Unit circle
POLAR COORDINATES

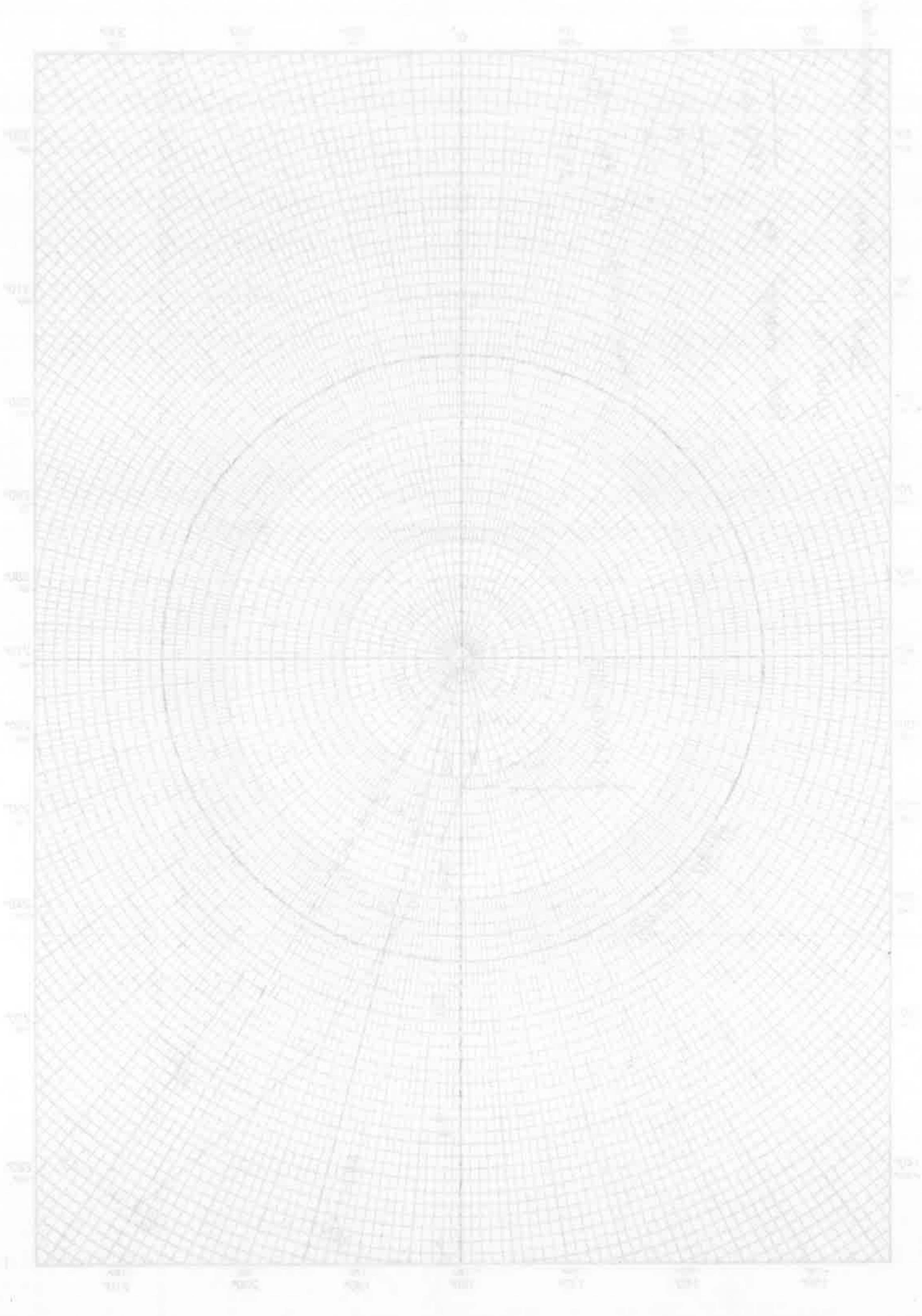


Table 1: Magnitude and phase of $G(j\omega)$ at various frequencies

ω rad/sec	0·35	0·4	0·45	0·5	0·6	0·7	1·0
$\lvert G(j\omega)\rvert$	2·2	1·8	1·5	1·2	0·9	0·7	0·3
$\angle G(j\omega)$ deg.	−144	−150	−156	−162	−171	−179·5 −180	−198

Table 2: Real and imaginary part of $G(j\omega)$ at various frequencies

ω rad/sec.	0·35	0·4	0·45	0·5	0·6	0·7	1·0
$G_R(j\omega)$	−1·78	−1·56	−1·37	−1·14	−0·89	−0·7	−0·29
$G_I(j\omega)$	−1·29	−0·9	−0·61	−0·37	−0·14	0	0·07

Result

Gain margin $K_g = \frac{1}{|G(j\omega)|_{pc}}$

$= \frac{1}{0\cdot 7}$

$= 1\cdot 4286$

phase margin $\gamma = 180^\circ = \phi_{gc}$

$= 180^\circ - 168^\circ$

$= 12$

Problem 2:

Consider a unity feedback system having an open loop T.F $G(s) = \frac{K}{s(1+0\cdot 2s)(1+0\cdot 05s)}$. Sketch the polar plot and determine the value of K so that
i) Gain margin is 18 db ii) Phase margin is 60°.

Sol:

Given that $G(s) = \dfrac{K}{s(1+0.2s)(1+0.05s)}$

put $K = 1$. and $s = j\omega$ in $G(s)$

$$G(j\omega) = \frac{1}{j\omega(1+j0.2\omega)(1+j0.05\omega)}$$

Corner frequencies are $\omega_{c1} = \dfrac{1}{0.2} = 5$ rad/sec.

$\omega_{c2} = \dfrac{1}{0.05} = 20$ rad/sec.

The magnitude and phase angle of $G(j\omega)$ are calculated for various frequencies and tabulated in table-1.

Using polar to rectangular conversion the polar co-ordinates listed in table-1 are converted to rectangular co-ordinates and tabulated in table-2.

The polar plot using polar co-ordinates is sketched on a polar graph sheet as &.

The polar plot using ~~pol~~ rectangular co-ordinates is sketched on an ordinary graph sheet.

$$G(j\omega) = \frac{1}{j\omega(1+j0.2\omega)(1+j0.05\omega)}$$

$$= \frac{1}{\omega\angle 90^\circ \sqrt{1+(0.2\omega)^2}\angle \tan^{-1}0.2\omega \sqrt{1+(0.05\omega)^2}\angle \tan^{-1}0.05\omega}$$

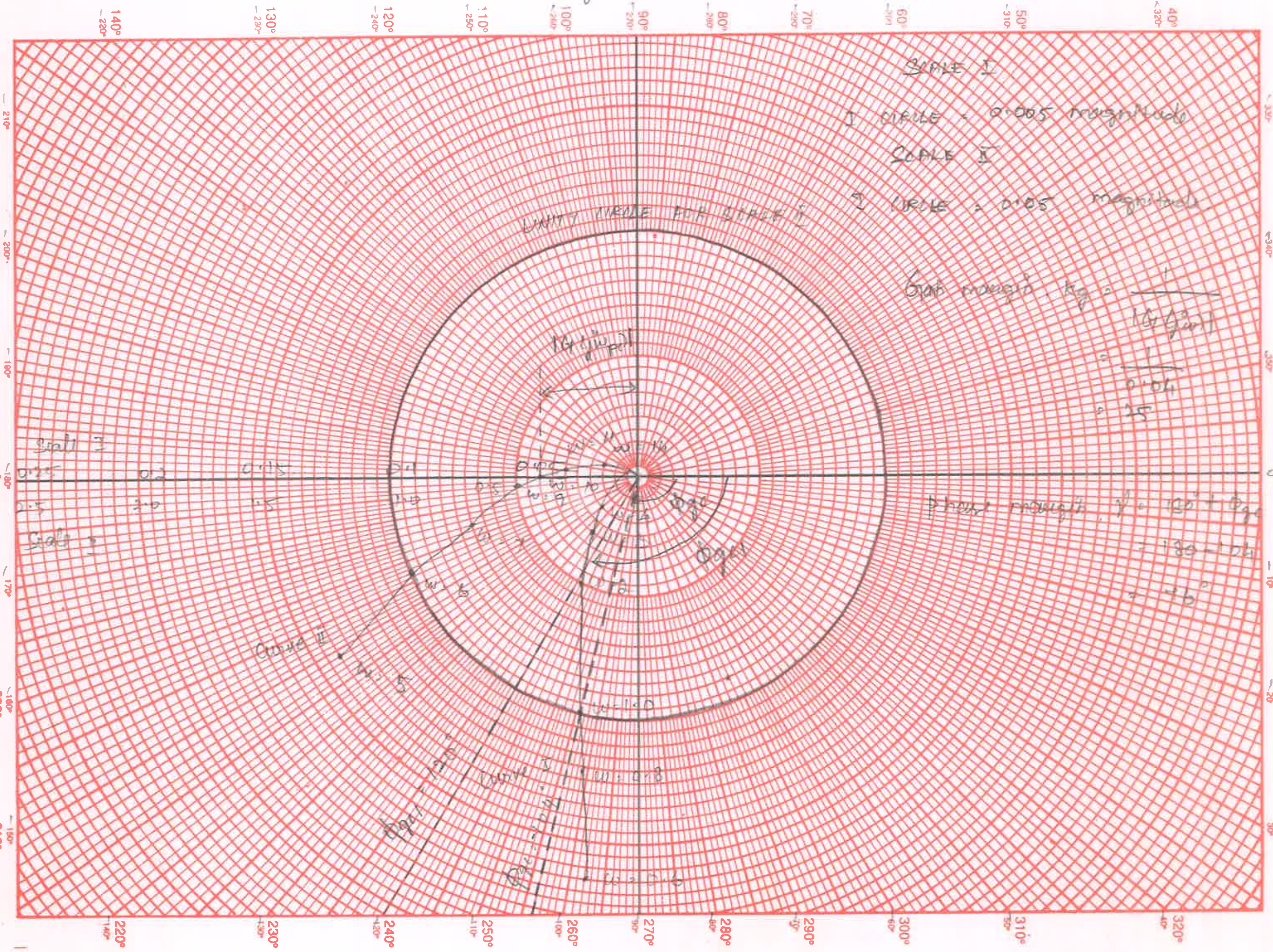
$$= \frac{1}{\omega\sqrt{1+(0.2\omega)^2}\sqrt{1+(0.05\omega)^2}} \angle(-90^\circ - \tan^{-1}0.2\omega - \tan^{-1}0.05\omega)$$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{1+(0.2\omega)^2}\sqrt{1+(0.05\omega)^2}}$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}0.2\omega - \tan^{-1}0.05\omega$$

Using POLAR COORDINATES

PROBLEM - 2



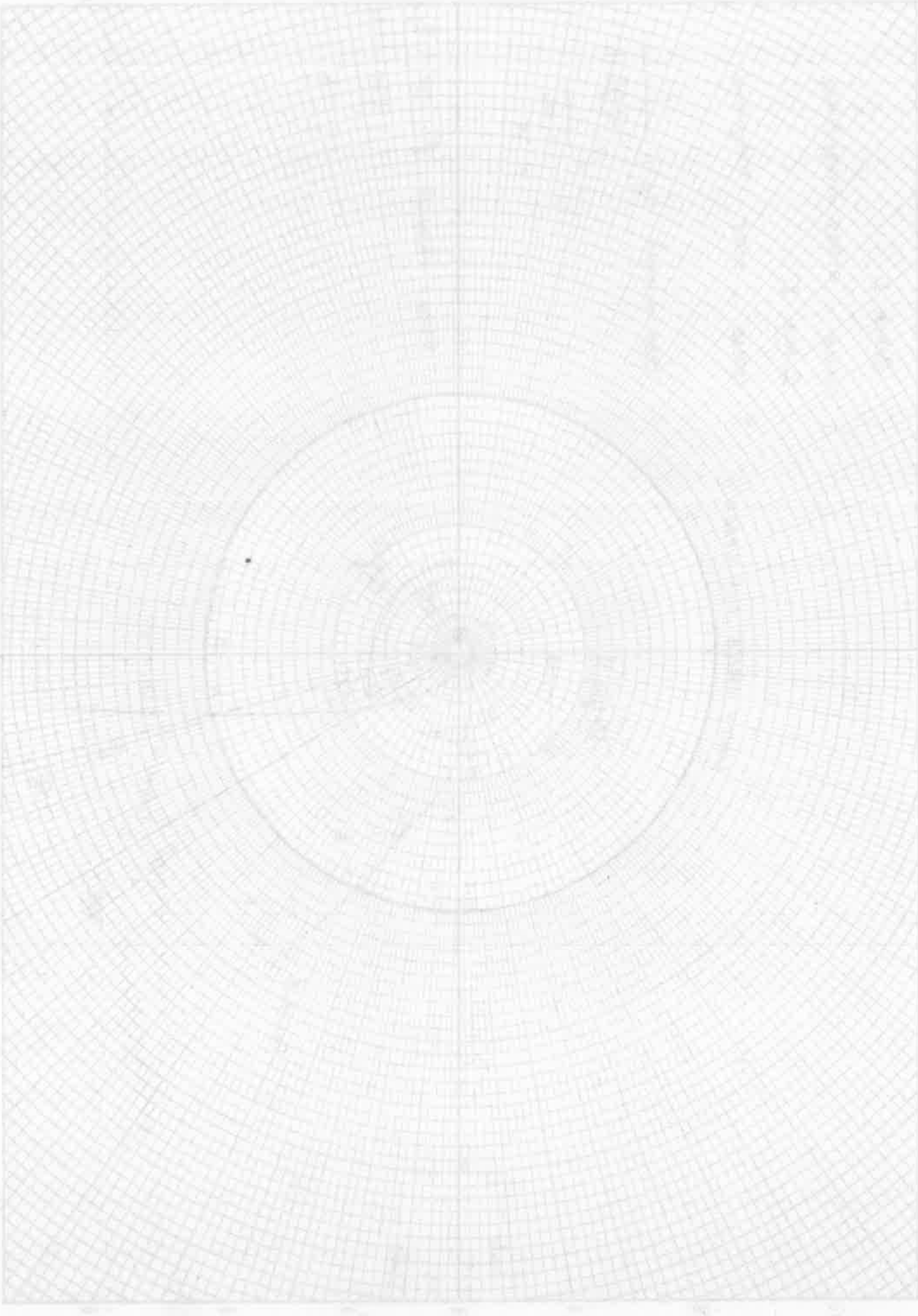


TABLE 1: Magnitude and phase of $G(j\omega)$ at various frequencies

ω rad/sec	0.6	0.8	1	2	3	4
$\lvert G(j\omega)\rvert$	1.65	1.23	1.0	0.5	0.3	0.2
$\angle G(j\omega)$ deg	−98	−101	−104	−117.5	−129.4	−140

ω rad/sec	5	6	7	9	10	11	14
$\lvert G(j\omega)\rvert$	0.14	0.1	0.07	0.05	0.04	0.03	0.01
$\angle G(j\omega)$ deg.	−149	−157	−164	−176	−180	−184	−195

TABLE 2: Real and imaginary part of $G(j\omega)$ at various frequencies

ω rad/sec.	0.6	0.8	1	2	3	4
$G_R(j\omega)$	−0.23	−0.23	−0.24	−0.23	−0.19	−0.15
$G_I(j\omega)$	−1.63	−1.21	−0.97	−0.44	−0.23	−0.13

ω rad/sec	5	6	7	9	10	11	14
$G_R(j\omega)$	−0.120	−0.092	−0.067	−0.050	−0.04	−0.03	−0.019
$G_I(j\omega)$	−0.072	−0.039	−0.019	−0.0034	0	0.002	0.005

In polar plot there are two plots, marked as curve-I and curve II. these two loci are sketched with different scales to clearly determine the gain margin and phase margin.

From the polar plot, with $k = 1$.

$$\text{Gain margin}, \; K_g = \frac{1}{0.04} = 25$$

$$\text{Gain margin in db} = 20 \log 25$$
$$= 28 \text{ db.}$$

$$\text{phase margin}, \; \gamma = 76^\circ.$$

Case i:

With $K = 1$, let $G(j\omega)$ cut the $-180°$ axis at point B and gain corresponding to that point be G_B. From the polar plot $G_B = 0.04$. The gain margin of 28 db with $K = 1$ has to be reduced to 18 db and so K has to be increased to a value greater than one.

Let G_A be the gain at $-180°$ for a gain margin of 18 db.

Now, $$20 \log \frac{1}{G_A} = 18$$

$$\log \frac{1}{G_A} = \frac{18}{20}$$

$$\frac{1}{G_A} = 10^{18/20}$$

$$G_A = \frac{1}{10^{18/20}} = 0.125$$

The value of K is given by,

$$K = \frac{G_A}{G_B} = \frac{0.125}{0.04}$$

$$\boxed{K = 3.125}$$

Case (ii) :

with $k = 1$, the gain margin is $76°$. This has to be reduced to $60°$. Hence gain has to be increased

Let ϕ_{gc2} be the phase of $G(j\omega)$ for a phase margin of $60°$.

$$\therefore 60 = 180° + \phi_{gc2}$$

$$\phi_{gc2} = 60° - 180°$$

$$= -120°$$

In the polar plot the $-120°$ line cut the locus of $G(j\omega)$ at point C and cut the unity circle at point D.

Let, G_C = Magnitude of $G(j\omega)$ at point C.

G_D = magnitude of $G(j\omega)$ at point D.

From the polar plot, $G_C = 0.425$

$$G_D = 1$$

Now, $$k = \frac{G_D}{G_C} = \frac{1}{0.425}$$

$$\boxed{k = 2.353}$$

Result :

i) when $k = 1$, Gain margin, $K_g = 25$

Gain margin in db = 28 db.

ii) when $k = 1$, Phase margin, $\gamma = 76°$

iii) For a gain margin of 18 db, $k = 3.125$

iv) For a phase margin of $60°$, $k = 2.353$.

NICHOLS PLOT :

The Nichols plot is a frequency response plot of the open loop transfer function of a system. The Nichols plot is a graph between magnitude of $G(j\omega)$ in db and the phase of $G(j\omega)$ in degree, plotted on a ordinary graph sheet.

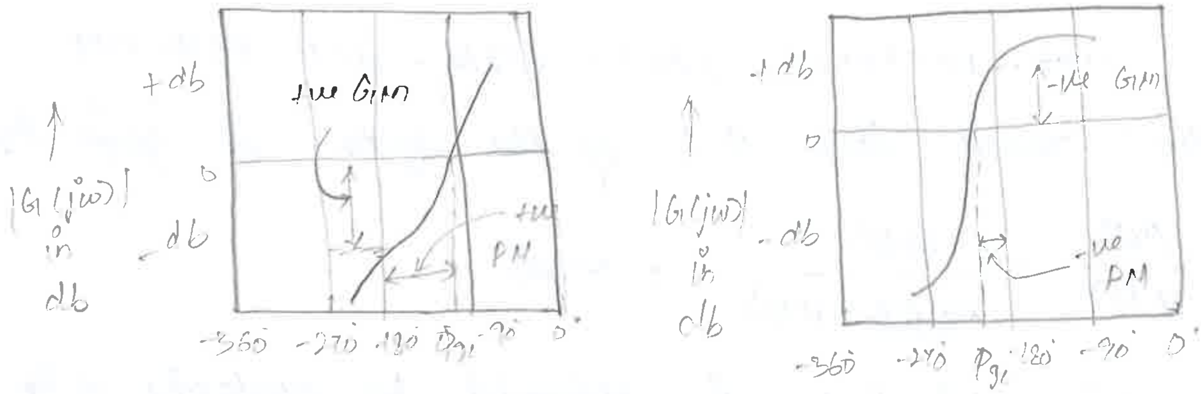
In ~~ordinary~~ to plot the Nichols plot, the magnitude of $G(j\omega)$ in db and phase of $G(j\omega)$ in deg are computed for various values of ω and tabulated. Usually the choice of frequencies are corner frequencies. Choose appropriate scales for magnitude on y-axis and phase on x-axis. Fix all the points on ordinary graph sheet and join the points by smooth curve, and mark frequency corresponding to each point.

In another method, first the Bode plot of $G(j\omega)$ is sketched. From the Bode plot the magnitude and phase for various values of frequency, ω are noted and tabulated. Using these values the Nichols plot is sketched as explained earlier.

DETERMINATION OF GAIN MARGIN AND PHASE MARGIN FROM NICHOLS PLOT :

The gain margin in db is given by the negative of db magnitude of $G(j\omega)$ at the phase crossover frequency, ω_{pc}. The ω_{pc} is the frequency at which phase of $G(j\omega)$ is $-180°$. If the db magnitude of $G(j\omega)$ at ω_{pc} is negative then gain margin is positive and vice versa.

Let ϕ_{gc} be the phase angle of $G(j\omega)$ at gain cross over frequency ω_{gc}. The ω_{gc} is the frequency at which the db magnitude of $G(j\omega)$ is zero. Now the phase margin, γ is given by $\gamma = 180° + \phi_{gc}$. If ϕ_{gc} is less negative than $-180°$ then phase margin is positive and vice versa. The positive and negative gain margins are illustrated.



Nichols plot showing phase margin (PM) and gain margin (GM)

GAIN ADJUSTMENT IN NICHOLS PLOT :

In the open loop T.F. $G(j\omega)$ the constant K contributes only magnitude. Hence by changing the value of K the system gain can be adjusted to meet the desired specifications. The desired specifications are gain margin and phase margin.

In a system T.F, if the value of K required to be estimated, in order to satisfy a desired specification, then draw the Nichols plot of the system with $K = 1$. The constant K can add $20 \log K$ to every point of the plot. Due to this addition, the Nichols plot will shift vertically up or down. Hence shift the plot vertically up or down to meet the desired specification. Equate the vertical distance by which the Nichols plot is shifted to 20 log k and solve for k.

Let, x = change in db (x is +ve if the plot is shifted up and vice versa).

Now, $20 \log K = x$

$\log k = \frac{x}{20}$ $\boxed{k = 10^{x/20}}$

CLOSED LOOP RESPONSE FROM OPEN LOOP RESPONSE:

The closed loop T.F of the system is given by,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\,H(s)} = M(s)$$

The sinusoidal T.F is obtained by replacing s by jω.

$$M(j\omega) = \frac{G(j\omega)}{1+G(j\omega)\,H(j\omega)}$$

Let, $M(j\omega) = M\angle\alpha$

where, M = Magnitude of closed loop T.F.

α = Phase of closed loop T.F.

The magnitude and phase of closed loop system are functions of frequency, ω. The sketch of magnitude and phase of closed loop system with respect to ω is closed loop frequency response plot. The magnitude and phase of closed loop system for various values of frequency can be evaluated analytically or graphically. The analytical ~~meth~~ method of determining the frequency response involves tedious calculations. Two graphical methods are available to determine the closed loop frequency response from open loop frequency response. They are,

1. M and N circles.

2. Nichols chart.

M. and N CIRCLES :

The magnitude of closed loop T.F with unity feedback can be shown to be in the form of circle for every value of M. These circles are called M-circles.

If the phase of closed loop T.F with unity feedback is α, then it can be shown that $\tan\alpha$ will be in the form of circle for every value of α. These circles are called N-circles.

The M and N circles are used to find the closed loop frequency response graphically from the open loop frequency response $G(j\omega)$ without calculating the magnitude and phase of the closed loop T.F at each frequency.

The M and N circles are available as standard chart. The chart consists of M and N circles superimposed on ordinary graph sheet. Using ordinary graph the locus of $G(j\omega)$ (Polar plot) is sketched. The locus of $G(j\omega)$ will cut the M-circles and N-circles at various points. The intersection of $G(j\omega)$ locus with M and N circles gives the magnitude and phase of the closed loop system at frequencies corresponding to the cutting point of $G(j\omega)$.

The M and α for various values of ω are tabulated. The magnitude and phase response of closed loop system are sketched on semilog graph sheet by taking ω on the logarithmic scale on x-axis. [The closed loop frequency response has two plots. They are M Vs ω and α Vs ω].

<u>M circles</u> :

Consider the closed loop T.F of unity feedback system,

$$M(s) = \frac{G(s)}{1+G(s)}$$

Put $s = j\omega$, $M(j\omega) = \frac{G(j\omega)}{1+G(j\omega)}$

Let, $G(j\omega) = X + jY$

where, X = Real part of $G(j\omega)$

Y = Imaginary part of $G(j\omega)$.

$$M(j\omega) = \frac{X+jY}{1+X+jY} = \frac{\sqrt{X^2+Y^2}\ \angle \tan^{-1}\frac{Y}{X}}{\sqrt{(1+X)^2+Y^2}\ \angle \tan^{-1}\frac{Y}{1+X}}$$

$$= \frac{\sqrt{X^2+Y^2}}{\sqrt{(1+X)^2+Y^2}} \angle \left(\tan^{-1}\frac{Y}{X} - \tan^{-1}\frac{Y}{1+X}\right)$$

Let, M = Magnitude of $M(j\omega)$

$$M = \frac{\sqrt{X^2+Y^2}}{\sqrt{(1+X)^2+Y^2}}$$

On Squaring the above eq. we get,

$$m^2 = \frac{x^2+y^2}{(1+x)^2+y^2} \Rightarrow m^2((1+x)^2+y^2) = x^2+y^2$$

$$m^2(1+x^2+2x+y^2) = x^2+y^2$$

$$m^2+m^2x^2+m^22x+m^2y^2-x^2-y^2=0$$

$$x^2(m^2-1)+m^22x+m^2+y^2(m^2-1)=0 \longrightarrow ①$$

when $m=1$, the above eq. represents a straight line.

when $m=1$, the above eq. is

$$x^2(1-1)+2x+1+y^2(1-1)=0$$

$$2x+1=0$$

$$x=-1/2$$

Hence when $m=1$, eq ① represents a straight line passing through $x=-1/2$ & $y=0$

when $m\neq 1$, the eq ① represents a family of circles.

when $m\neq 1$, eq ① can be rearranged in the form of eq. of a circle as shown below.

$$x^2(m^2-1)+m^22x+m^2+y^2(m^2-1)=0$$

$\div$ the above eq. throughout by (m^2-1).

$$x^2+\frac{m^2}{m^2-1}2x+\frac{m^2}{m^2-1}+y^2=0$$

Add (+) $\frac{m^2}{(m^2-1)^2}$ on both sides of the above eq.

$$x^2+\frac{m^2}{m^2-1}2x+\frac{m^2}{m^2-1}+\frac{m^2}{(m^2-1)^2}+y^2=\frac{m^2}{(m^2-1)^2}$$

$$x^2+\frac{m^2}{m^2-1}2x+\frac{m^2(m^2-1)+m^2}{(m^2-1)^2}+y^2=\frac{m^2}{(m^2-1)^2}$$

Consider the eq. for N-circle when $x = 0$ and $y = 0$

$$\left(x+\frac{1}{2}\right)^2+\left(y-\frac{1}{2N}\right)^2 = \frac{1}{4}+\frac{1}{(2N)^2}$$

$$\left(\frac{1}{2}\right)^2+\left(-\frac{1}{2N}\right)^2 = \frac{1}{4}+\frac{1}{(2N)^2}$$

$$\frac{1}{4}+\frac{1}{4N^2} = \frac{1}{4}+\frac{1}{4N^2}$$

Consider the eq. of N-circle, when $x = -1$ and $y = 0$.

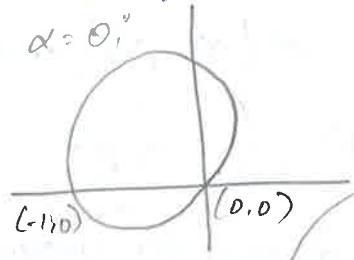
$$\left(x+\frac{1}{2}\right)^2+\left(y-\frac{1}{2N}\right)^2 = \frac{1}{4}+\frac{1}{2N^2}$$

$$\left(-1+\frac{1}{2}\right)^2+\left(-\frac{1}{2N}\right)^2 = \frac{1}{4}+\frac{1}{(4N)^2}$$

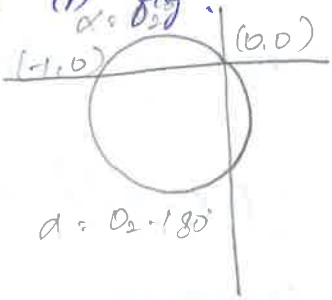
$$\frac{1}{4}+\frac{1}{4N^2} = \frac{1}{4}+\frac{1}{4N^2}$$

The above analysis shows that the eq. of N-circle is satisfied at point (0,0) and (-1,0).

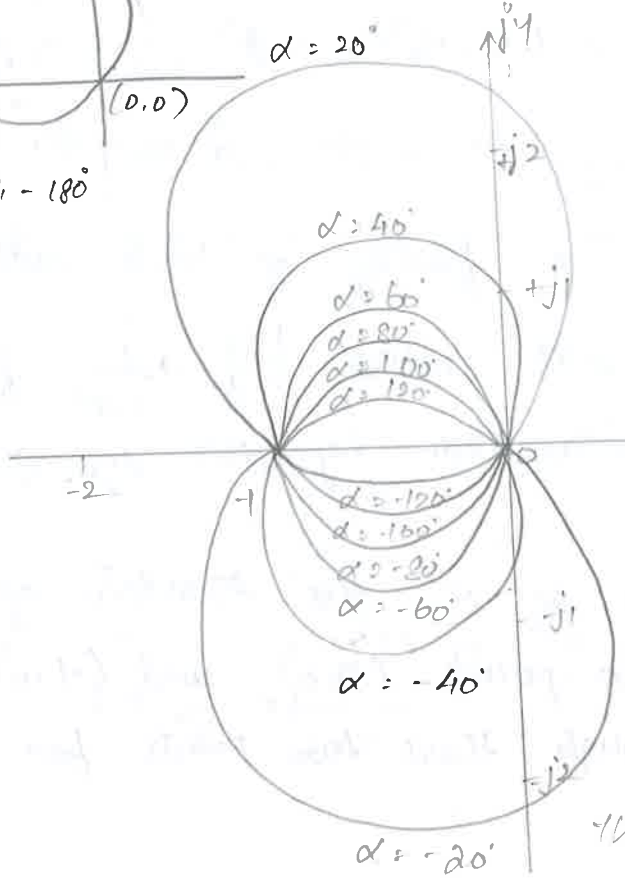
When $\alpha = 180°$ the circle becomes a straight line passing through real axis. It is also observed that the circle for $\alpha = 0° - 180°$ above the real axis will be a part of circle for $\alpha = 0°$ below the real axis, as shown in fig. The family of N circles are shown in fig.



$\alpha = 0_1 - 180°$



$\alpha = 0_2 - 180°$



Complex plane Rectangular co-ordinates

The family of constant N-circles

LAG COMPENSATOR :

A compensator having the charactoristics of a lag network is called a lag compensator. If a sinusoidal signal is applied to a lag network, then in steady state the output will have a phase lag with respect to input.

Procedure : (using Bode plot)

Step 1: Choose the value of k in uncompensated system to meet the steady state error requirement.

Step 2: Sketch the bode plot of uncompensated system

Step 3: Determine the phase margin of uncompensated system from the bode plot. If the phase margin does not satisfy the requirement then lag compensation is required.

Step 4: Choose a suitable value for the phase margin of the compensated system.

Let γ_d = Desired phase margin as given in specification

γ_n = Phase margin of compensated system.

Now, $\gamma_n = \gamma_d + \varepsilon$

where ε = Additional phase lag to compensate for shift in gain crossover frequency.

Choose an initial value of $\varepsilon = 5°$.

Step 5: Determine the new gain crossover frequency, ω_{gn}. The new ω_{gn} is the frequency corresponding to a phase margin of γ_n on the bode plot of uncompensated system.

16 Marks & Tutorial:

1. Bode plot - Problems.

i) $G(s) = \dfrac{100(1+0.1s)}{s(1+0.2s)(1+0.5s)}$

ii) $G(s) = \dfrac{30(1+0.1s)}{s(1+0.001s)(1+s)}$

2. Polar plot and Nichols chart.

i) $G(s) = \dfrac{10(1+s)}{(s+10)^2}$

ii) $G(s) = \dfrac{1}{s(s+4)(s+8)}$

3. Derive constant M and N circles.

4. Procedure of Lag, Lead and Lag-Lead compensator.

5. Tutorial problems of compensator i) Lag, ii) Lead iii) Lag-Lead.

UNIT - IV CONCEPTS OF STABILITY ANALYSIS

Concept of stability - Bonded - Input Bounded - Output stability - Routh stability criterion - Relative stability - Root locus concept - Guidelines for sketching root locus - Nyquist stability criterion.

Stability :

The term stability refers to a stable working condition of a control system. Every working system is designed to be stable. In a stable system, the response or output is predictable, finite and stable for a given input.

i/p - change
system parameter - change

* A system is stable, if its output is bounded (finite) for any bounded (finite) input.

* A system is asymptotically stable, if in the absence of i/p, the o/p tends towards zero (or equilibrium state) irrespective of initial conditions.

* A system is stable if for a bounded disturbing input signal the o/p vanishes ultimately as t approaches infinity.

* A system is unstable if for a bounded disturbing input signal the output is of infinite amplitude or oscillatory.

* For a bounded input signal, if the output has constant amplitude oscillations then the system may be stable or unstable under some limited constraints. Such a system is called limitedly stable.

If a system output is stable for all variations of its parameters, then the system is called absolutely system stable system.

If a system output is stable for a limited range of variations of its parameters, then the system is called conditionally stable system.

Bounded Input Bounded output stability (BIBO):

A linear relaxed system is said to have BIBO stability if every bounded (finite) input results in a bounded (finite) output. A condition for BIBO stability can be obtained from convolution operation defined by eq.

$$\text{Response, } c(t) = \int_0^\infty m(\tau)\, r(t-\tau)\, d\tau$$

Stability of system depending on the location of roots of characteristics equation.

Stable system - i) All roots of CE has -ve real parts.

Unstable system - ii) All roots of CE has +ve real parts or repeated roots on imaginary axis

Limitedly or marginally stable } - i) Satisfied except for the presence of one or more non repeated roots on imaginary axis.

coefficients of characteristics polynomial.

coefficients ⇒ +ve, No zeroes → All roots are in left half of s-plane.

coefficient $\Rightarrow 0 \rightarrow$ Roots may be imaginary axis or right half of s-plane.

coefficient $\Rightarrow -ve \rightarrow$ atleast one root in Right half of s-plane.

ROUTH HURWITZ CRITERION :

It is an analytical procedure for determining whether all the roots of a polynomial have negative real part or not.

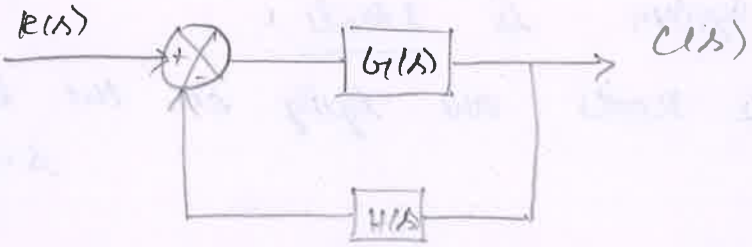
TYPES :

Case I : Normal Routh Array.
(No zero element in 1st column of routh array)

Case II : A row of all zeros.

Case III : First element of a row is zero but some or other element are not zero.

Formation of Routh Array :



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$CE = 1+G(s)H(s) = 0. \Rightarrow$ characteristics equation

Problem 1 :
Using Routh criterion determine the stability of the system represented by the CE, $s^4+8s^3+18s^2+16s+5=0$ Comment on the location of the roots of CE

Sol:

Roots ⇒ 4.

C.E ⇒ $s^4 + 8s^3 + 18s^2 + 16s + 5 = 0$.

$$\begin{array}{lllll} s^4 & : & 1 & 18 & 5 \\ s^3 & : & 8 & 16 & \div 8 \end{array}$$

$$\begin{array}{lll} s^4 & : & \boxed{1} & 18 & 5 \\ s^3 & = & 1 & 2 & \\ s^2 & = & 16 & 5 & \\ s^1 & = & 1.7 & & \\ s^0 & = & 5 & & \end{array}$$

↳ 1st column.

$s^2 = \frac{1 \times 18 - 2 \times 1}{1} \quad \frac{5 \times 1 - 0 \times 1}{1}$

$s^2 = 16 \quad 5$

$s^1 = \frac{16 \times 2 - 5 \times 1}{16}$

$= 1.68 \approx 1.7$

$s^0 = \frac{1.7 \times 5 - 16 \times 0}{1.7}$

All elements in 1st column are +ve.

Result:

1. The System is stable.
2. All 4 roots are lying on the left half of s-plane.

Problem 2: (N/D - 2014, M/J - 2013)

Construct Routh array and determine the stability of the system whose C.E is $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$.

Sol:

Root = 6.

C.E : $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$.

s^6 : 1 8 20 16

s^5 : 2 12 16 $\div 2$

s^6 :	1	8	20	16
s^5 :	1	6	8	0
s^4 :	1	6	8	0
s^3 :	0	0	0	
s^3 :	1	3		
s^2 :	3	8		
s^1 :	0.33			
s^0 :	8			

Auxiliary polynomial.

$$s^4 + 6s^2 + 8 = 0$$

$$s^2 = x$$

$$x^2 + 6x + 8 = 0$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4 \times 8}}{2}$$

$$= -3 \pm 1$$

$$x = -2 \text{ or } -4.$$

$$s = \pm\sqrt{x}$$

$$= \pm\sqrt{-2} \text{ and } \pm\sqrt{-4}$$

$$s = +j\sqrt{2},\ -j\sqrt{2},\ +j\sqrt{4},\ -j\sqrt{4}$$

$$= +j\sqrt{2},\ -j\sqrt{2},\ +j2,\ -j2$$

s^4 : $\frac{1\times 8 - \cancel{0}\times 1}{1}$ $\frac{1\times 20 - 1\times 8}{1}$ $\frac{1\times 16 - 1\times 0}{1}$

: 2 12 16

s^3 : $\frac{1\times 6 - 1\times 6}{1}$ $\frac{1\times 8 - 1\times 8}{1}$ $\frac{1\times 0 - 1\times 0}{1}$

: 0 0 0

$= s^4 + 6s^2 + 8$

$$\frac{dA}{ds} = 4s^3 + 12s$$

4 , 12 $\div 4$

1 , 3

s^2 $= \frac{1\times 8 - 3\times 1}{1}$ $\frac{1\times 8 - 0\times 1}{1}$

= 3 8

s^1 = $\frac{3\times 3 - 1\times 8}{3}$

$= \frac{9-8}{3} = \frac{1}{3}$

$= 0.33$

s^0 : $\frac{0.33\times 8 - 0\times 3}{0.33}$

= 8

Result :

1. System is limitedly or marginally stable.
2. 4 Roots are lying on imaginary axis & remaining two roots are lying on left half of s-plane.

Problem 2:

Construct Routh array and determine the stability of the system represented by CE $s^5+s^4+2s^3+2s^2+3s+5=0$. Comment on the location of the roots of C.E (M/J - 2014)

Sol:

Roots $=5$

C.E $s^5+s^4+2s^3+2s^2+3s+5=0$.

$$\begin{array}{llll} s^5 & : 1 & 2 & 3 \\ s^4 & : 1 & 2 & 5 \\ s^3 & : \epsilon & -2 & - \\ s^2 & : \frac{2\epsilon+2}{\epsilon} & 5 & - \\ s^1 & : \frac{-(5\epsilon^2+4\epsilon+4)}{2\epsilon+2} & - & - \\ s^0 & : 5 & & \end{array}$$

s^3 : $\frac{1\times2-2\times1}{1}$ $\frac{1\times3-5\times1}{1}$

: 0 -2

Replace 0 by ϵ.

s^3 : ϵ -2

s^2 : $\frac{\epsilon\times2+2}{\epsilon}$ $\frac{\epsilon5-0}{\epsilon}$

$\frac{2\epsilon+2}{\epsilon}$ 5

s^1 : $\frac{2\left(\frac{2\epsilon+2}{\epsilon}\right)-5\epsilon}{2\frac{\epsilon+2}{\epsilon}}$

$= \frac{-4\epsilon-4-5\epsilon^2/\cancel{\epsilon}}{2\epsilon+2/\cancel{\epsilon}}$

s^0 : $\frac{5\left(\frac{-(5\epsilon^2+4\epsilon+4)}{2\epsilon+2}\right)-0}{\frac{-5\epsilon^2+4\epsilon+4}{2\epsilon+2}}$

Sub. $\epsilon=0$.

$$\begin{array}{llll} s^5 & , & \boxed{1} & 2 & 3 \\ s^4 & : & 1 & 2 & 5 \\ s^3 & : & 0 & -2 & \\ s^2 & : & \infty & 5 & \\ s^1 & : & -2 & & \\ s^0 & : & 5 & & \end{array}$$

Result:

i) The system is unstable.

ii) Two roots are lying on right half of s-plane and

Problem 4 : (N/D - 2013)

The OLTF of a unity feedback system is given by $G(s) = \frac{k}{(s+2)(s+4)(s^2+6s+25)}$. By applying Routh criterion discuss the stability of closed loop system as a function of K. Also determine the value of k which will cause sustained oscillation in the closed loop system. Determine the oscillating frequencies.

Sol :

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{k/[(s+2)(s+4)(s^2+6s+25)]}{1+k/(s+2)(s+4)(s^2+6s+25)}$$

$$= \frac{k}{(s+2)(s+4)(s^2+6s+25)+k}$$

C.E is $(s+2)(s+4)(s^2+6s+25)+k = 0$

$$(s^2+\underbrace{4s+2s}_{6s}+8)(s^2+6s+25)+k = 0$$

$$s^4+6s^3+25s^2+6s^3+36s^2+150s+8s^2+48s+200+k = 0$$

$$s^4+12s^3+69s^2+198s+200+k = 0$$

s^4 : 1   69   200+k

s^3 : 12   198

s^2 : 52.5   200+k

s^1 : $\frac{7995-12k}{52.5}$

s^0 : 200+k

s^2 : $\frac{12\times69-1\times198}{12}$ $\frac{12(200+k)-0}{12}$

: 52.5   200+k

s^1 : $\frac{52.5\times198-12(200+k)}{52.5}$

: $\frac{7995-12k}{52.5}$

s^0 : $\frac{\frac{7995-12k}{52.5}(200+k)-0}{\frac{7995-12k}{52.5}}$

1^{st} column should be +ve.

s^1 row $\frac{7995 - 12K}{52.5} > 0$

$7995 - 12K > 0$

$-12K > -7995$

$\boxed{K < 666.25}$

s^0 row. $200 + K > 0$

$K > -200$

Normally K value starts from 0.

$0 < K < 666.25$

Sub, $K = 666.25$

s^4 : 1 69 866.25

s^3 : 12 198 -

s^2 : 52.5 866.25 -

s^1 : 0 -

s^0 : 866.25

s^1 row is zero when $K = 666.25$

Auxiliary polynomial $52.5 s^2 + 866.25 = 0$

$52.5 s^2 = -866.25$

$s^2 = -16.5$

$s = \sqrt{-16.5}$

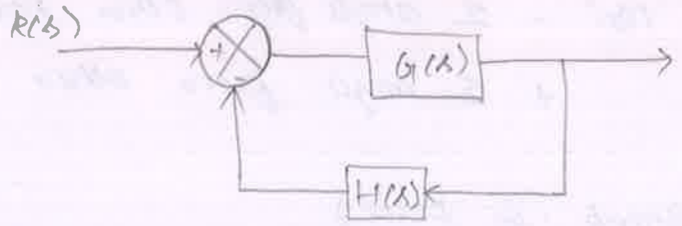
$s = \pm j4.06$

Frequency of oscillation $s = j\omega = \pm j4.06$

$\boxed{\omega = 4.06 \text{ rad/sec.}}$

ROOT LOCUS :

The path taken by a root of characteristics equation when open loop gain K is varied from 0 to ∞ is called root locus.



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

C.E is $1+G(s)H(s) = 0$

$$G(s)H(s) = -1$$

Magnitude criterion $|G(s)H(s)| = 1$

Angle criterion $\angle G(s)H(s) = \pm 180^\circ(2q+1)$

where $q = 0, 1, 2, 3 \ldots$

Procedure :

Step 1 : Locate the pole and zeros for the given T.F

Step 2 : Determine the root locus on real axis.

Step 3 : Calculate the angle of asymptotes and centroid

$$\text{Centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m}$$

$$\angle \text{Asymptotes} = \pm \frac{180^\circ(2q+1)}{n-m}$$

where $q = 0, 1, 2 \ldots (n-m)$

Step 4 : Find the break away and break in points.

$$\frac{dK}{ds} = 0$$

Step 5: Find the angle of Departure and angle of arrival.

Angle of Departure: (Breakaway point)

If there is complex pole. => Angle of Departure Find.

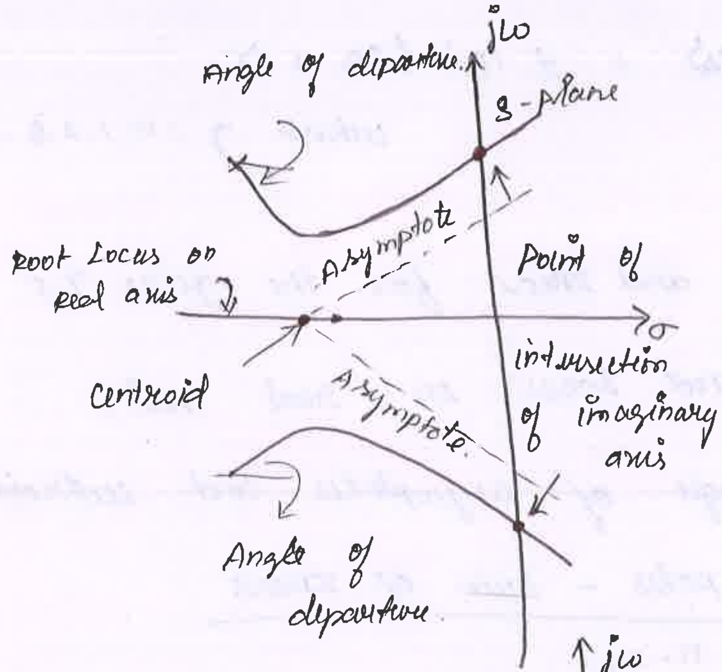
Angle of Departure = $180^\circ - \Sigma$ angle from other poles $+ \Sigma$ angle from other zeros.

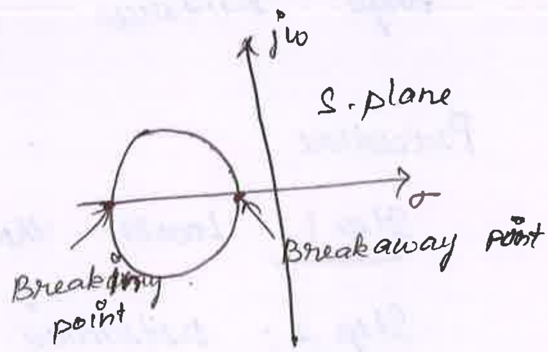
Angle of arrival: (Break in point)

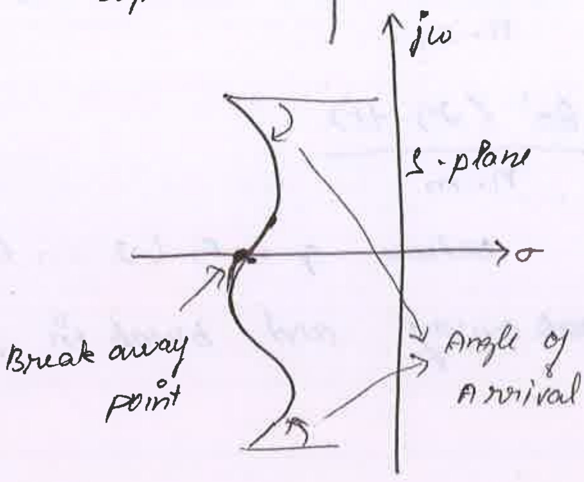
If there is complex zero => Angle of arrival.

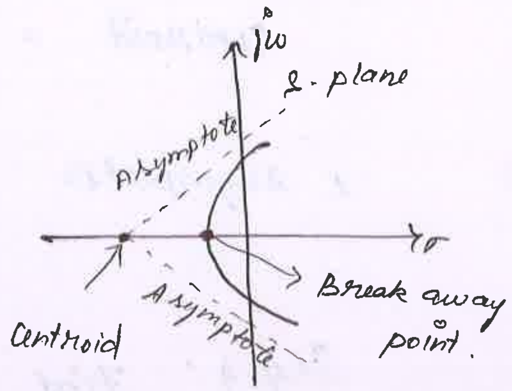
Angle of arrival = $180^\circ - \Sigma$ angle from other zeros $+ \Sigma$ angle from other poles.

Step 6: Find crossing point of Imaginary axis.









Problem 1:

A unity feedback control system has an open loop T.F $G(s) = \dfrac{K}{s(s^2+4s+13)}$. Sketch the root locus.

K → zeros → m.

$s(s^2+4s+13)$ → poles → n

Sol:

Step 1: To locate poles and zeros.

Poles of OLTF are roots of eq. $s(s^2+4s+13)=0$

$$s = \frac{-4 \pm \sqrt{4^2-4\times 13}}{2} = -2 \pm j3$$

poles are $s = 0, -2+j3, -2-j3$

Denote poles ⇒ P_1, P_2 & P_3

$P_1 = 0, \quad P_2 = -2+j3, \quad P_3 = -2-j3$

Poles ⇒ X.

Step 2: To find root locus on real axis

Test point between	No. of poles and zeros on right side of test point	Real axis between
0 to $-\infty$	odd no. of poles & zeros ⇒ $s=0$ 1 pole.	0 to $-\infty$ (Root locus)

Step 3: To find angle of asymptotes and centroid:

No. of poles $= 3$ so $n = 3$.

$n - m = 3 - 0 = 3.$

$$\text{Angle of asymptotes} = \frac{\pm 180^\circ (2q+1)}{n-m} \qquad q = 0, 1, \ldots \frac{n-m}{3}.$$

$q = 0, 1, 2, 3.$

$n = 3, \quad m = 0, \quad q = 0, 1, 2, 3.$

when $q = 0$, Angles $= \pm \frac{180}{3} = \pm 60^\circ$

$q = 1$, Angles $= \pm \frac{180(\cancel{3})}{\cancel{3}} = \pm 180^\circ$

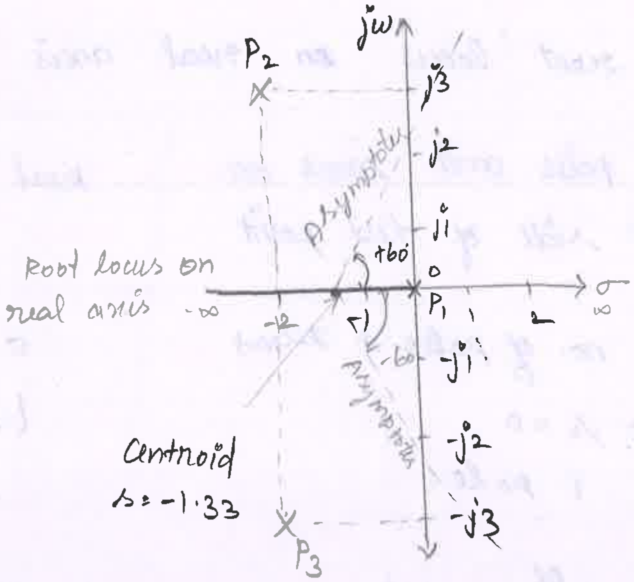
$q = 2$, Angles $= \pm \frac{\overset{60}{\cancel{180}}(5)}{\cancel{3}} = \pm 300 = \pm 60^\circ$

$q = 3$, Angles $= \pm \frac{\overset{60}{\cancel{180}}(7)}{\cancel{3}} = \pm 420 = \pm 60^\circ$

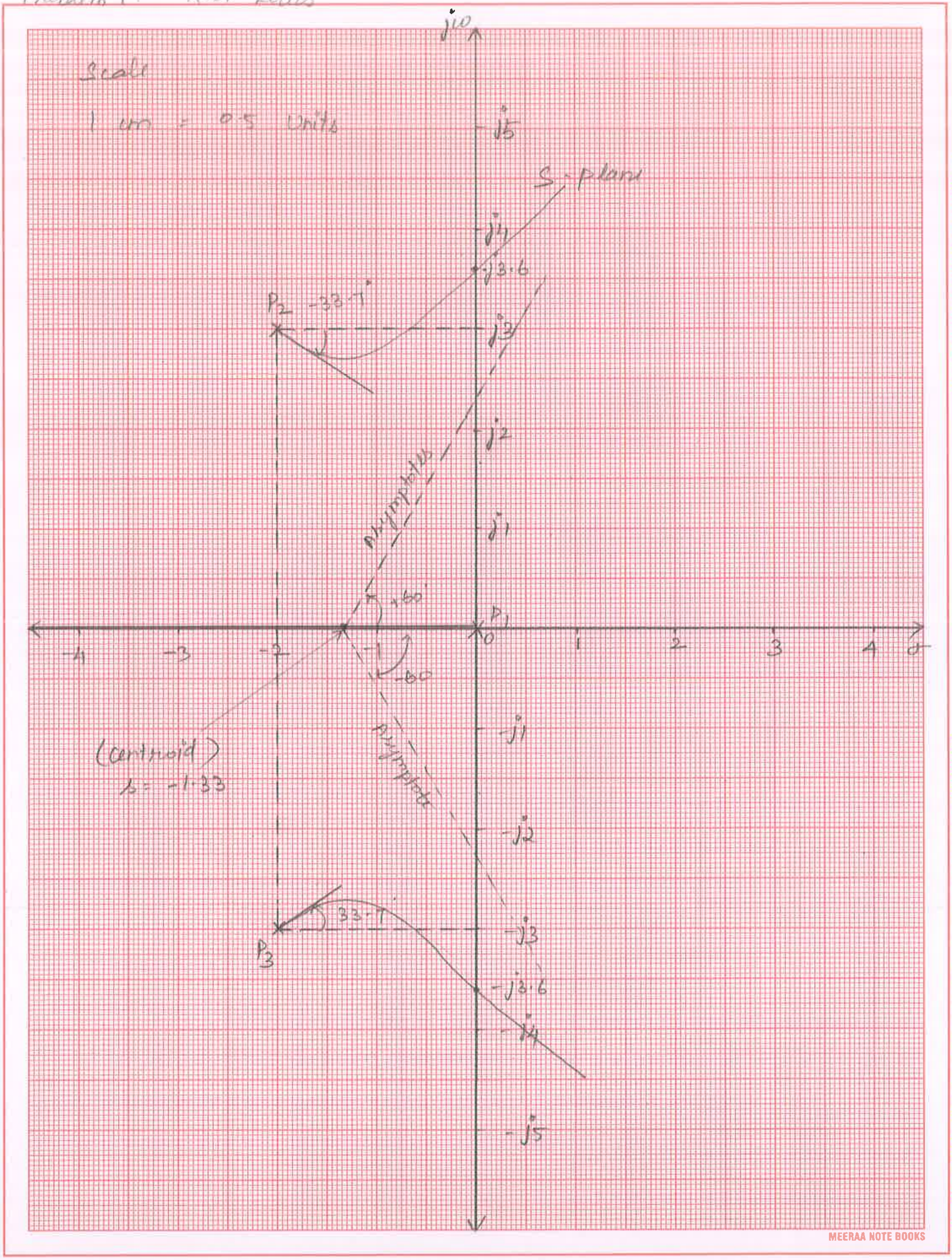
$$\text{centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m}$$

$$= \frac{0 - 2 + \cancel{j\sqrt{3}} - 2 - \cancel{j\sqrt{3}} - 0}{3}$$

$$= -\frac{4}{3} = -1.33$$



Problem 1: Root Locus



Step 4: To find the breakaway and breakin points.

$$CLTF \Rightarrow \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{G(s)}{1+G(s)}$$

$$= \frac{K/s(s^2+4s+13)}{1+K/s(s^2+4s+13)}$$

$$= \frac{K}{s(s^2+4s+13)+K} \longrightarrow C.E$$

C.E : $s(s^2+4s+13)+K=0$

$$s^3+4s^2+13s+K=0$$

$$K = -(s^3+4s^2+13s)$$

Diff above eq. with respect to s, we get,

$$\frac{dK}{ds} = -(3s^2+8s+13)$$

Put $\frac{dK}{ds}=0$

$$-(3s^2+8s+13)=0 \Rightarrow 3s^2+8s+13=0$$

$$s = \frac{-8 \pm \sqrt{8^2-4\times 13\times 3}}{2\times 3} = -1.33 \pm j1.6$$

$$s_1 = -1.33+j1.6$$

$$s_2 = -1.33-j1.6$$

sub s_1 in K eq.

$$K = -(s^3+4s^2+13s)$$

$$= -[(-1.33+j1.6)^3+4(-1.33+j1.6)^2+13(-1.33+j1.6)]$$

$$= 12.59 - j8.17$$

Sub s_2 in K eq.

$$K = -\left[(-1.33 - j1.6)^3 + 4(-1.33 - j1.6)^2 + 13(-1.33 \mp j1.6)\right]$$

$$= 12.59 \mp j8.17$$

K is not the and real.

Hence there is neither break away point nor break in point.

<u>Step 5</u>: To find angle of departure.

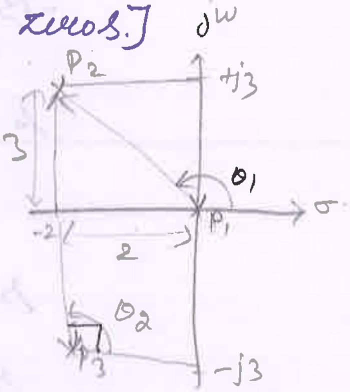
Let us consider pole P_2.

$$\theta_1 = 180^\circ - \tan^{-1}\left(\frac{3}{2}\right)$$

$$= 123.7^\circ$$

$$\theta_2 = 90^\circ$$

Angle of departure from complex pole P_2 $\Big\}$ = 180° − [Sum of angle of complex pole P_2 from other poles] + [Sum of angle of complex pole P_2 from zeros.]



$$= 180 - (\theta_1 + \theta_2)$$

$$= 180 - (123.7 + 90^\circ)$$

$$= -33.7^\circ$$

Angle of departure from Pole P_3 $\Big\}$ = − (Angle of departure from P_2)

$$= 33.7^\circ$$

7

Mark AOD at Complex pole using protector.

Step 6 : To find the crossing point on imaginary axis

C.E $\Rightarrow s^3 + 4s^2 + 13s + K = 0$

Put $s = j\omega$

$$(j\omega)^3 + 4(j\omega)^2 + 13(j\omega) + K = 0$$

$$-j\omega^3 - 4\omega^2 + 13j\omega + K = 0$$

Equating imaginary part. 1st Find

$$13\omega - \omega^3 = 0$$

$$-\omega^3 = -13\omega$$

$$\omega^2 = 13$$

$$\omega = \pm\sqrt{13}$$

$$\boxed{\omega = \pm 3.6}$$

Equating real part to 0 After

$$-4\omega^2 + K = 0$$

$$4\omega^2 = K$$

$$4(13) = K$$

$$\boxed{K = 52}$$

Crossing point of root locus $= \pm j3.6$

Value of K at this crossing point, $K = 52$.

This is limiting value of K for the stability of system.

Problem 2 :

Sketch the root locus of the system whose open loop transfer function is $G(s) = \dfrac{K}{s(s+2)(s+4)}$. Find the value of K so that the damping ratio of the closed loop system is 0.5.

Sol :

Step 1 : To locate poles and zeros :

poles $\Rightarrow$ OLTF $\Rightarrow s(s+2)(s+4) = 0$.

poles are lying at, $s = 0, -2, -4$.

poles as P_1, P_2 and P_3

$P_1 = 0$, $P_2 = -2$, $P_3 = -4$

poles marked by x.

Step 2: To find the root locus on real axis.

Test point between	No. of poles and zeros on real axis between right side of test point	
0 to -2	odd no. = 1 poles $s=0$	0 to -2 Root locus
-2 to -4	even no = 2 poles $(s=0)$ $(s=-2)$	-2 and -4 Not root locus
-4 to $-\infty$	odd no = 3 poles $(s=0)$, $s=-2$, $s=-4)$	-4 to $-\infty$ Root locus.

Step 3: Find angle of asymptotes and centroid

Asymptotes $= n - m = 3 - 0 = 3$.

$$\text{Angle of asymptotes} = \frac{\pm 180(2q+1)}{n-m} \qquad n = 3, \quad m = 0$$

At $q = 0$, Angle of asymptotes $= \frac{\pm 180(1)}{3}$ $\quad q = 0, 1, 2, 3$

$= \pm 60$

At $q = 1$ Angle of asymptotes $= \frac{\pm 180^\circ(3)}{3} = \pm 180^\circ$

At $q = 2$ Angle of asymptotes $= \frac{\pm 180(5)}{3} = \pm 300^\circ$

At $q = 3$ Angle of asymptotes $= \frac{\pm 180^\circ(7)}{3} = \pm 420^\circ$

$$\text{centroid} = \frac{\text{sum of poles} - \text{sum of zeros}}{n-m}$$

$$= \frac{0-2-4-0}{3} = -2$$

$$\boxed{\text{centroid} = -2}$$

Step 4: To find breakaway and breakin points

Closed loop Transfer Function

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\,H(s)} = \frac{K/s(s+2)(s+4)}{1+K/s(s+2)(s+4)}$$

$$= \frac{K}{s(s+2)(s+4)+K}$$

C.E $\quad s(s+2)(s+4)+K = 0$

$$s^3+6s^2+8s+K=0$$

$$K = -(s^3+6s^2+8s) \rightarrow ①$$

$$\frac{dK}{ds} = -(3s^2+12s+8)$$

Sub $\frac{dK}{ds} = 0$; $\quad 3s^2+12s+8=0$

$$s = -0.845, \quad s = -3.15$$

Sub $s = -0.845$ ①

$$K = -[(-0.845)^3 + 6(-0.845)^2 + 8(-0.845)]$$

$$= -(-0.603 + 4.284 - 6.76)$$

$\boxed{K = 3.08}$ => Real and Positive.

Sub $s = -3.15$ in ①

$$K = -[(-3.15)^2 + 6(-3.15)^2 + 8(-3.15)]$$

$\boxed{K = -3.08}$ => Not real and +ve.

Step 5: Angle of arrival and angle of departure.

C.E => $s^3 + 6s^2 + 8s + K = 0$

sub $s = j\omega$

$$(j\omega)^3 + 6(j\omega)^2 + 8j\omega + K = 0$$

$$\Rightarrow -j\omega^3 - 6\omega^2 + j8\omega + K = 0$$

$$(K - 6\omega^2) + j(8\omega - \omega^3) = 0$$

Equating imaginary part

$$-j\omega^3 + j8\omega = 0$$

$$-j\omega^3 = -j8\omega$$

$$\omega^2 = 8$$

$$\omega = \pm\sqrt{8}$$

$$\boxed{\omega = \pm 2.8}$$

Equating real part.

$$-6\omega^2 + K = 0$$

$$K = 6\omega^2$$

$$K = 6 \times 8$$

$$\boxed{K = 48}$$

Crossing point of root locus is $\pm j2.8$

$K = 48$.

To find value of K corresponding to $\zeta = 0.5$

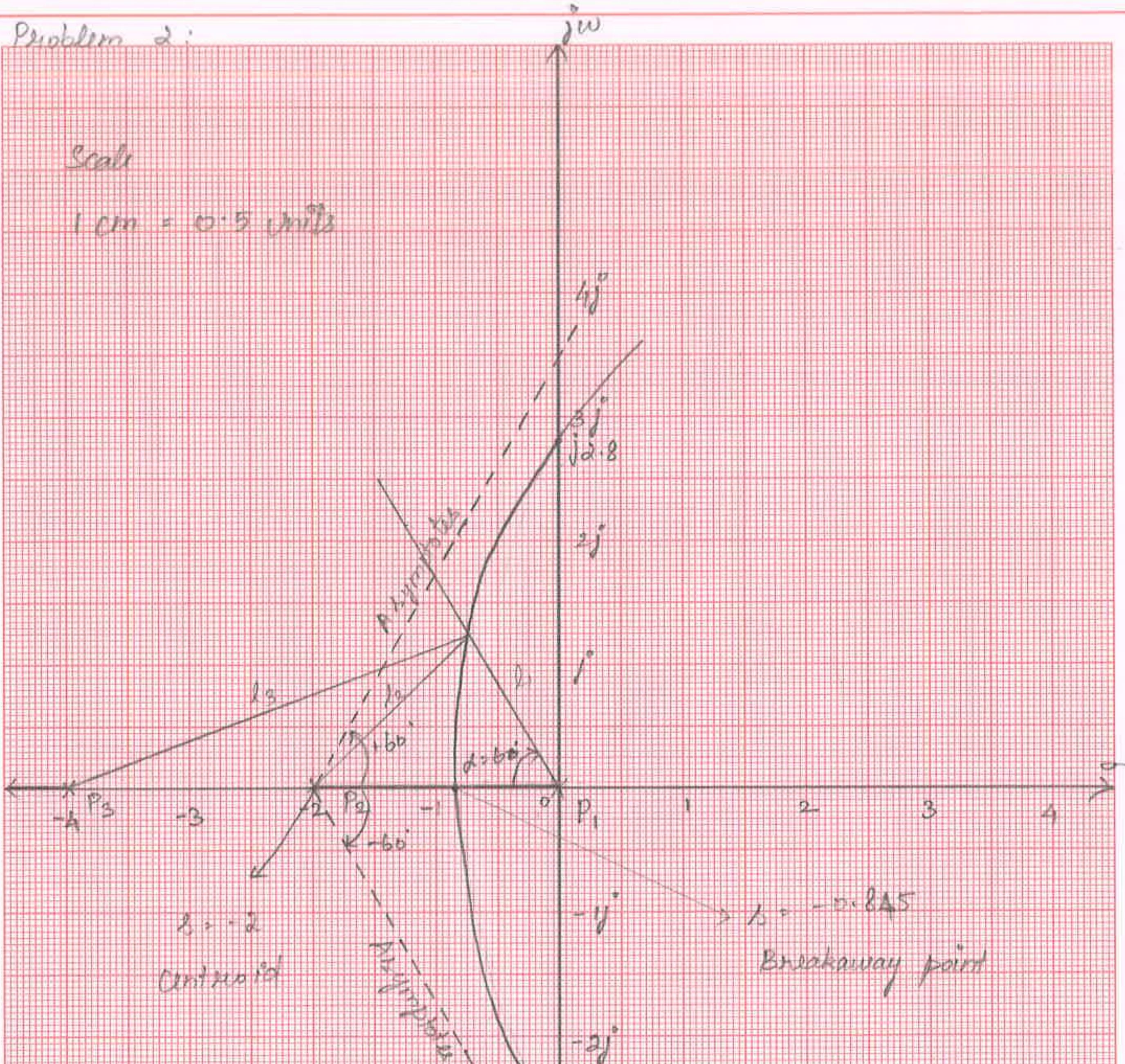
$\zeta = 0.5$.

$$\alpha = \cos^{-1}\zeta = \cos^{-1}0.5 = 60^\circ$$

Problem 2:

Scale

1 cm = 0.5 units



$\alpha = \cos^{-1}\zeta = \cos^{-1} 0.5 = 60^\circ$

$l_1 = 2.6$ cm $= 2.6 \times 0.5 = 1.3$ Units

$l_2 = 3.5$ cm $= 3.5 \times 0.5 = 1.75$ Units

$l_3 = 7$ cm $= 7 \times 0.5 = 3.5$ Units

$K = \dfrac{l_1 \times l_2 \times l_3}{1}$

$= 1.3 \times 1.75 \times 3.5$

$= 7.96 \approx 8$

Draw angle between OP and negative real axis is 60° $(\alpha = 60^\circ)$ ⇒ Dominant pole ⇒ S_d.

$$K_{sd} = \frac{\text{Product of length of Vector from all poles to the point, } s = s_d}{\text{Product of length of Vector from all zeros to the point, } s = s_d}$$

$$= \frac{l_1 \times l_2 \times l_3}{1}$$

$$= \frac{1.3 \times 1.75 \times 3.5}{1}$$

$$= 7.96 \approx 8.$$

Problem 3:

Sketch the root locus for the unity feedback system whose OLTF $G(s) = \dfrac{K}{s(s+4)(s^2+4s+20)}$ (MJ -2014)

Sol:

Step 1: To locate poles and zeros

$$s(s+4)(s^2+4s+20) = 0$$

$$s = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 20}}{2} = -2 \pm j4$$

poles ⇒ $s = 0$

$s = -4$

$s = -2 + j4$

$s = -2 - j4$

zeros are lying at infinity.

poles $\Rightarrow P_1 \quad P_2 \quad P_3 \quad P_4$.

Where $P_1 = 0$

$P_2 = -4$

$P_3 = -2+j4$

$P_4 = -2-j4$

Step 2 : To find root locus on real axis.

$s=0$ $s = 0$ to -4 Root locus.

odd (1)

$s = -4$, $s = -4$ $s = -4$ to $-\infty$ Not root locus

Total $\Rightarrow$ (2)

even.

Step 3: To find asymptotes and centroid.

$n = 4, \quad m = 0.$

$n - m = 4.$

$q = 0, 1 \ldots n-m \Rightarrow q = 0, 1, 2, 3, 4$

At $q = 0 \Rightarrow$ Angles $= \pm \dfrac{180(2(0)+1)}{4}$

$= \pm \dfrac{180}{4}$

$q = 1. \Rightarrow$ Angles $= \pm \dfrac{180(2+1)}{4} = \dfrac{540}{4} = 135°$

$q = 2 \Rightarrow$ Angles $= \pm \dfrac{(180(5))}{4} \Rightarrow 225 = 45$

$q = 3 \Rightarrow$ Angles $= \pm \dfrac{180(7)}{4} \Rightarrow 315$

centroid = $\dfrac{\text{sum of poles} - \text{sum of zeros}}{n-m}$

$= \dfrac{0-4-2+j4-2-j4-0}{4-0} = \dfrac{-8}{4} = -2$

Step 4: To find the breakaway and breakin point.

CLTF $\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)H(s)}$

$= \dfrac{K/s(s+4)(s^2+4s+20)}{1+K/s(s+4)(s^2+4s+20)}$

$= \dfrac{K}{s(s+4)(s^2+4s+20)+K}$

C.E

C.E $\Rightarrow s(s+4)(s^2+4s+20)+K=0$

$K = -[(s^2+4s)(s^2+4s+20)] \rightarrow (1)$

$= -[s^4+4s^3+20s^2+4s^3+16s^2+80s]$

$K = -[s^4+8s^3+36s^2+80s]$

$\dfrac{dK}{ds} = -[4s^3+24s^2+72s+80]$

By Lin's method or by calc.

$s_1 = -2$

$s_2 = -2+2.45j$

$s_3 = -2-2.45j$

For check. Sub s value in ①

$$s = -2 \Rightarrow k = -[s^4 + 8s^3 + 36s^2 + 80s]$$

$$= -[(-2)^4 + 8(-2)^3 + 36(-2)^2 + 80(-2)]$$

$$= 64.$$

$$s = -2-2j \Rightarrow k = -[(-2-2j)^3(-2-2j) + 8(-2-2j)^3 + 36(-2-2j)^2 + 80(-2-2j)]$$

$$= 99.99 \cong 100$$

$$s = -2+2j \Rightarrow k = -[(-2+2j)^3(-2+2j) + 8(-2+2j)^3 + 36(-2+2j)^2 + 80(-2+2j)]$$

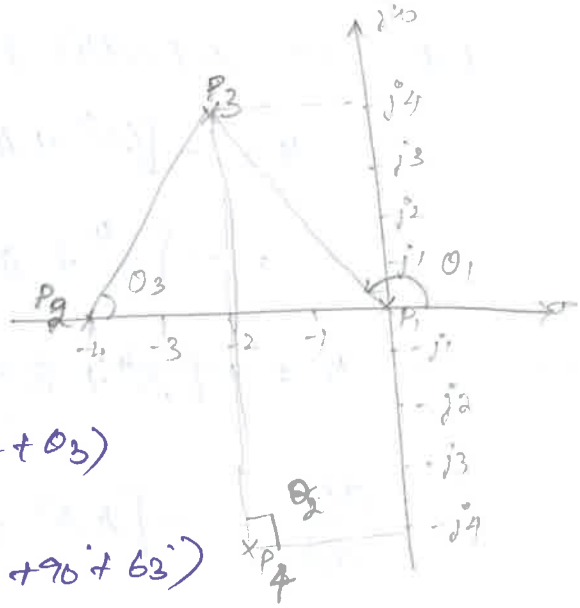
$$= 100$$

Step 5 : To find angle of departure :

$$\theta_1 = 180° - \tan^{-1}\frac{4}{2} = 117.$$

$$\theta_2 = 90°$$

$$\theta_3 = \tan^{-1}\frac{4}{2} = 63$$



Angle of departure from complex pole P_3 $\Big\}$ $= 180 - (\theta_1 + \theta_2 + \theta_3)$

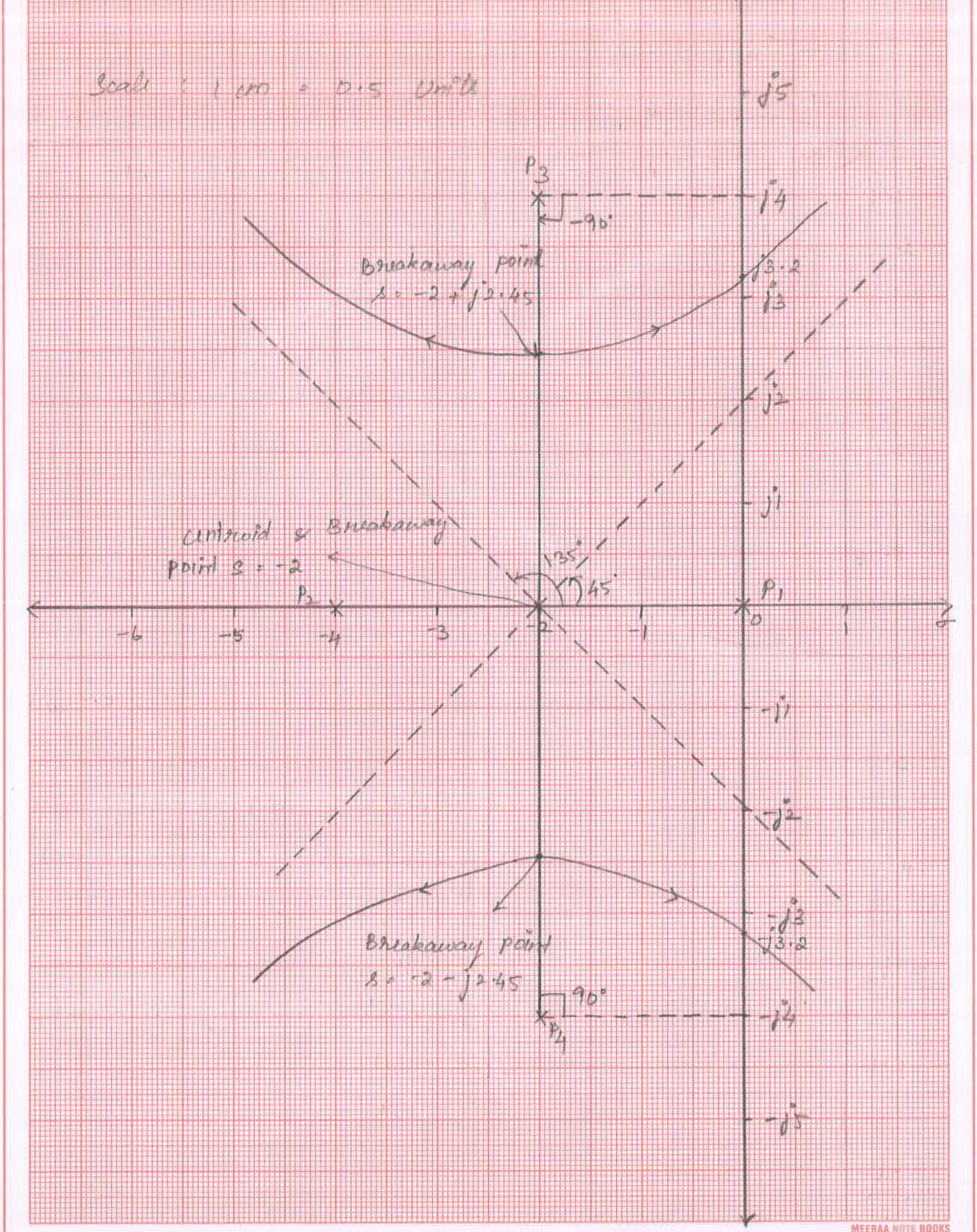
$$= 180° - (117 + 90° + 63°)$$

$$= -90°$$

Angle of departure from complex pole P_4 $\Big\}$ $= -[P_3]$

$$= +90°$$

Problem 3 : ROOT LOCUS



Step 6: To find crossing point on imaginary axis.

$$C.E \Rightarrow s^4 + 8s^3 + 36s^2 + 80s + k = 0$$

$s = j\omega$

$$(j\omega)^4 + 8(j\omega)^3 + 36(j\omega)^2 + 80(j\omega) + k = 0$$

$$\omega^4 - 8\omega^3 j - 36\omega^2 + 80j\omega + k = 0$$

Imaginary part

$$-8\omega^3 j + 80 j\omega = 0$$

$$8\omega^3 j = 80 j\omega$$

$$\omega^2 = 10$$

$$\omega = \pm\sqrt{10} = \pm 3.2$$

$$\boxed{\omega = \pm 3.2}$$

Real part.

$$\omega^4 - 36\omega^2 + k = 0$$

$$(\omega^2)^2 - 36\omega^2 + k = 0$$

$$k = 36(10) - (10)^2$$

$$= 360 - 100$$

$$\boxed{k = 260}$$

RELATIVE STABILITY:

The relative stability indicates the closeness of the system to stable region. It is an indication of the strength of degree of stability.

GAIN MARGIN:

Gain margin is a factor by which the system gain can be increased to drive the system to the verge of instability.

$$\text{Gain margin}, k_g = \frac{1}{|G(j\omega)H(j\omega)|}\bigg|_{\omega=\omega_{pc}} = \frac{1}{G_A}$$

$$\text{Gain margin in db} = 20\log\frac{1}{|G(j\omega)H(j\omega)|} = 20\log\frac{1}{G_A}$$

$$= -20\log G_A$$

PHASE MARGIN :

phase margin is defined as the amount of additional phase lag at gain cross over frequency required to bring the system to verge of instability.

$$\phi_{gc} = \angle G(j\omega) H(j\omega) \big|_{\omega = \omega_{gc}} \Rightarrow -180^\circ + \gamma = \phi_{gc}$$

phase margin, $\gamma = 180^\circ + \phi_{gc}$

Problem 1:

The open loop T.F of a system is $G(s) = \frac{K}{s(1+0.1s)(1+s)}$

i) Determine the value of K so that gain margin is 6db.

ii) Determine the value of K so that phase margin is 40°.

Sol:

i) to find K for specified gain margin

$$G(s) = \frac{K}{s(1+0.1s)(1+s)}$$

$s = j\omega$

$$G(j\omega) = \frac{K}{j\omega(1+j0.1\omega)(1+j\omega)}$$

$$= \frac{K}{j\omega(1+j1.1\omega - 0.1\omega^2)}$$

$$= \frac{K}{-1.1\omega^2 + j\omega(1-0.1\omega^2)}$$

Imaginary part $= 0$.

$\omega = \omega_{pc} \Rightarrow \omega_{pc}(1 - 0.1\omega^2) = 0$.

$$1 - 0.1\omega_{pc}^2 = 0$$

$$-0.1\omega_{pc}^2 = -1$$

$$\omega_{pc} = \frac{1}{\sqrt{0.1}} = 3.162 \text{ rad/sec}$$

$$|G(j\omega)|_{\omega=\omega_{pc}} = \left|\frac{K}{-1.1\omega^2}\right|_{\omega=\omega_{pc}}$$

$$= \frac{K}{1.1 \times 3.162^2} = 0.0909\,K$$

Gain margin = 6 db.

$$20 \log K_g = 6$$

$$\log K_g = \frac{6}{20}$$

$$\text{Gain margin}, \; K_g = 10^{6/20} = 1.9953$$

By definition of GM,

$$GM, \; K_g = \frac{1}{|G(j\omega)|_{\omega=\omega_{pc}}}$$

$$1.9953 = \frac{1}{0.0909K}$$

$$K = \frac{1}{0.0909 \times 1.9953}$$

$$\boxed{K = 5.5135}$$

ii) To find K for specified phase margin.

$$G(s) = \frac{K}{s(1+0.1s)(1+s)}$$

$s = j\omega$

$$G(j\omega) = \frac{K}{j\omega(1+j0.1\omega)(1+j\omega)}$$

$$= \frac{K}{\omega\angle 90^\circ \sqrt{1+(0.1\omega)^2}\angle \tan^{-1}0.1\omega \sqrt{1+\omega^2}\angle \tan^{-1}\omega}$$

$$|G(j\omega)| = \frac{K}{\omega\sqrt{1+0.01\omega^2}\sqrt{1+\omega^2}}$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}0.1\omega - \tan^{-1}\omega \Rightarrow \phi_{gc}$$

P.M $\gamma = 180^\circ + \phi_{gc}$

$$40^\circ = 180^\circ - 90^\circ - \tan^{-1}0.1\omega_{gc} - \tan^{-1}\omega_{gc}.$$

$$\tan^{-1}0.1\omega_{gc} + \tan^{-1}\omega_{gc} = 180^\circ - 90^\circ - 40^\circ$$

$$\tan^{-1}0.1\omega_{gc} + \tan^{-1}\omega_{gc} = 50^\circ \qquad \left[\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\tan B}\right.$$

put tan on either. Side,

$$\tan[\tan^{-1}0.1\omega_{gc} + \tan^{-1}\omega_{gc}] = \tan 50^\circ$$

$$\frac{\tan(\tan^{-1}0.1\omega_{gc}) + \tan(\tan^{-1}\omega_{gc})}{1-\tan(\tan^{-1}0.1\omega_{gc}) \times \tan(\tan^{-1}\omega_{gc})} = \tan 50$$

$$\frac{0.1\omega_{gc} + \omega_{gc}}{1-0.1\omega_{gc}\times\omega_{gc}} = 1.192 \Rightarrow \frac{1.1\omega_{gc}}{1-0.1\omega_{gc}^2} = 1.192$$

$$1.1\omega_{gc} = 1.192(1-0.1\omega_{gc}^2) \Rightarrow 0.1192\omega_{gc}^2 + 1.1\omega_{gc} - 1.192 = 0$$

$$\omega_{gc}^2 + \frac{1.1}{0.1192}\omega_{gc} - \frac{1.192}{0.1192} = 0$$

$$\omega_{gc}^2 + 9.228\omega_{gc} - 10 = 0$$

$$\omega_{gc} = \frac{-9.228 \pm \sqrt{9.228^2 + 4\times10}}{2} = \frac{-9.228 \pm 11.1873}{2}$$

Take +ve Value,

$$\omega_{gc} = \frac{-9.228 + 11.1873}{2} = 0.98 \text{ rad/sec}$$

At $\omega = \omega_{gc}$, $|G(j\omega)| = 1$

$$|G(j\omega)|_{\omega=\omega_{gc}} = \frac{K}{\omega_{gc}\sqrt{1+0.01\omega_{gc}^2}\sqrt{1+\omega_{gc}^2}} = 1$$

$$K = \omega_{gc}\sqrt{1+0.01\,\omega_{gc}^2}\sqrt{1+\omega_{gc}^2}$$

$$= 0.98\sqrt{1+0.01\times 0.98^2}\sqrt{1+0.98^2}$$

$$\boxed{K = 1.3787}$$

Result :

For a GM of 6 db , $K = 5.5135$

For a PM of 40°, $K = 1.3787$.

NYQUIST STABILITY CRITERION :

Consider CLTF $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$

C.E $\Rightarrow$ $1+G(s)H(s) = 0$

Let $F(s) = 1+G(s)H(s)$

LTF $G(s)H(s)$ can be expressed as

$$G(s)H(s) = \frac{K(s+z_1)(s+z_2)\dots(s+z_m)}{(s+P_1)(s+P_2)\dots(s+P_n)} \quad \text{where } m \le n. \rightarrow (1)$$

$$F(s) = 1+G(s)H(s) = 1 + \frac{K(s+z_1)(s+z_2)\dots(s+z_m)}{(s+P_1)(s+P_2)\dots(s+P_n)}$$

$$= \frac{(s+P_1)(s+P_2)\dots(s+P_n) + K(s+z_1)(s+z_2)\dots(s+z_m)}{(s+P_1)(s+P_2)\dots(s+P_n)}$$

$$= \frac{(s+z_1')(s+z_2')\dots(s+z_n')}{(s+P_1)(s+P_2)\dots(s+P_n)} \rightarrow (2)$$

6. what is auxillary polynomial ?

In the construction of routh array a row of all zero indicates the existence of an even polynomial as a factor of the given characteristics equation. This even polynomial factor is called auxillary polynomial.

7. what is quadrantal symmetry ?

The symmetry of roots with respect to both real and imaginary axis is called quadrantal symmetry.

8. what is limitedly stable system ?

For a bounded input signal, if the output has constant amplitude oscillations then the system may be stable or unstable under some limited constraints, such systems is called limitedly stable.

9. what is nyquist stability criterion ?

If $G(s)H(s)$ - contour in $G(s)H(s)$ -plane corresponding to nyquist contour in S-plane encircles the point $-1+j0$ in the anti-clockwise direction as many as the number of right half of S-plane poles of $G(s)H(s)$. Then the closed loop system is stable.

10. what is root locus ?

The path taken by a root of characteristics equation when open loop gain K, is varied from 0 to ∞ is called root locus.

11. what is magnitude criterion?

The magnitude condition states that $s = s_a$ will be a point on root locus if for that value of s magnitude of $G(s).H(s)$ is equal to 1.

12. what is angle criterion?

The angle criterion states that $s = s_a$ will be a point on root locus if for that value of s the argument or phase of $G(s).H(s)$ is equal to an odd multiple of $180°$.

13. what are asymptote?

Asymptote are straight lines which are parallel to root locus going ~~to~~ (∞) infinity and meet the root locus at infinity.

$$\text{Angles of asymptote} = \frac{\pm 180° (2q+1)}{n-m} \; ; \; q = 0, 1, 2 \ldots (n-m)$$

14. what is centroid?

The meeting point of asymptote with real axis is called centroid. The centroid is given by

$$\text{centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m}$$

15. what are Breakaway and Break in point?

At break away point the root locus breaks from the real axis to enter into the complex plane.

At breakin point the root locus enters the real axis from the complex plane.

UNIT - V CONTROL SYSTEM ANALYSIS USING STATE VARIABLE METHODS

State Variable representation - conversion of state Variable models to transfer functions - conversion of transfer functions to state Variable models - Solution of state equations - concepts of controllability and observability Stability of linear systems - Equivalence between transfer function and state variable representations - State Variable analysis of digital control system - Digital control design using state feedback.

STATE SPACE ANALYSIS :

The State Variable approach is a powerful technique for the analysis and design of control systems.

The state space analysis is a modern approach and also easier for analysis using digital computers. It can be applied for any type of systems. The analysis can be carried with initial conditions and can be carried on multiple input and multiple output systems.

STATE SPACE FORMULATION :

The state of a dynamic system is minimal set of variables such that the knowledge of these Variables at $t = t_0$ together with the knowledge of the inputs for $t \geq t_0$, completely determines the behaviour of the system for $t > t_0$

ii) computation of e^{At} using L.T.

$$e^{At} = \phi(t)$$

$$\Rightarrow e^{At} = L^{-1}\left[(SI - A)^{-1}\right]$$

Problem 1:

Consider the matrix A. Compute e^{At} by two methods.

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

Sol:

METHOD 1: by matrix exponential.

$$e^{At} = \left[I + At + \frac{1}{2!} A^2 t^2 + \frac{1}{3!} A^3 t^3 + \cdots\right]$$

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$A^2 = AA = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -3 \\ 6 & 7 \end{bmatrix}$$

$$A^3 = A^2 \times A = \begin{bmatrix} -2 & -3 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 7 \\ -14 & -15 \end{bmatrix}$$

$$A^4 = A^3 \times A = \begin{bmatrix} 6 & 7 \\ -14 & -15 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -14 & -15 \\ 30 & 31 \end{bmatrix}$$

$$\therefore\ e^{At} = I + At + \frac{1}{2!}A^2t^2 + \frac{1}{3!}A^3t^3 + \frac{1}{4!}A^4t^4 + \dots$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} t + \begin{bmatrix} -2 & -3 \\ 6 & 7 \end{bmatrix} \frac{t^2}{2} + \begin{bmatrix} 6 & 7 \\ -14 & -15 \end{bmatrix} \frac{t^3}{6} + \begin{bmatrix} -14 & -15 \\ 30 & 31 \end{bmatrix} \frac{t^4}{24}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & t \\ -2t & -3t \end{bmatrix} + \begin{bmatrix} -2t^2/2 & -3t^2/2 \\ 3t^2 & 7t^2/2 \end{bmatrix} + \begin{bmatrix} t^3 & \frac{7t^3}{6} \\ -\frac{7t^3}{3} & -\frac{5t^3}{2} \end{bmatrix} + \begin{bmatrix} -\frac{7t^4}{12} & -\frac{5t^4}{8} \\ \frac{5t^4}{4} & \frac{31t^4}{24} \end{bmatrix}$$

$$= \begin{bmatrix} 1 - t^2 + t^3 - \frac{7t^4}{12} + \dots & t - \frac{3t^2}{2} + \frac{7t^3}{6} - \frac{5t^4}{8} + \dots \\ -2t + 3t^2 - \frac{7t^3}{3} + \frac{5t^4}{4} + \dots & 1 - 3t + \frac{7t^2}{2} - \frac{5t^3}{2} + \frac{31t^4}{24} + \dots \end{bmatrix}$$

By power series,

$$e^{At} = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix} //$$

Method 2: by Laplace Transform method.

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$e^{At} = \phi(t) = L^{-1}\left[(sI - A)^{-1}\right]$$

Transform this state model into a canonical state model. Also compute the state transition matrix e^{At}.

Sol :

To find eigen values.

$$\lambda I - A = \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 6 & 11 & \lambda+6 \end{bmatrix}$$

$$|\lambda I - A| = \lambda[\lambda(\lambda+6)+11] + 1(6)$$

$$= \lambda(\lambda^2 + 6\lambda + 11) + 6$$

$$= \lambda^3 + 6\lambda^2 + 11\lambda + 6$$

Eigen values, $\lambda = -1, -2, -3$

The λ value are distinct.

$$M = \begin{bmatrix} 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \end{bmatrix}$$ by Vander monde matrix.

$$= \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix}$$

$$M^{-1} = \frac{1}{|M|} \text{adj } M.$$

$$\text{Adj } M = [\text{cof } M]^T$$

$$\text{cof. of } 1 = -18 + 12 = -6$$

$$\text{cof. of } 1 = -(-9+3) = 6$$

$$1 = -4+2 = -2$$

$$-1 = -(9-4) = -5$$

$$-2 = 9-1 = 8$$

$$-3 = -(4-1) = -3$$

$$1 = -3+2 = -1$$

$$4 = -(-3+1) = 2$$

$$9 = -2+1 = -1$$

$$\text{cof. } M = \begin{bmatrix} -6 & 6 & -2 \\ -5 & 8 & -3 \\ -1 & 2 & -1 \end{bmatrix}$$

$$[\text{cof. } M]^T = \text{Adj } M = \begin{bmatrix} -6 & -5 & -1 \\ 6 & 8 & 2 \\ -2 & -3 & -1 \end{bmatrix}$$

$$|M| = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix}$$

$$= 1(-18+12) - 1(-9+3) + 1(-4+2)$$

$$= -6 + 6 - 2 = -2$$

$$|M| = -2 \#$$

$$= \begin{bmatrix} 3e^{-t}-3e^{-2t}+e^{-3t} & 2.5e^{-t}-4e^{-2t}+1.5e^{-3t} & 0.5e^{-t}-e^{-2t}+0.5e^{-3t} \\ -3e^{-t}+6e^{-2t}-3e^{-3t} & -2.5e^{-t}+8e^{-2t}-4.5e^{-3t} & -0.5e^{-t}+2e^{-2t}-1.5e^{-3t} \\ 3e^{-t}-12e^{-2t}+9e^{-3t} & 2.5e^{-2t}-16e^{-2t}+13.5e^{-3t} & 0.5e^{-t}-4e^{-2t}+4.5e^{-3t} \end{bmatrix}$$

CONCEPT OF CONTROLLABILITY AND OBSERVABILITY :

CONTROLLABILITY :

A system is said to be completely state controllable if it is possible to transfer the system state from any initial state $x(t_0)$ to any other desired state $x(t_d)$ in specified finite time by a control vector $u(t)$.

There are two methods.

i) Gilberts method

In a transformed state model either

matrix has distinct eigenvalue,

$$\dot{z} = \Lambda z + \tilde{B}U$$

$$y = \tilde{C}z + DU$$

matrix has repeated eigen values.

$$\dot{z} = Jz + \tilde{B}U$$

$$y = \tilde{C}z + DU.$$

The system is completely controllable if the elements of any row of $\tilde{B}$ that corresponds to the last row of each Jordan block are not all zeros and the row corresponding to the state variable must not have all zero.

In $\tilde{B}$, it should have all row zero.

ii) Kalman's method :

$$Q_c = \left[B \quad AB \quad A^2B \ldots \quad A^{n-1}B \right]$$

$n \Rightarrow$ order (or) no. of state variables.

$|Q_c| \neq 0$ condition of controllability.

OBSERVABILITY :

A system is said to be completely observable if every state $x(t)$ can be completely identified by measurements of the output $y(t)$ over a finite time interval.

There are two methods,

i) Gilbert's method :

The necessary and sufficient condition for completely observability is that none of the columns of matrix $\tilde{C}$ be zero.

If any of the columns of $\tilde{C}$ has all zero's then the corresponding state variable is not observable.

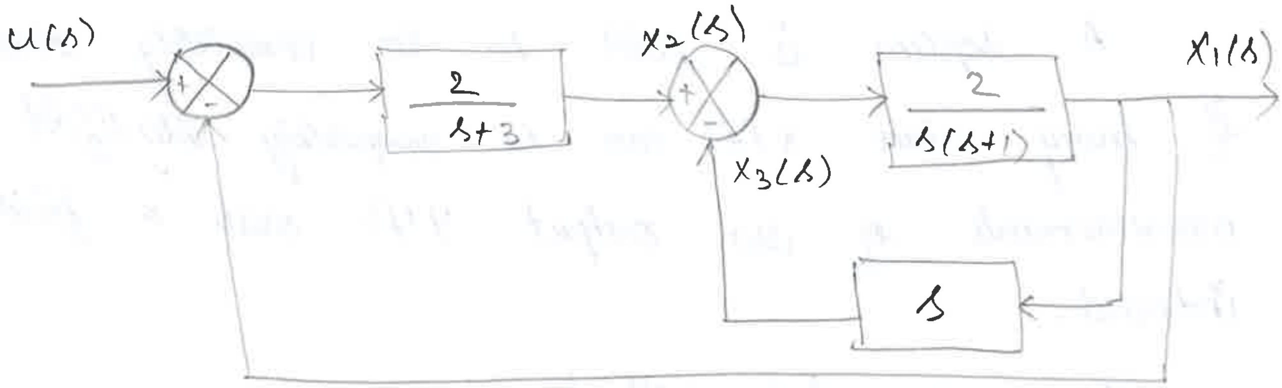
ii) Kalman's test:

$$Q_0 = \left[C^T \quad A^T C^T \quad (A^T)^2 C^T \ldots\ldots (A^T)^{n-1} C^T \right]$$

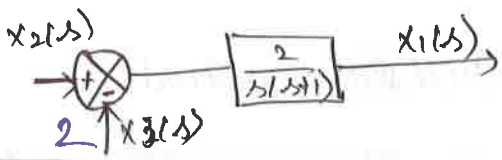
& $|Q_0| \neq 0$ is the condition for observability.

Problem 1:

Write the state equations for the given system in which X_1 X_2 and X_3 constitute a state vector. Determine the system is controllable and observable.



Sol:



From dig,

$$X_1(s) = \left[X_2(s) - X_3(s) \right] \frac{2}{s(s+1)}$$

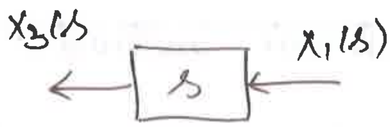
$$X_1(s) \left[s(s+1) \right] = 2X_2(s) - 2X_3(s)$$

$$s^2 X_1(s) + sX_1(s) - 2X_2(s) + 2X_3(s) = 0$$

ILT:

$$\ddot{x}_1 + \dot{x}_1 - 2x_2 + 2x_3 = 0 \quad \rightarrow (1)$$

From dig:

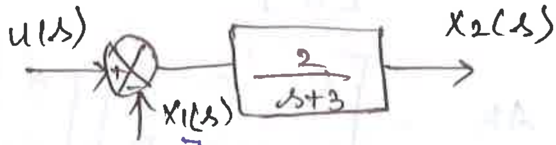


$$X_3(s) = s\,X_1(s)$$

ILT:

$$X_3(s) = \dot{x}_1(s) \rightarrow ②$$

From dig:



$$X_2(s) = \frac{2}{s+3}\left[U(s) - X_1(s)\right]$$

$$X_2(s)\left[s+3\right] = 2U(s) - 2X_1(s)$$

$$sX_2(s) + 3X_2(s) + 2X_1(s) = 2U(s)$$

ILT:

$$\dot{x}_2 + 3x_2 + 2x_1 = 2U \rightarrow ③$$

From ①, ② & ③

State eq:

$$\dot{x}_1 = x_3$$

$$\dot{x}_2 = -2x_1 - 3x_2 + 2u$$

$$\dot{x}_3 = 2x - 3x_3$$

o/p eq: $y = x_1$

State model in matrix form is,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 0 & 1\\ -2 & -3 & 0\\ 0 & 2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 2\\ 0\end{bmatrix}\left[u\right]$$

$$y = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

Both H_1 and H_2 should be positive otherwise is the condition of stability.

BILINEAR TRANSFORMATION :

In Bilinear transformation,

$$r = \frac{z-1}{z+1} \quad \text{or} \quad z = \frac{1+r}{1-r}$$

It is substituted in characteristics polynomial and with Routh array stability is found.

Problem 1:

Check for stability of the sampled data control system represented by the following characteristics equation

a) $5z^2 - 2z + z = 0$

b) $z^4 - 1 - 7z^2 + 1.04z - 0.268z + 0.024 = 0$.

Sol s

a) $5z^2 - 2z + z = 0$

$$a_2 z^2 + a_1 z + a_0$$

necessary condition

$$F(z) = 5z^2 - 2z + z$$

$$F(1) = 5 - 2 + 2 = 5 > 0$$

$$(-1)^n F(-1) = (-1)^2 [5 + 2 + 2] = 9 > 0$$

$\therefore$ Necessary conditions are satisfied.

DIGITAL CONTROL SYSTEM:

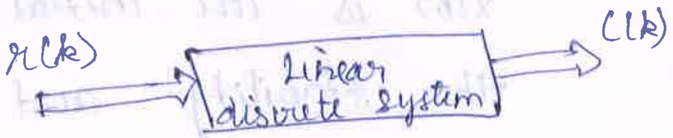
The signals which are varying continuously with respect to time are called continuous time signals.

The discrete time signals are defined only at discrete instants of time. It is defined only for integer values of independent variable, time.

The system using the combination of continuous time signals and discrete time signals are called Discrete data systems or Digital control systems.

Open loop sampled data systems:

$$G(z) = \frac{C(z)}{R(z)}$$



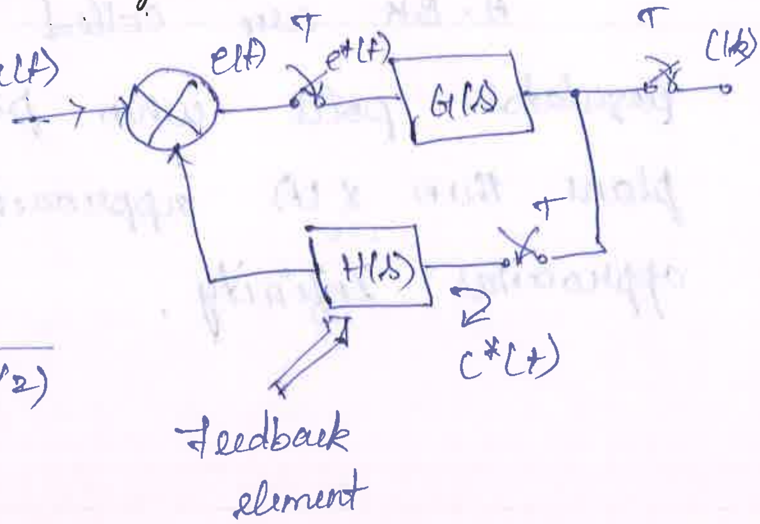
$$= \frac{Z[c(k)]}{Z[r(k)]}$$

Closed loop sampled Data System:

$$T(z) = \frac{C^*(s)}{R^*(s)}$$

$$= \frac{C(z)}{R(z)}$$

$$= \frac{G(z)}{1 + G(z)H(z)}$$



Concept of State Feedback:

Consider a control system,

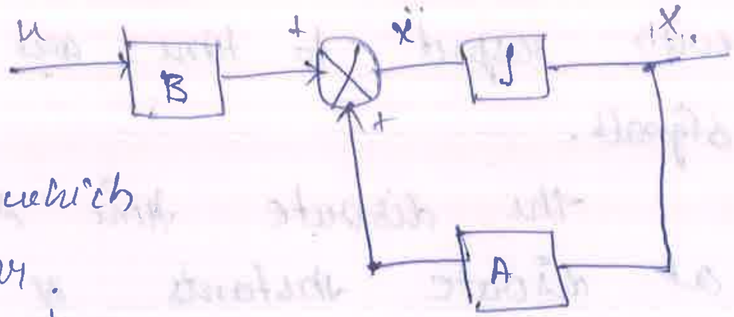
$\dot{x} = Ax + Bu$

x is state vector.

u is control signal which is scalar.

A is $n \times n$ state matrix

B is $n \times 1$ constant matrix.

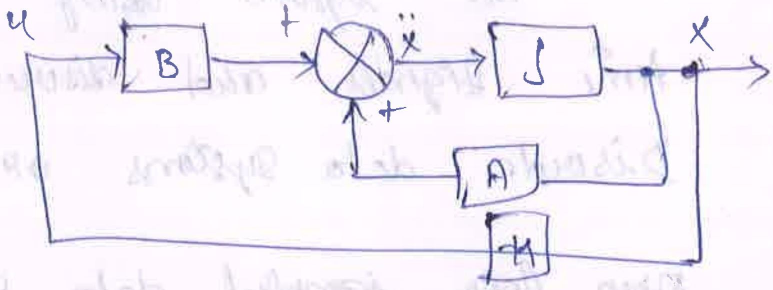


open loop control system.

$u = -Kx$ state

$\dot{x} = Ax + B(-Kx)$

$= (A - BK)x$.



System with state feedback.

$x(t) = e$

$x(0)$ is the initial state.

The stability and the transient response characteristics are determined by the eigen values of matrix $A - BK$. $[x(0) \neq 0$.

$A - BK$ are called regulator poles. These regulator poles when placed in left half of s plane then $x(t)$ approaches zero as term t approaches infinity.

$a_0 z^2 + a_1 z^1 + a_2$

$$X = \begin{bmatrix} 5 & -2 & 2 \\ 0 & 5 & -2 \\ 0 & 0 & 5 \end{bmatrix} \quad ; \quad Y = \begin{bmatrix} 2 & 0 & 0 \end{bmatrix}$$

$H_1 = X + Y \quad ; \quad H_2 = X - Y$

$$H_1 = \begin{bmatrix} 7 & -2 & 2 \\ 0 & 5 & -2 \\ 0 & 0 & 5 \end{bmatrix} \Rightarrow |H_1| = 175$$

$$H_2 = \begin{bmatrix} 3 & -2 & 2 \\ 0 & 5 & -2 \\ 0 & 0 & 5 \end{bmatrix} \Rightarrow |H_2| = 75$$

H_1 & H_2 are positive innerwise

So the system is stable.

b.) $F(z) = z^3 - 0.2z^2 - 0.25z + 0.05 = 0$

$$F(r) = \left(\frac{1+r}{1-r}\right)^3 - 0.2\left(\frac{1+r}{1-r}\right)^2 - 0.5\left(\frac{1+r}{1-r}\right) + 0.05 = 0$$

$$= 0.9r^3 + 3.6r^2 + 2.9r + 0.6 = 0$$

r^3 :	0.9	2.9
r^2 :	3.6	0.6
r^1 :	2.75	
r^0 :	0.6	

The system is stable.

3. compute e^{At}

i) $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ ii) $A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}$

4. A linear time invariant system is described by the following state model. Obtain the canonical form of the system model and check the system is controllable and observable by both method.

i) $$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -2 & -2 & 1 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u \quad \text{and}$$

$$y = \begin{bmatrix} 3 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

ii) $$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} u \quad \text{and}$$

$$y = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Reg. No. :

Question Paper Code: E3080

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2010

Fourth Semester

Electronics and Communication Engineering

EC2255 — CONTROL SYSTEMS

(Regulation 2008)

Time: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A — ($10 \times 2 = 20$ Marks)

1. Compare the open loop system with closed loop system.

2. Draw the analogous electrical network for the mechanical system in Figure 1 using force-voltage analogy.

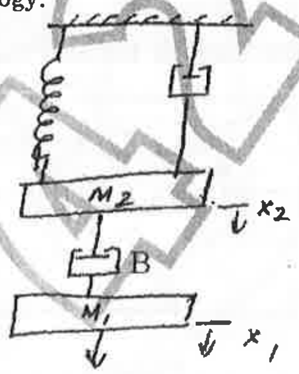


Figure 1

3. The block diagram shown in the Figure 2 represents a heat treating oven. The set paint is 1000° C. What is the steady-state temperature?

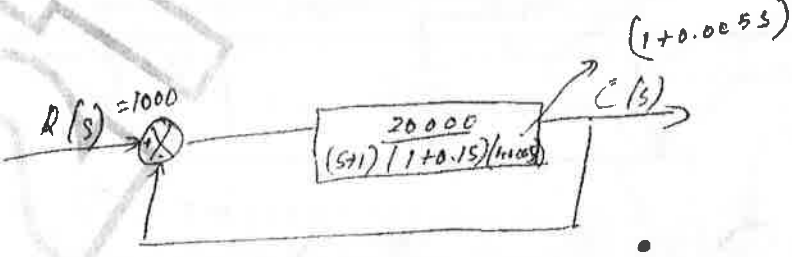


Figure 2

14. (a) Draw the root locus for the open-loop transfer function of a unity feedback control system give below and determine.

(i) the value of K for $G = 0.5$

(ii) the value of K for marginal stability

(iii) the value of K at $S = -4$.

$$G(s) = \frac{K}{s(s+1)(s+3)}.$$

Or

(b) (i) Using Routh criterion, investigate the stability of a unity feedback control system whose open-loop transfer function is given by

$$G(s) = \frac{e^{-st}}{s(s+2)}.$$

(ii) A Closed loop control system has the characteristic equation given by $s^3 + 4.5s^2 + 3.5s + 1.5 = 0$.

15. (a) (i) Using cascade method decompose the transfer function $\dfrac{Y(s)}{U(s)} = \dfrac{s+3}{(s+1)(s+2)}$ and obtain the state model.

(ii) Obtain state space representation for the electrical network shown in Figure 7 below.

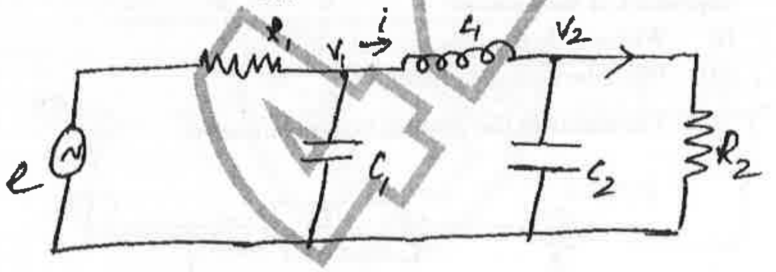


Figure 7

Or

(b) (i) Determine the transfer matrix from the data given below:

$$A = \begin{bmatrix} -3 & 1 \\ 0 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix} \quad D = 0.$$

(ii) The transfer function of a control system is given by

$$\frac{Y(s)}{U(s)} = \frac{s+2}{s^3 + 9s^2 + 26s + 24}.$$

Check for controllability.

Reg. No.

Question Paper Code: 55336

B.E./B.Tech. DEGREE EXAMINATIONS, NOV./DEC. 2011
Regulations 2008

Fourth Semester

Electronics and Communication Engineering

EC 2255 Control Systems

Time: Three Hours

Maximum: 100 marks

Answer ALL Questions

Part A - (10 x 2 = 20 marks)

1. Define open-loop and closed-loop control systems.

2. What is meant by 'block diagram' of a control system? What are the basic components of a block diagram?

3. The damping ratio and natural frequency of oscillation of a second order system is 0.5 and 8 rad/sec respectively. Calculate resonant peak and resonant frequency.

4. With reference to time response of a control system, define 'Rise time'.

5. Name the parameters which constitute frequency domain specifications.

6. Write the MATLAB command for plotting Bode diagram $\frac{Y(s)}{U(s)} = \frac{4s+6}{s^3+3s^2+8s+6}$.

7. State any two limitations of Routh-stability criterion.

8. Define stability of a system.

9. What are the advantages of State-Space approach?

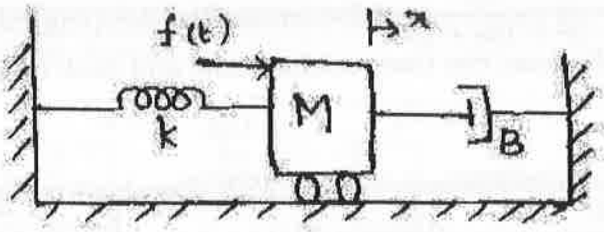
10. What is 'alias' in sampling process?

(ii) List the advantages of Routh's array method of examining stability of a control system. (4)

OR

14. (b) Sketch the Nyquist plot for a system with open loop transfer function $G(s)H(s) = \dfrac{K(1+0.4s)(s+1)}{(1+8s)(s-1)}$ and determine the range of K for which the system is stable. (16)

15. (a) (i) Find the state variable equation for a mechanical system (spring-mass-damper system) shown below. (8)



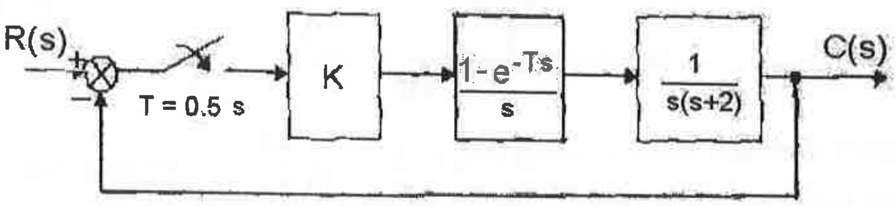
(ii) A LTI system is characterized by the state equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

where u is a unit step function. Compute the solution of these equation assuming initial condition $x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Use inverse Laplace transform technique. (8)

OR

15. (b) A sampled data control systems is shown in the figure below:



Find the open loop pulse transfer function, if the controller gain is unity with sampling time 0.5 seconds. (16)

Reg. No. :

Question Paper Code : 10296

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2012.

Fourth Semester

Electronics and Communication Engineering

EC 2255/147405/EC 46/EE 1256 A/10144 EC 406/080290023 — CONTROL SYSTEMS

(Regulation 2008)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. What are the advantages of the closed loop control system?

2. What are the properties of signal flow graphs?

3. List the advantages of generalized error coefficients.

4. Why derivative controller is not used in control system?

5. Draw the polar plot of $G(s) = \frac{1}{(1+sT)}$.

6. State the uses of Nicholas chart.

7. What is root locus?

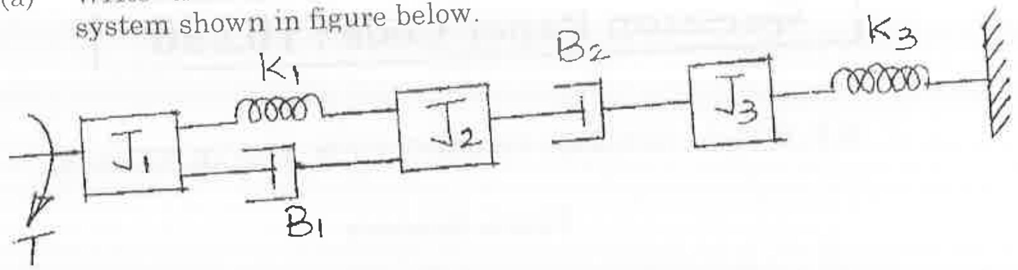
8. State Nyquist stability criterion.

9. How the modal matrix is determined?

10. What is meant by quantization?

PART B — (5 × 16 = 80 marks)

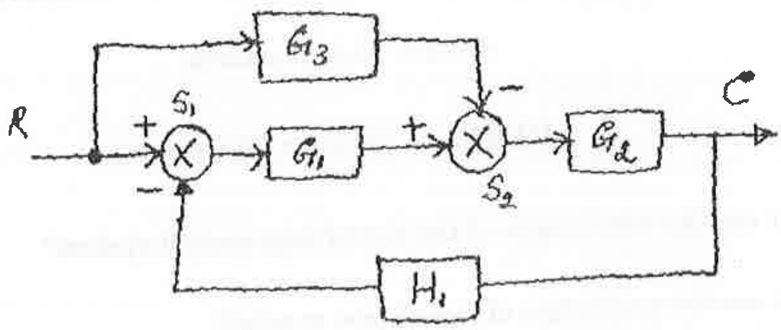
11. (a) Write the differential equations governing the mechanical rotational system shown in figure below.



Draw the torque-voltage and torque-current electrical analogous circuits and verify by writing mesh and node equations. (16)

Or

(b) (i) Using block diagram reduction technique, find the closed-loop transfer function C/R of the system whose block diagram is shown below. (8)



(ii) Construct the signal flow graph for the following set of simultaneous equations.

$$X_2 = A_{21}X_1 + A_{23}X_3$$
$$X_3 = A_{31}X_1 + A_{32}X_2 + A_{33}X_3$$
$$X_4 = A_{42}X_2 + A_{43}X_3$$

and obtain the overall transfer function using Mason's gain formula. (8)

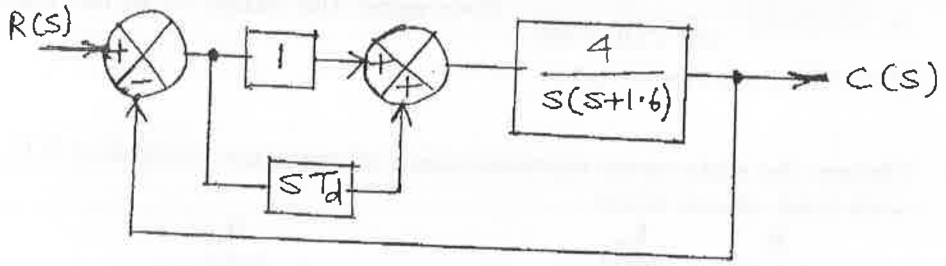
12. (a) (i) The open loop transfer function of a unity feedback control system is given by $G(s) = \dfrac{K}{s(sT+1)}$ where K and T are positive constants. By what factor should the amplifier gain be reduced so that the peak over-shoot of unit step response of the system is reduced from 75% to 25%. (8)

(ii) A certain unity negative feedback control system has the following forward path transfer function $G(s) = \frac{K(s+2)}{s(s+5)(4s+1)}$. The input applied is $r(t) = 1 + 3t$. Find the minimum value of K so that the steady state error is less than 1. (8)

Or

(b) (i) Discuss the effect of derivative control on the performance of a second order system. (8)

(ii) Figure shows PD controller used for a system.



Determine the value of T_d so that system will be critically damped. Calculate its settling time. (8)

13. (a) (i) For the following transfer function,

$$G(s) = \frac{K(s+3)}{s(s+1)(s+2)}$$

sketch the Bode magnitude plot by showing slope contributions from each pole and zero. (8)

(ii) For an unity feedback system with closed loop transfer function $\frac{G(s)}{1+G(s)}$ derive the equations for the locus of constant M circles and constant N circles. (8)

Or

(b) (i) Write the procedure to obtain Nichol's chart from Constant M circles. (8)

(ii) Write a MATLAB program to examine stability using Bode plot for the given transfer function $G(s) = \frac{20_e^{-0.2s}}{s(s+2)(s+8)}$. Explain the code (statements) as to what the variables and numbers mean and also what action is caused by each statement. State also how you will interpret the result. (8)

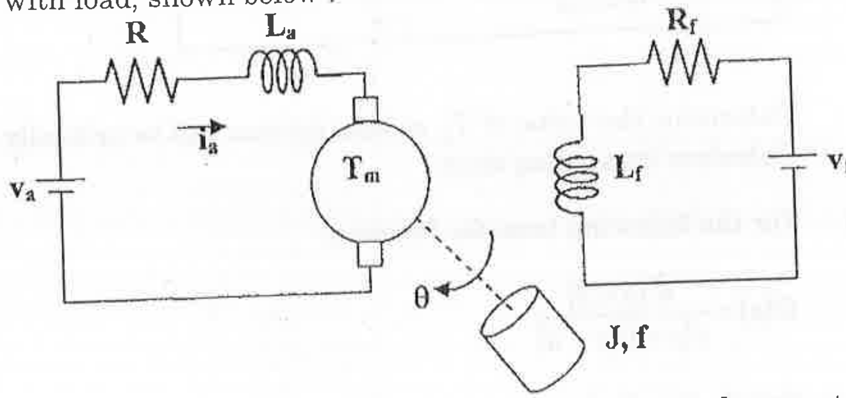
14. (a) (i) Construct Routh array and determine the stability of the system whose characteristic equation is $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$. (6)

(ii) Sketch the root locus of the system whose open loop transfer function is $G(s) = \dfrac{K}{s(s+2)(s+4)}$. Find the value of K so that the damping ratio of the closed loop system is 0.5. (10)

Or

(b) Draw the Nyquist plot for the system whose open loop transfer function is $G(s)H(s) = \dfrac{K}{s(s+2)(s+10)}$. Determine the range of K for which the closed loop system is stable. (16)

15. (a) Obtain the state space representation of armature controlled D.C. motor with load, shown below :



Choose the armature current i_a, the angular displacement of shaft θ, and the speed $\dfrac{d\theta}{dt}$ as state variables and θ as output variables. (16)

Or

(b) (i) The state model matrices of a system are given below :

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 3 & 4 & 1 \end{bmatrix}$$

Evaluate the observability of the system using Gilbert's test. (10)

(ii) Find the controllability of the system described by the following equation :

$$\dot{X} = \begin{bmatrix} -1 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U(t).$$ (6)

Reg. No. :

Question Paper Code : 11330

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2012.

Fourth Semester

Electronics and Communication Engineering

EC 2255/147405/EC 46/EE 1256 A/10144 EC 406/080290023 — CONTROL SYSTEMS

(Regulation 2008)

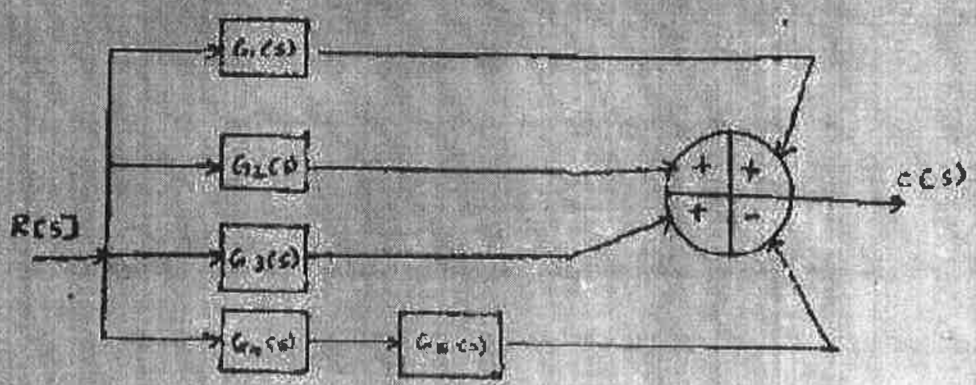
Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What are the advantages of the closed loop control system?

2. Write down the transfer function of the system whose block diagram is shown below.



3. What are transient and steady state responses of a control system?

4. With reference to time response of a control system, define 'peak time'.

5. What is meant by 'Corner frequency' in frequency response analysis?

6. What is Nichols chart?

7. State any two limitations of Routh stability criterion.

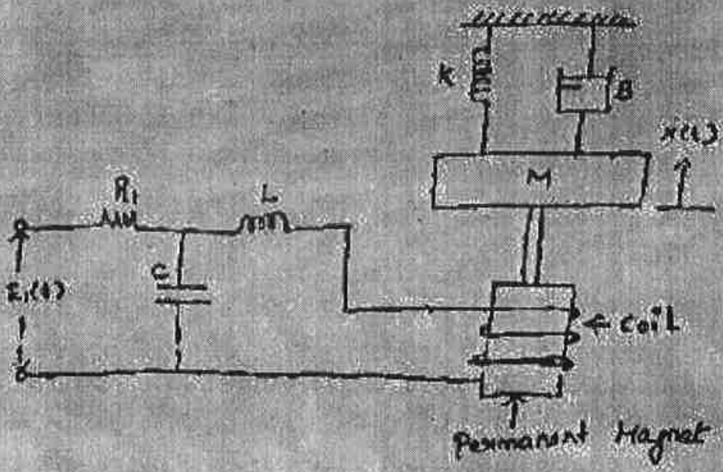
8. State the advantages of Nyquist stability criterion over that of Routh's criterion.

9. Define 'state' and 'state—variables'.

10. What is meant by Sampled—data control systems?

PART B — (5 × 16 = 80 marks)

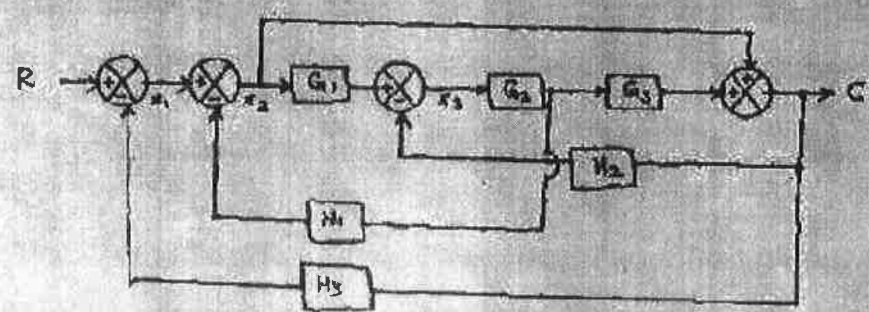
11. (a) In the system shown in figure below, R, L and C are electrical parameters while K, M and B are mechanical parameters.



Find the transfer function $X(s)/E_1(s)$ for the system, where $E_1(t)$ is input voltage while x(t) is the output displacement. (16)

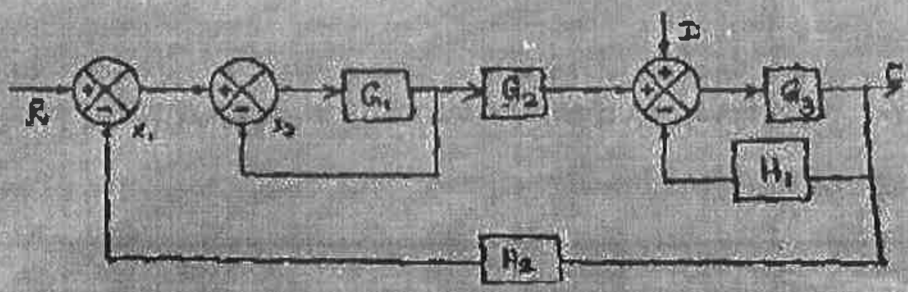
Or

(b) (i) A block diagram shown below.



Construct the equivalent Signal Flow Graph and obtain $\frac{C}{R}$ using Mason's formula. (8)

(ii) For the block diagram shown below, find the output C due to R and disturbance D. (8)



12. (a) (i) The unity feedback system is characterized by an open loop transfer function $G(s) = \frac{K}{s(s+10)}$. Determine the gain K, so that the system will have a damping ratio of 0.5. For this value of K, determine settling time, peak overshoot and time to peak overshoot for a unit step input. (8)

(ii) A unity feedback system has the forward transfer function $G(s) = \frac{K_1(2s+1)}{s(5s+1)(1+s)^2}$. The input $r(t) = (1+6t)$ is applied to the system. Determine the minimum value of K_1, if the steady error is to be less than 0.1. (8)

Or

(b) With suitable block diagrams and equations, explain the following types of controllers employed in control systems:

(i) Proportional controller (4)

(ii) Proportional-plus-integral controller (4)

(iii) PID controller (4)

(iv) Integral controller. (4)

13. (a) Given $G(s) = \frac{Ke^{-0.2s}}{s(s+2)(s+8)}$, find K for the following two cases:

(i) Gain margin equal to 6 db

(ii) Phase margin equal to 45°. (16)

Or

(b) Draw the pole-zero diagram of a lead compensator. Propose lead compensation using electrical network. Derive the transfer function. Draw the Bode plots. (16)

14. (a) (i) Determine the range of K for stability of unity feedback system whose open loop transfer function is $G(s)=\frac{K}{s(s+1)(s+2)}$ using Routh stability criterion. (6)

(ii) Draw the approximate root locus diagram for a closed loop system whose loop transfer function is given by $G(s)H(s)=\frac{K}{s(s+5)(s+10)}$ Comment on the stability. (10)

Or

(b) Sketch the Nyquist plot for a system with open loop transfer function $G(s)H(s)=\frac{K(1+0.4s)(s+1)}{(1+8s)(s-1)}$ and determine the range of K for which the system is stable. (16)

15. (a) The state space representation of a system is given below:

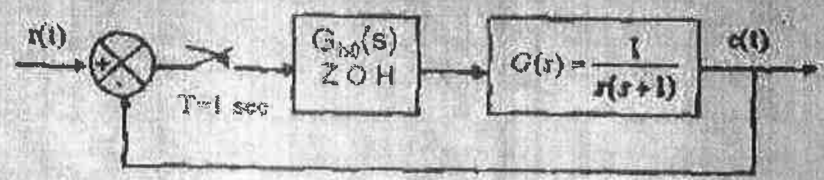
$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{pmatrix}=\begin{pmatrix}-2 & 1 & 0\\ 0 & -3 & 1\\ -3 & -4 & -5\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}+\begin{pmatrix}0\\ 0\\ 1\end{pmatrix}u \quad y=(0\ \ 1\ \ 0)\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$$ obtain the transfer function. (16)

Or

(b) (i) Determine the controllability and observability of the following system:

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{pmatrix}=\begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -2 & -3\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}+\begin{pmatrix}0\\ 0\\ 10\end{pmatrix}u \quad y=[1\ \ 0\ \ 0]\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}.$$ (8)

(ii) Obtain the z - domain transfer function of the system shown below. (8)



Reg. No. :

Question Paper Code : 21360

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2013.

Fourth Semester

Electronics and Communication Engineering

EC 2255/EC 46/EE 1256 A/10144 EC 406/080290023 — CONTROL SYSTEMS

(Regulation 2008/2010)

Time : Three hours — Maximum : 100 marks

Graph sheet and Semi-log sheet are to be provided

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Name any two dynamic models used to represent control systems.

2. Write the Mason's gain formula of signal flow graph.

3. The closed loop transfer function of a second order system is given by $\frac{400}{s^2+2s+400}$. Determine the damping ratio and natural frequency of oscillation.

4. Give the steady state errors to a various standard inputs for type-2 system.

5. Draw the polar plot of an integral term transfer function.

6. Write the MATLAB statement to draw the Bode plot of the given system.

7. Write the necessary and sufficient condition for stability in Routh stability criterion.

8. Define Nyquist stability criterion.

9. What are the advantages of state space representation?

10. Define state and state variable.

Reg. No. :

Question Paper Code : 31360

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Fourth Semester

Electronics and Communication Engineering

EC 2255/EC 46/ EE 1256 A/ 080290023/10144 EC 406 — CONTROL SYSTEMS

(Regulation 2008/2010)

(Bode plot, Graph sheet, Semi – log, Nichol's chart are permitted)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Define Transfer function.

2. Define resistance and capacitance of liquid level system.

3. What are the units of K_p, K_v and K_a?

4. What is the effect of PI controller on the system performance?

5. Define phase margin.

6. State Nyquist stability criterion for a closed loop system when the open loop system is stable.

7. What are constant M and N circles?

8. State the property of a lead compensator.

9. Define state equation.

10. Give the concept of controllability.

Reg. No. :

Question Paper Code : 51402

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2014.

Fourth Semester

Electronics and Communication Engineering

EC 2255/EC 46/EE 1256 A/080290023/10144 EC 406 — CONTROL SYSTEMS

(Regulation 2008/2010)

Time : Three hours Maximum : 100 marks

(Bode plot, Graph Sheet, Semi–log, Nichol's chart are permitted)

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. What are the characteristics of negative feedback?

2. State Mason's Gain formula.

3. What are type 0 and type 1 systems?

4. What is meant by rise time?

5. The damping ratio and the undamped natural frequency of a second order system are 0.5 and 5 respectively. Calculate the Resonant frequency.

6. What is Corner frequency?

7. What is meant by relative stability?

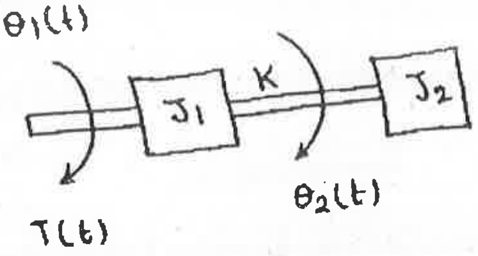
8. Define phase margin.

9. Draw the circuit diagram of sample and hold circuit.

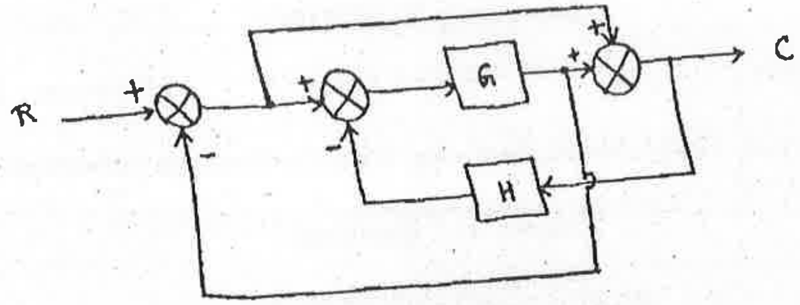
10. What are the properties of State Transition matrix?

PART B — (5 × 16 = 80 marks)

11. (a) (i) Write the torque equation of the rotational system shown below and derive the expression for transfer function $\theta_1(s)/T(s)$. (8)



(ii) Determine the overall transfer function of the system represented by the block diagram. (8)



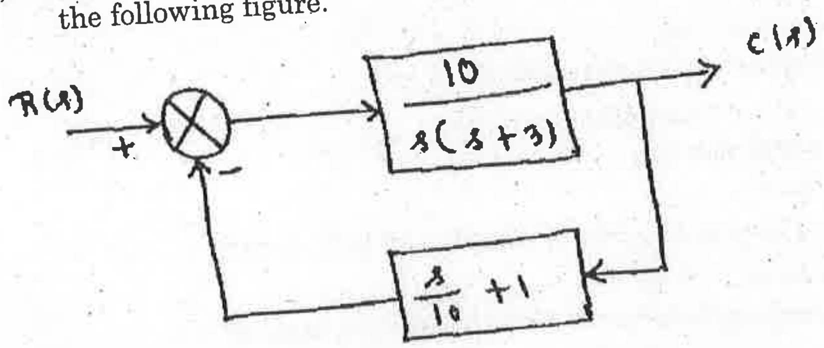
Or

(b) (i) Derive the expression for the Transfer function of Armature controlled DC motor. (8)

(ii) Draw the signal flow graph for the following system and obtain the Transfer function using Mason Gain formula. (8)

$$x_2 = a_{12}x_1 + a_{22}x_2 + a_{32}x_3$$
$$x_3 = a_{23}x_2 + a_{43}x_4$$
$$x_4 = a_{24}x_2 + a_{34}x_3 + a_{44}x_4$$
$$x_5 = a_{25}x_2 + a_{45}x_4$$

12. (a) (i) Determine the unit step response of the control system shown in the following figure. (8)

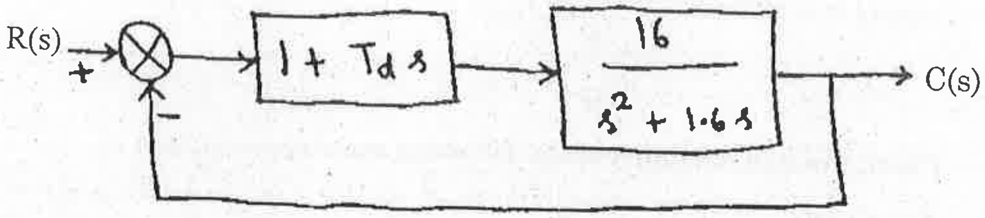


(ii) The open loop transfer function of a unity feedback system is given by $G(s) = \frac{20}{s(s+2)}$. The input function is r(t) = 2 +3t +t². Determine generalized error coefficient and steady state error. (8)

Or

(b) (i) A unity feedback system is characterized by an open loop transfer function $G(s) = \frac{k}{s(s+10)}$. Determine the gain k so that the system will have a damping ratio of 0.5. For this value of k, determine peak overshoot and peak time for a unit step input. (8)

(ii) The following diagram shows a unity feedback system with derivative control. By using this derivative control the damping ratio is to be made 0.5. Determine the value of Td. (8)



13. (a) The open loop transfer function of a system is given by

$$G(s)H(s) = \frac{30}{s(1+0.5s)(1+0.08s)}$$

Draw the Bode plot and determine Gain margin and Phase margin. (16)

Or

(b) (i) Sketch the polar plot of the unity feedback system with open loop transfer function $G(s) = \frac{1}{s(s+1)^2}$. Also find the frequency at which $|G(jw)| = 1$. (10)

(ii) What are the advantages and disadvantages of frequency response analysis? (6)

14. (a) Draw the root locus plot for the system whose open loop transfer function is given by $G(s)H(s) = \frac{k}{s(s+4)(s^2+4s+13)}$.

Find the marginal value of k which causes sustained oscillations and the frequency of these oscillations. (16)

Or

(b) (i) The open loop transfer function is given by

$$G(s) = \frac{k}{s(1+0.1s)(1+s)}$$

For this unity feedback system, determine the value of k so that the gain margin is 6dB. (8)

(ii) By using Routh Criterion, determine the stability of the system represented by following characteristic equation (8)

$$s^5 + s^4 + 2s^3 + 2s^2 + 11s + 10 = 0$$

15. (a) (i) Obtain the state model of the system described by the following transfer function. $\frac{y(s)}{u(s)} = \frac{5}{s^3 + 6s + 7}$. (8)

(ii) Obtain the state transition matrix for the state model whose system matrix A is given by $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. (8)

Or

(b) (i) Check the controllability of the following state space system (8)

$$\dot{x}_1 = x_2 + u_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = -2x_2 - 3x_3 + u_1 + u_2$$

(ii) Obtain the transfer function model for the following state space system. (8)

$$A = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \end{bmatrix}$$

Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 91404

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2014.

Fourth Semester

Electronics and Communication Engineering

EC 2255/EC 46/EE 1256 A/ 080290023/10144 EC 406 — CONTROL SYSTEMS

(Regulation 2008/2010)

(Common to 10144 EC 406 – Control Systems for B.E. (Part-Time) Third Semester – ECE – Regulation 2010)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Differentiate open and closed loop control system.

2. Write Mason's gain Formula.

3. What are the various time domain specifications?

4. What is the effect of PI controller on the system performance?

5. Define Gain and phase margin.

6. State the usage of Nichol's chart in control system analysis.

7. Define BIBO stability.

8. What is meant by dominant pole?

9. Define state model of n^{th} order system.

10. What are sampler and hold circuits?

PART B — (5 × 16 = 80 marks)

11. (a) Write the differential equations governing the mechanical translational system as shown in figure 11(a). Draw the Force – Voltage and Force – Current electrical analogous circuits and verify by mesh and node equations. (16)

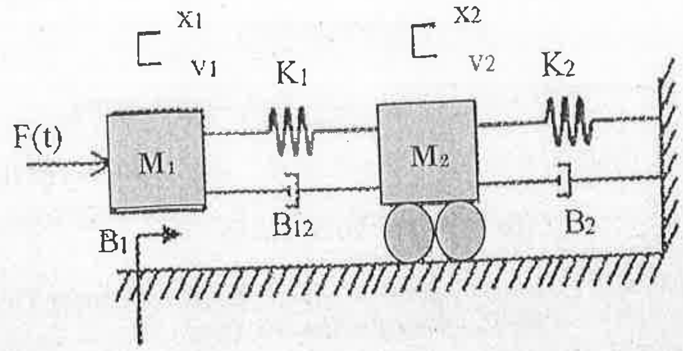


Fig.11(a)

Or

(b) (i) The signal flow graph for a feedback control system is shown in figure 11(b)(i). Determine the closed loop transfer function $C(s)/R(s)$. (12)

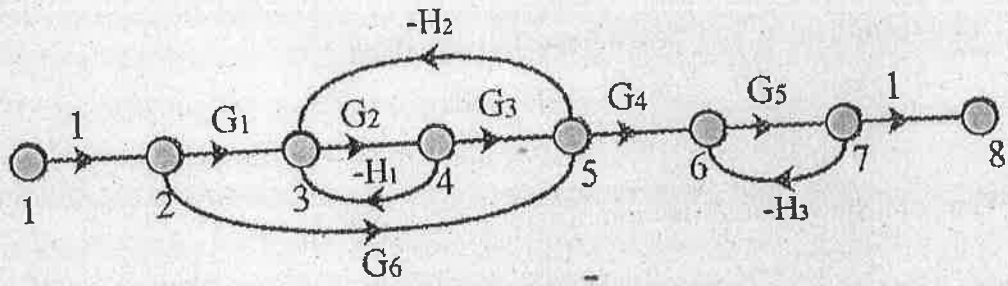


Fig.11(b)(i)

(ii) State any four block diagram reduction rules. (4)

12. (a) (i) What are the various standard test signals? Draw the characteristic diagram and obtain the mathematical representation of all. (8)

(ii) Calculate the following parameters for the system whose natural frequency of oscillations is 10 rad/sec and damping factor is 0.707

(1) Delay time

(2) Rise time

(3) Peak overshoot

(4) Settling time. (8)

Or

(b) (i) Determine the steady state errors for the following inputs 5u(t), 5tu(t), $5t^2u(t)$ to a system whose open-loop transfer function is given by $G(s) = \frac{[100(s+2)(s+6)]}{[(s(s+3)(s+4))]}$. (8)

(ii) With its block diagram explain the concepts of PI and PD compensation. (8)

13. (a) The open loop transfer function of a unity feedback system is given by $G(s) = 1/[s(1+s)(1+2s)]$.

Sketch the polar plot and determine the gain and phase margin. (16)

Or

(b) (i) Describe about Lead-lag compensators design procedure. (8)

(ii) Write short notes on constant M and N circles. (8)

14. (a) (i) Obtain Routh array for the system whose characteristic polynomial equation is

$$s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$$

Check the stability. (8)

(ii) Define Nyquist stability criterion and explain the different situations of it. (8)

Or

(b) Sketch the root locus for the open loop transfer function of unity feedback control system given below. (16)

$$G(s) = k/[s(s^2 + 4s + 13)]$$

15. (a) Test the controllability and observability of the system whose state space representation is given as (16)

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ -2 & -3 & 0 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} u$$

$$y = [1 \quad 0 \quad 0] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}.$$

Or

(b) (i) State and explain sampling theorem. (4)

(ii) A discrete system is described by the difference equation (12)

$$y(k+2) + 5y(k+1) + 6(yk) = u(k)$$

$$y(0) = y(1) = 0;\ T = 1\,\text{sec}.$$

Determine the state model in canonical form. Draw the block diagram.

Reg. No. :

Question Paper Code : 71451

B.E/B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

Fourth Semester

Electronics and Communication Engineering

EC 2255/EC 46/EE 1256 A/080290023/10144 EC 406 — CONTROL SYSTEMS

(Regulation 2008/2010)

(Common to 10144 EC 406 — Control Systems for B.E. (Part – Time) Third Semester ECE – Regulation 2010)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the transfer function of the network given in fig. 1.

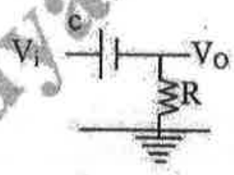


Fig. 1

2. State Mason's gain formula.

3. How do you find the type of a system?

4. Find the unit impulse response of system $H(S) = 5/(s+4)$ with zero initial conditions.

5. What is the use of Nichol's chart?

6. List the advantages and disadvantages of phase lag network.

7. Find the range of K for closed loop stable behavior of system with characteristic equation $4s^4 + 24s^3 + 44s^2 + 24s + K$ using Routh Hurwitz stability criterion.

8. What is the angle of asymptotes in the Root Locus of a system with n poles and m zeros?

9. Define controllability of a system.

10. State Sampling theorem.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Explain the functional blocks of closed loop feedback control system. (6)

$x_1(t)\{\ \ ] = 0$

(ii) Derive the transfer function of system shown in fig. 2. (10)

x_1 $M_1 \frac{d^2 x_2}{dt^2} + B \frac{dx_2}{dt} = 0$ B

x_2 $M_2 \frac{d^2 x_2}{dt^2} + K_2 x_2 + B \frac{d(x_2 - x_1)}{dt} = 0$ B_1

$M_3 \frac{d^2 x_1}{dt^2} + B_1 \frac{d(x_1 - x_2)}{dt} = 0$

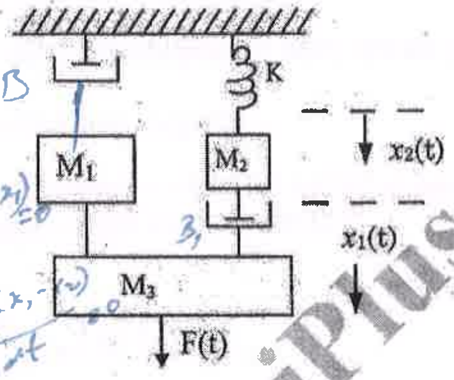


Fig. 2

Or

(b) Find the transfer function of the system shown in fig. 3 using block diagram reduction technique and signal flow graph technique.

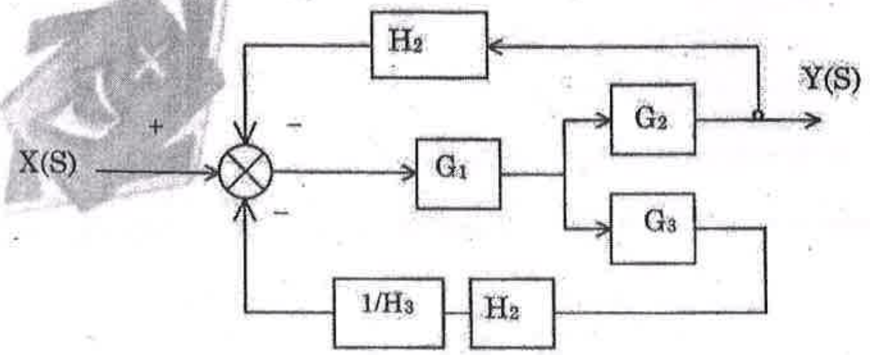


Fig. 3

12. (a) (i) For the system shown in figure 4. find the error using dynamic error coefficient method for input $r(t) = 5 + 4t + 7t^2$.

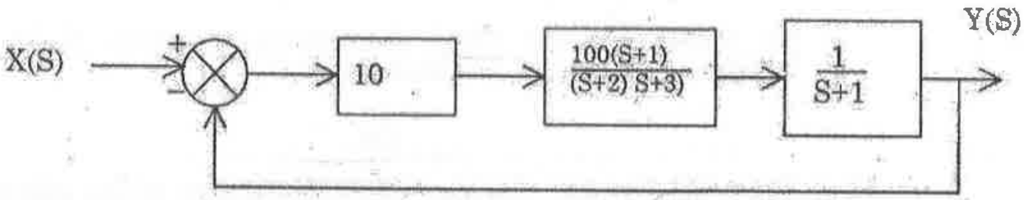


Fig. 4

(ii) Briefly discuss about transient response specifications.

Or

(b) (i) For the system shown in fig. 5. find the effect of PD controller with Td = 1/10 on peak overshoot and settling time when it is excited by unit step input.

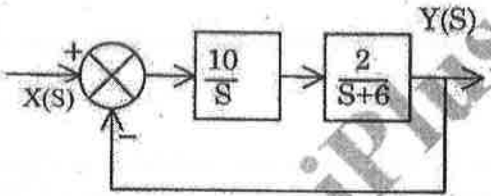


Fig. 5

(ii) Discuss the effect of PI controller in the forward path of a system.

13. (a) A robotic arm has a joint control loop transfer function $G_c(s)G(s) = \dfrac{300(s+100)}{s(s+10)(s+40)}$. Prove that the frequency equals 28.3 rad/s when the phase angle is $-180°$. Find the magnitude at that frequency.

Or

(b) Derive the transfer function of the compensating network and the type of compensation given in figure 6 and draw the Bode plot.

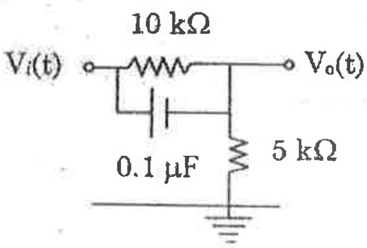


Fig. 6

14. (a) A single loop negative feedback system has a loop transfer function $G_c(s)G(s) = \frac{k(s+2)^2}{s(s^2+1)(s+8)}$. Sketch the root locus as a function of K. Find the range of K for which the system is stable. K for which purely imaginary roots exist and find the roots.

Or

(b) Draw the Nyquist plot and find the stability of the following open loop transfer function of unity feedback control system. $G(s)H(s) = K(s+4)/\left(s^2(s+2)\right)$ If the system is conditionally stable, find the range of K for which the system is stable.

15. (a) Consider a system with state-space model given below.

$$x = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} u; y = \begin{bmatrix} 2 & -4 & 0 \end{bmatrix} x + (0)u$$

Verify that the system is observable and controllable.

Or

(b) Explain the functional modules of closed loop sampled data system, and compare its performance with open loop sampled data system.

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Reg. No. : | | | | | | | | | | | |

Question Paper Code : 21451

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Fourth Semester

Electronics and Communication Engineering

EC 2255/EC 46/EE 1256 A/080290023/10144 EC 406 — CONTROL SYSTEMS

(Regulations 2008/2010)

(Common to 10144 EC 406 — Control Systems for B.E. (Part – Time) Third Semester ECE — Regulations 2010)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. What is the main advantages of closed loop system over open loop systems?
2. Write the mathematical expressions for step input and impulse input.
3. Mention few applications of Bode plot.
4. State Routh Hurwitz criterion.
5. Define State space.
6. What is meant by sample and hold?
7. Define Mason's gain formula.
8. Define rise time and peak overshoot.
9. Define Nyquist stability criterion.
10. What is gain margin and phase margin?

PART B — (5 × 16 = 80 marks)

11\. (a) Derive the transfer function of a RLC series circuit.

Or

(b) With a neat diagram, derive the transfer function of a field controlled dc motor.

12\. (a) Derive an expression for unit step response of a second order control system.

Or

(b) Write explanatory notes on PI and PD controllers.

13\. (a) Define all the frequency domain specifications of a second order control system after plotting the response.

Or

(b) Describe the procedure for obtaining the polar plot for a system whose loop transfer function is $\frac{4}{(s+2)(s+4)}$.

14\. (a) Describe the procedure for obtaining the root locus for a system.

Or

(b) Determine the closed loop stability of the system using Nyquist stability criterion $G(s) = \frac{2}{s^2(s+2)}$.

15\. (a) Explain how controllability and observability for a system can be tested, with an example.

Or

(b) Write the explanatory notes on open loop and closed loop sampled data systems.

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Reg. No. | | | | | | | | | | | | |

Question Paper Code : 51451

B.E./B. Tech. DEGREE EXAMINATION, MAY/JUNE 2016

Fourth Semester

Electronics and Communication Engineering

EC 2255/EC 46/EE 1256 A/080290023/10144 EC 406 – CONTROL SYSTEMS

(Regulations 2008/2010)

(Common to 10144 EC 406 – Control Systems for B.E. (Part-time) Third Semester ECE – Regulations 2010)

Time : Three Hours **Maximum : 100 Marks**

(Provide Graph Sheet, Semilog sheet)

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. Draw the equivalent block diagrams for the figures 1 and 2 given below :

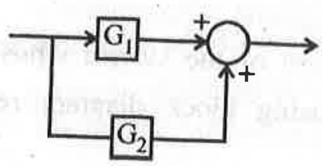
(i) 

Figure-1

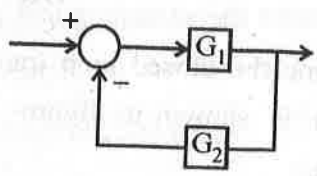
(ii) 

Figure-2

2. List any two properties of signal flow graph.

3. Define steady state error.

4. Write the expression for the transfer function of PI Controller.

5. Define phase margin.

6. What is the use of M and N circles ?

7. State Routh-Hurwitz stability criterion.

8. List any two advantages of Nyquist stability criterion.

9. Define observability.

10. State sampling theorem.

PART – B (5 × 16 = 80 Marks)

11. (a) (i) A certain system is described by the differential equation, $\frac{d^2y}{dt^2} + 14\frac{dy}{dt} + 40y = 5$. Find the expression for y(t), assuming initial conditions to be zero. **(8)**

(ii) Find the transfer function of the electric circuit shown in figure 11(a) (ii). **(8)**

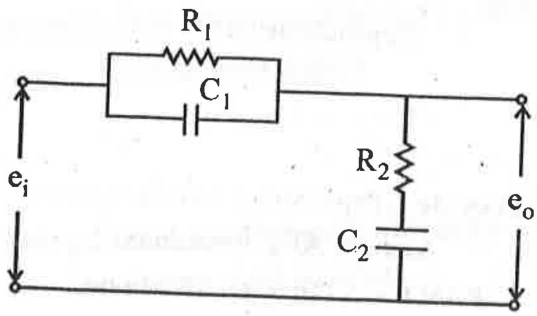


Figure-11(a) (ii)

OR

(b) (i) Determine the closed loop transfer function of the system whose block diagram is shown in figure 11(b)(i), using block diagram reduction technique. **(8)**

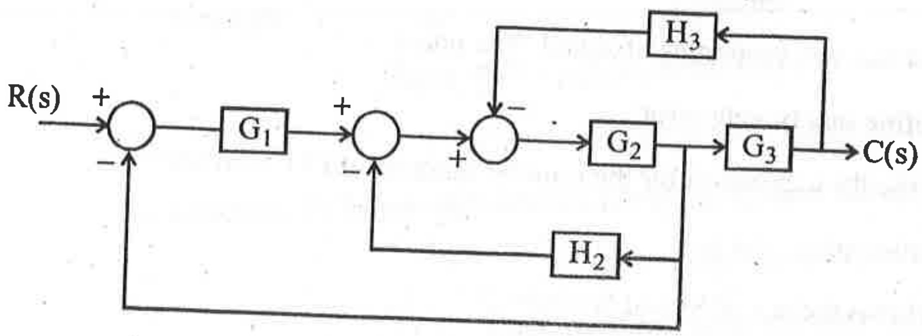


Figure-11(b) (i)

(ii) Determine the closed loop transfer function of the system whose signal flow graph is shown in figure 11(b) (ii), using Maron's gain formula. **(8)**

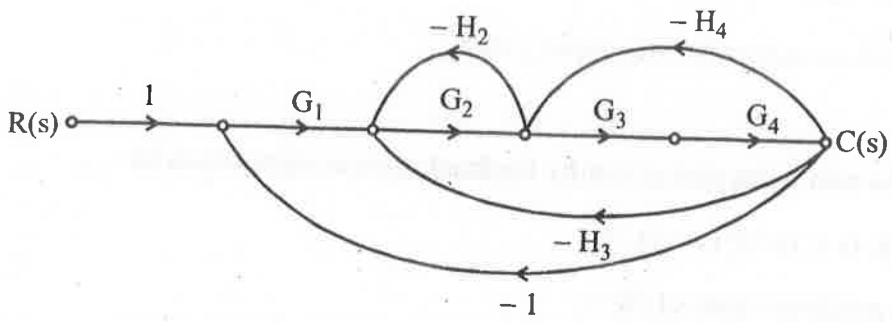


Figure-11(b) (ii)

12. (a) Derive expressions for the following, for a second order, under damped unity feedback system when excited by a unit step input.

(1) Output response c(t)

(2) Peak time (t_p)

(3) Rise time (t_r) **(10 + 3 + 3)**

OR

(b) (i) The open loop transfer function of a unity feedback system is given by

$G(s) = 40/(s(0.2s + 1))$

Determine the steady state error using error series approach for the input, $r(t) = 3t + 4t^2$ **(10)**

(ii) Define the following time domain specifications :

(a) Peak time (b) Rise time (c) Peak overshoot **(2 + 2 + 2)**

13. (a) (i) List any four frequency domain specifications. **(4)**

(ii) Draw the bode magnitude and phase plot for the unity feedback system with $G(s) = \frac{40}{s(1 + 0.1s)}$ and hence determine phase margin and gain margin. **(6 + 6)**

OR

(b) A unity feedback, type-2 system has a open loop transfer function, $G(s) = K/s^2$. Design a lead compensator to meet the following specifications :

(i) Settling time, $t_s \leq 4s$

(ii) Peak overshoot for step input $\leq 20\%$. **(16)**

14. (a) Draw the root locus plot of a unity feedback system represented by

$G(s) = K(s+1) / s^2(s+9)$

For the positive values of 'K'. **(16)**

OR

(b) For the feedback system whose open loop transfer function is ,

$G(s)H(s) = K/s(s+3)(s+5)$, investigate the stability of the system for various values of 'K' using Nyquist stability criteria. **(16)**

15. (a) (i) List any four advantages of state space representation of a system. **(4)**

(ii) For the state variable representation given below, determine the transfer function of the system.

$$[\dot{X}] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -40 & -44 & -14 \end{bmatrix} [X] + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} [U]$$

$$Y = [0 \ \ 1 \ \ 0]\,[X]$$ **(12)**

OR

(b) (i) Obtain the state equation and output equation of a system described by the differential equation $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = u$. **(4)**

(ii) A control system represented in state space form has the following data :

$$[A] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix};\ B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};\ C = [\,3 \ \ 4 \ \ 1\,]$$

Examine its observability. **(12)**

Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 60451

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

Fourth Semester

Electronics and Communication Engineering

EC 2255/EC 46/EE 1256 A/080290023/10144 EC 406 — CONTROL SYSTEMS

(Regulations 2008/2010)

(Common to 10144 EC 406 – Control Systems for B.E. (Part-Time) Third Semester – ECE – Regulations 2010)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What is the main advantages of closed loop system over open loop systems?
2. Write the mathematical expressions for step input and impulse input.
3. How do you find the type of a system?
4. Find the unit impulse response of system $H(S) = 5/(s + 4)$ with zero initial conditions.
5. What is meant by 'Corner frequency' in frequency response analysis?
6. What is Nichols chart?
7. Write the necessary and sufficient condition for stability in Routh stability criterion.
8. Define Nyquist stability criterion.
9. Define state equation.
10. Give the concept of controllability.

PART B — (5 × 16 = 80 marks)

11. (a) Derive the transfer function of a RLC series circuit.

Or

(b) With a neat diagram, derive the transfer function of a field controlled dc motor.

12. (a) (i) For the system shown in figure 12(a)(i) find the error using dynamic error coefficient method for input $r(t) = 5 + 4t + 7t^2$.

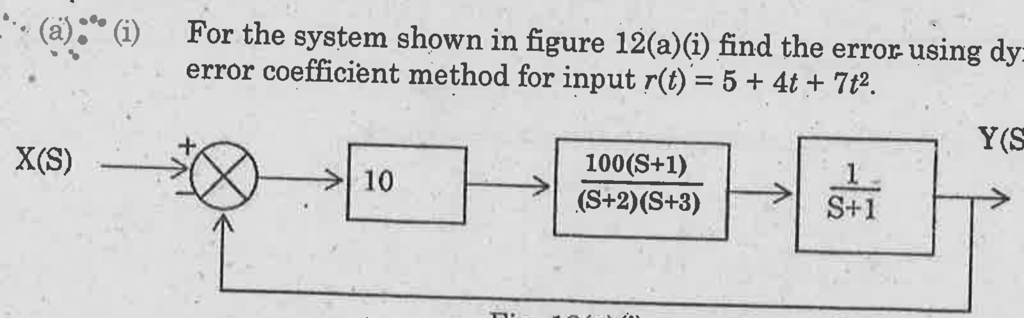


Fig. 12(a)(i)

(ii) Briefly discuss about transient response specifications.

Or

(b) (i) For the system shown in fig. 12(b)(i) find the effect of PD controller with Td = 1/10 on peak overshoot and settling time when it is excited by unit step input.

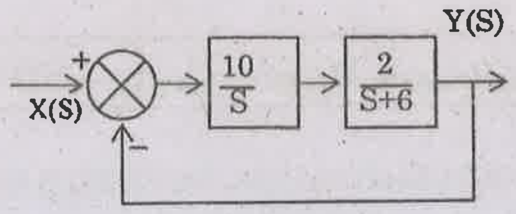


Fig. 12(b)(i)

(ii) Discuss the effect of PI controller in the forward path of a system.

13. (a) Consider a unity feedback open loop transfer function $G(s)=\dfrac{100}{s(1+0.1s)(1+0.2s)}$. Draw the Bode plot and find the phase and gain cross over frequencies, phase and gain margin and the stability of the system.

Or

(b) Explain in detail the design procedure of lead compensator using Bode plot.

14. (a) (i) Determine the range of K for stability of unity feedback system whose open loop transfer function is $G(s)=\dfrac{K}{s(s+1)(s+2)}$ using Routh stability criterion. (6)

(ii) Draw the approximate root locus diagram for a closed loop system whose loop transfer function is given by $G(s)H(s)=\dfrac{K}{s(s+5)(s+10)}$ Comment on the stability. (10)

Or

(b) Sketch the Nyquist plot for a system with open loop transfer function $G(s)H(s)=\dfrac{K(1+0.4s)(s+1)}{(1+8s)(s-1)}$ and determine the range of K for which the system is stable. (16)

15. (a) For the given state variable representation of a second order system given below find the state response for a unit step input and $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 2\end{bmatrix}[u]\begin{bmatrix}x_1 & (0)\\ x_2 & (0)\end{bmatrix}=\begin{bmatrix}0\\ 1\end{bmatrix}$ by using the discrete – time approximation.

Or

(b) Consider the system with the state equation.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -6 & -11 & -6\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u.$$

Check the controllability of the system.

Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 27196

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Fourth Semester

Electronics and Communication Engineering

EC 6405 — CONTROL SYSTEM ENGINEERING

(Common to Mechatronics Engineering and Medical Electronics Engineering)

(Regulations 2013)

Time : Three hours   Maximum : 100 marks

(Provide Semilog sheet, Polar graph and ordinary graph sheet)

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. List the advantages of Closed loop System?

2. What is Block diagram? What are its basic components?

3. State some standard test signals used in time domain analysis.

4. What is a steady state error?

5. Give the specifications used in frequency domain analysis.

6. What are Constant M and N circles?

7. What is dominant pole?

8. Define about Nyquist stability criterion.

9. What are state variables?

10. Draw the Sampler and hold circuits.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Write the differential equations governing the mechanical translational system as shown in Fig.1. Draw the Force – Voltage and Force – Current electrical analogous circuits and verify by mesh and node equations. (12)

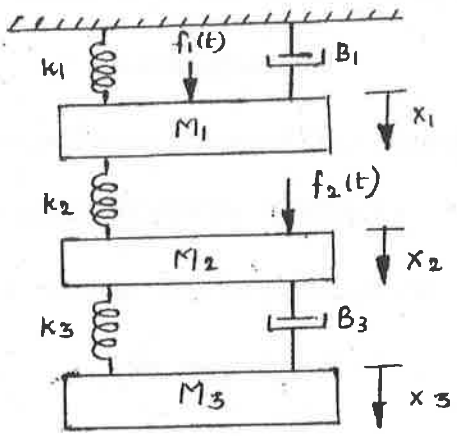


Fig. 1

(ii) What are the basic elements of mechanical rotational systems and write its torque balance equations. (4)

Or

(b) (i) Write the rule for eliminating negative and positive feedback in block diagram reduction. (4)

(ii) The signal flow graph for a feed back control system is shown in Fig. 2. Determine the closed loop transfer function $C(s)/R(s)$. (12)

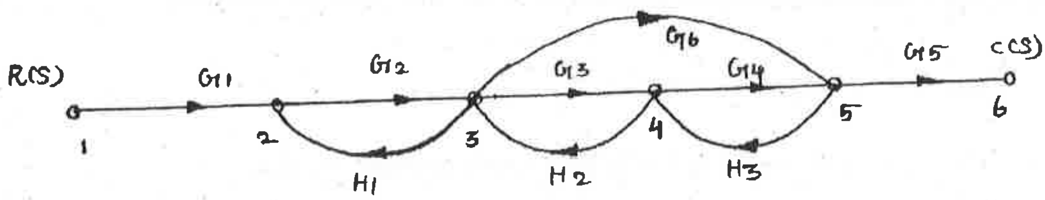


Fig. 2

12. (a) (i) Derive the time response analysis of a first order system for step and ramp input. (12)

(ii) What are the time domain specifications? Define any two. (4)

Or

(b) (i) Determine the type and order of the system with following transfer functions.

(1) $\dfrac{S+4}{(S-2)(S+3)}$

(2) $\dfrac{10}{S^3(S^2+2S+1)}$. (4)

(ii) With a neat diagram, explain the function of PID compensation in detail. (12)

13. (a) Sketch the Bode plot for the following transfer function and determine the Phase margin and gain margin.

$G(S) = \dfrac{20}{S(1+3S)(1+4S)}$. (16)

Or

(b) The open loop transfer function of a unity feedback system is given by $G(S) = \dfrac{1}{S^2(1+S)(1+2S)}$. Sketch the polar plot and determine the gain and phase margin. (16)

14. (a) (i) Using Routh Hurwitz criterion, determine the stability of a system representing the characteristic equation :

$S^5 + S^4 + 2S^3 + 2S^2 + 3S + 5 = 0$.

Comment on location of the roots of the characteristics equation. (8)

(ii) Write detailed notes on relative stability with its roots of s-plane. (8)

Or

(b) The open loop transfer function of a unity feedback system is given by

$G(S) = \dfrac{K(S+9)}{S(S^2+4S+11)}$

Sketch the root locus of the system. (16)

15. (a) (i) Consider the following system with differential equation given by

$\dddot{y} + 6\ddot{y} + 11\dot{y} + 6y = 6u$

Obtain the state model in diagonal canonical form. (12)

(ii) Draw the state model of a linear single-input-single-output system and obtain its corresponding equations. (4)

Or

(b) (i) State the properties of state transition matrix. (4)

(ii) Consider the system defined by

$$X = Ax + BU$$

$$Y = Cx$$

Where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}; \; B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}; \; C = \begin{bmatrix} 10 & 5 & 1 \end{bmatrix}.$$

Check the controllability and observability of the system. (12)

Reg. No. :

Question Paper Code : 77119

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

Fourth Semester

Electronics and Communication Engineering

EC 6405 — CONTROL SYSTEM ENGINEERING

(Common to Mechatronics Engineering and Medical Electronics Engineering)

(Regulation 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the transfer function of the network given in Fig. 1

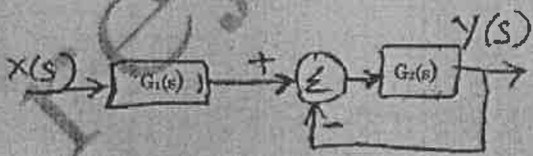


Fig. 1

2. State Mason's gain formula.

3. How do you find the type of a system?

4. Find the unit impulse response of system $H(s) = 5s/(s+2)$ with zero initial conditions.

5. What is the use of Nichol's chart?

6. What are the characteristics of phase lead network?

7. Find the range of K for closed loop stable behavior of system with characteristic equation $2s^4 + 12s^3 + 22s^2 + 12s + K$ using Routh Hurwitz stability criterion.

8. What is the value of gain K at any given point on root locus?

9. Define observability of a system.

10. State Sampling theorem.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Explain the features of closed loop feedback control system. (4)

(ii) Derive the transfer function of system shown in fig. 2. (12)

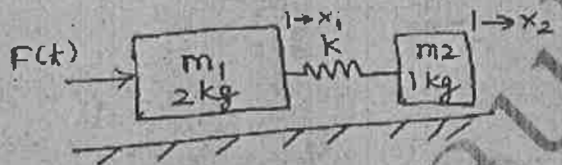


Fig. 2

Or

(b) Find the transfer function of the system shown in fig. 3 using block diagram reduction technique and signal flow graph technique.

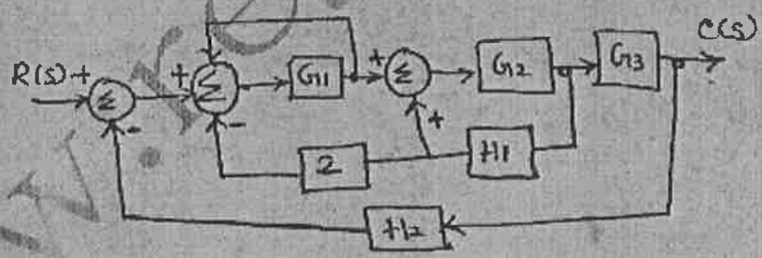


Fig. 3

12. (a) (i) A unity feedback system has the forward transfer function $G(S) = \dfrac{KS}{(1+S)^2}$. For the input $r(t) = 1 + 5t$, find the minimum value of K so that the steady state error is less than 0.1 (Use final value theorem).

(ii) Briefly discuss about step response analysis of second order system.

Or

(b) (i) For the system shown in Fig. 5 find the effect of PD controller with $T_d = 1/10$ on peak overshoot and setting time when it is excited by unit step input.

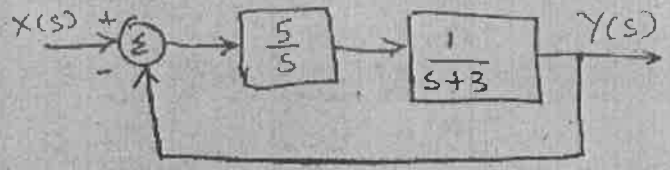


Fig. 5

(ii) Discuss- the effect of PID controller in the forward path of a system.

13. (a) For the following transfer function draw the Bode plot, find the gain and phase margin :

$$G(S)\,H(S) = \frac{5}{S(10+S)(20+S)}.$$

Or

(b) Design a lead compensator for the system $G(s) = 1/s(s+2)$ with damping coefficient equal to 0.45, velocity error constant > 20 and small setting time.

14. (a) A single loop negative feedback system has a loop transfer function $G_c(s)G(s) = \dfrac{K(s+6)^2}{s(s^2+1)(s+4)}$. Sketch the root locus as a function of K. Find the range of K for which the system is stable, K for which purely imaginary roots exist and find the roots.

Or

(b) Draw the Nyquist plot and find the stability of the following open loop transfer function of unity feedback control system $G(s)H(s) = K(s+1)/s^2(s+10)$. If the system is conditionally stable, find the range of K for which the system is stable.

15. (a) Consider a system with state-space model given below.

$$x = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -1 \end{bmatrix} X + \begin{bmatrix} 0 \\ 5 \\ -24 \end{bmatrix} u;\; y = [1\ 0\ 0]x + [0]u;$$

Verify that the system is observable and controllable.

Or

(b) Explain the functional modules of closed loop sampled data system and compare its performance with open loop sampled data system.

Reg. No.

Question Paper Code : 57288

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2016

Fourth Semester

Electronics and Communication Engineering

EC 6405 – CONTROL SYSTEM ENGINEERING

(Common to Mechatronics Engineering and Medical Electronics Engineering)

(Regulations 2013)

Time : Three Hours **Maximum : 100 Marks**

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. Distinguish between open loop and closed loop control systems.
2. Write Mason's gain formula.
3. List the standard test signals used in time domain analysis.
4. State the effect of PI compensation in system performance.
5. What are the frequency domain specifications ?
6. What are M & N circles ?
7. State the necessary conditions for stability.
8. How will you find root locus on real axis ?
9. Define the state and state variable of a model system.
10. What is zero order hold circuit ?

PART – B (5 × 16 = 80 Marks)

11. (a) (i) What are the basic elements of mechanical rotational systems ? Write its force balance equations. (4)

(ii) Write the differential equations governing the mechanical translational system shown in figure 11(a)(ii) and determine the transfer function. (12)

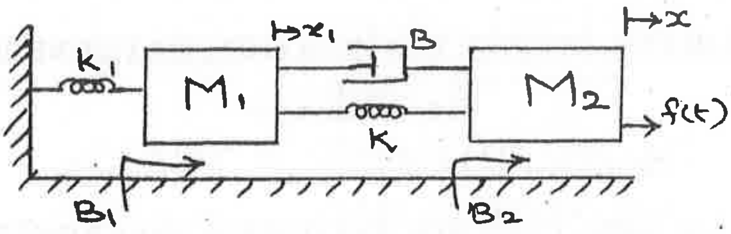


Figure 11(a)(ii)

OR

(b) Convert the block diagram shown in figure 11(b) to signal flow graph and find the transfer function using mason's gain formula. Verify with block diagram approach. (16)

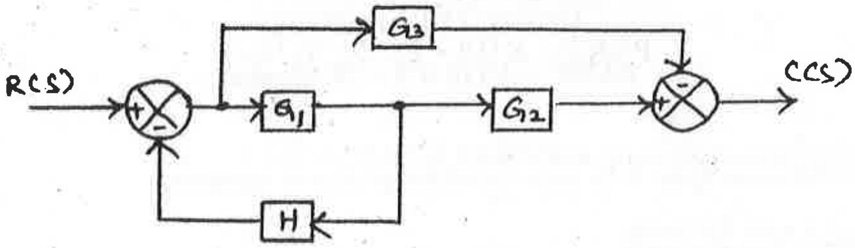


Figure 11(b)

12. (a) (i) Derive the time response of a first order system for unit step input. (8)

(ii) The unity feedback control system is characterized by an open loop transfer function G(s) = K / [s(s+10)]. Determine the gain K, so that the system will have damping ratio of 0.5 for this value of K. Determine the peak overshoot and peak time for a unit step input. (8)

OR

(b) With a neat block diagram and derivation explain how PI, PD and PID compensation will improve the time response of a system. (16)

13. (a) Sketch the Bode plot for the following transfer function and determine the system gain K for the gain cross over frequency to be 5 rad/sec.

$G(s) = Ks^2/[(1 + 0.2s)(1 + 0.02s)]$ (16)

OR

(b) (i) Write short notes on series compensation. (4)

(ii) Write down the procedure for designing Lead compensator using Bode plot. (12)

14. (a) (i) Using Routh Hurwitz criterion, determine the stability of a system representing the characteristic equation $S^6 + 2S^5 + 8S^4 + 12S^3 + 20S^2 + 16S + 16 = 0$ and comment on location of the roots of the characteristics equation. (8)

(ii) Describe about Nyquist Contour and its various segments (8)

OR

(b) A unity feedback control system has an open loop transfer function $G(s) = K/[s(s^2 + 4s + 13)]$. Sketch the root locus. (16)

15. (a) (i) Construct a state model for a system characterized by the differential equation $(d^3y/dt^3) + 6(d^2y/dt^2) + 11(dy/dt) + 6y + u = 0$ (8)

(ii) With the neat block diagram explain the sampled data control system and state its advantages. (8)

OR

(b) Test the controllability & observability of the system by any one method whose state space representation is given as (16)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -2 & -3 & 0 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} u \; ; \; y = [1\ 0\ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 80340

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

Fourth Semester

Electronics and Communication Engineering

EC 6405 — CONTROL SYSTEM ENGINEERING

(Common to Mechatronics Engineering and Medical Electronics Engineering)

(Regulations 2013)

Time : Three hours

Maximum : 100 marks

(Provide Semilog sheet, Polar graph and ordinary graph sheet)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What is a control system?
2. List the basic elements of translational mechanical systems.
3. Specify the time domain specifications.
4. What is meant by steady state error?
5. State the significance of Nichol's plot.
6. What is series compensation?
7. Define BIBO stability.
8. What is dominant pole?
9. List the main properties of a state transition matrix.
10. State sampling theorem.

13. (a) The open loop transfer function of a unity feedback system is given by $G(s)=1/\left[s(1+s)^2\right]$. Sketch the polar plot and determine the gain and phase margin. (16)

Or

(b) (i) Write down the procedure for designing Lag compensator using Bode plot. (12)

(ii) State about Parallel feedback compensation. (4)

14. (a) (i) State Nyquist stability criterion and explain the three situations while examining the stability of the linear control system. (8)

(ii) Construct R-H criterion and determine the stability of a system representing the characteristic equation

$S^5+S^4+2S^3+2S^2+3S+5=0$. Comment on location of the roots of the characteristics equation. (8)

Or

(b) With neat steps write down the procedure for construction of root locus. Each rule give an example. (16)

15. (a) A discrete time system is described by the difference equation $y(k+2)+5y(k+1)+6(yk)=u(k)$

$Y(0)=y(1)=0$ and $T=1$ sec, Determine

(i) State model in canonical form

(ii) State transition matrix. (16)

Or

(b) (i) Check the controllability of the system by Kalman's test whose state model is given as,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -2 & -3 & 0 \\ 0 & +2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} u; \; y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$ (8)

(ii) Write detailed notes on Sampler and hold circuits. (8)

CONTROL SYSTEM ENGINEERING - EC 6405

UNIT - III FREQUENCY RESPONSE ANALYSIS

1. BODE PLOT : (PROBLEM).

The bode plot is a frequency response plot of the sinusoidal T.F of the system. It consist of two plots.

1. Magnitude plot
2. Phase plot.

	Factors	Slope in dB/dec.	Phase angle in de.
1.	constant gain, K	0	0'
2.	Integral Factor		
	i) $\frac{K}{s} = \frac{K}{j\omega}$	-20	-90'
	ii) $\frac{K}{s^n} = \frac{K}{(j\omega)^n}$	-20n	-90n
3.	Derivative Factor,		
	i) $Ks = K(j\omega)$	+20	+90'
	ii) $Ks^n = K(j\omega)^n$	+20n	+90n'
4.	First order factor in Denominator		
	$\frac{1}{1+sT} = \frac{1}{1+j\omega T}$	-20	0 to -90'
	$\frac{1}{(1+sT)^n} = \frac{1}{(1+j\omega T)^n}$	-20n	0 to -90n
5.	1^{st} order factor in numerator		
	$(1+sT) = (1+j\omega T)$	+20	0 to 90'
	$(1+sT)^n = (1+j\omega T)^n$	+20n	0 to 90n'

6.	Quadratic factor in Denominator $\dfrac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \dfrac{1}{1+2\xi\dfrac{s}{\omega_n}+\left(\dfrac{s}{\omega_n}\right)^2}$	-40	0 to -180°
7.	Quadratic factor in numerator $\dfrac{s^2+2\xi\omega_n s+\omega_n^2}{\omega_n^2} = 1+2\xi\left(\dfrac{s}{\omega_n}\right)+\left(\dfrac{s}{\omega_n}\right)^2$	$+40$	0 to $+180^\circ$

Procedure for Bode Plot :

1. Replace 's' by 'jω'
2. Find the magnitude as a function of ω.

Term	corner frequency rad/sec.	Slope db/dec.	change in slope db/dec.
Increasing order			

3. Express the magnitude in dB by $20\log_{10}|G(j\omega)H(j\omega)|$
4. Find phase angle by using $\tan^{-1}\left[\dfrac{\text{Imaginary part}}{\text{Real part}}\right]$
$= \phi$ in deg.
5. with required approximation, Plot magnitude in dB and phase angle in degree against log ω by varying ω from 0 to ∞.

Gain margin (Kg)

$$K_g = \frac{1}{|G(j\omega)|_{\omega=\omega_{pc}}}$$

$$K_g = 20\log\frac{1}{|G(j\omega)|_{\omega=pc}}$$

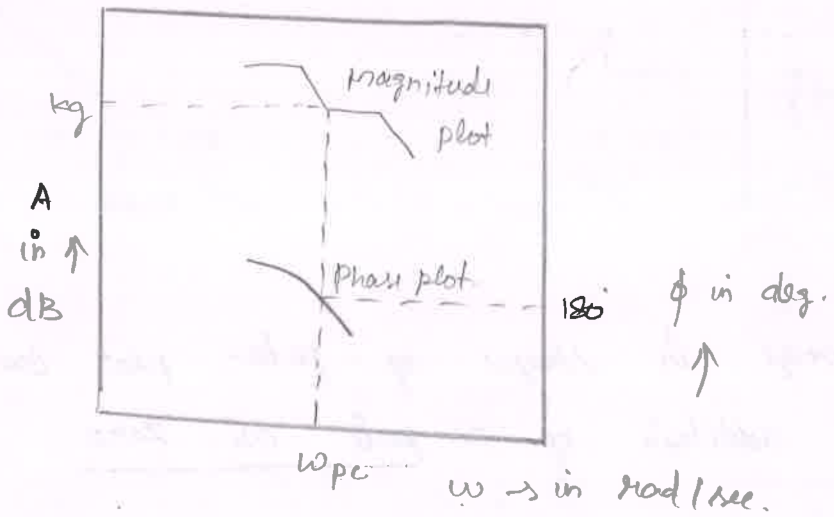
$$K_g = -20\log|G(j\omega)|_{\omega=\omega_{pc}}$$

Phase margin (γ) :

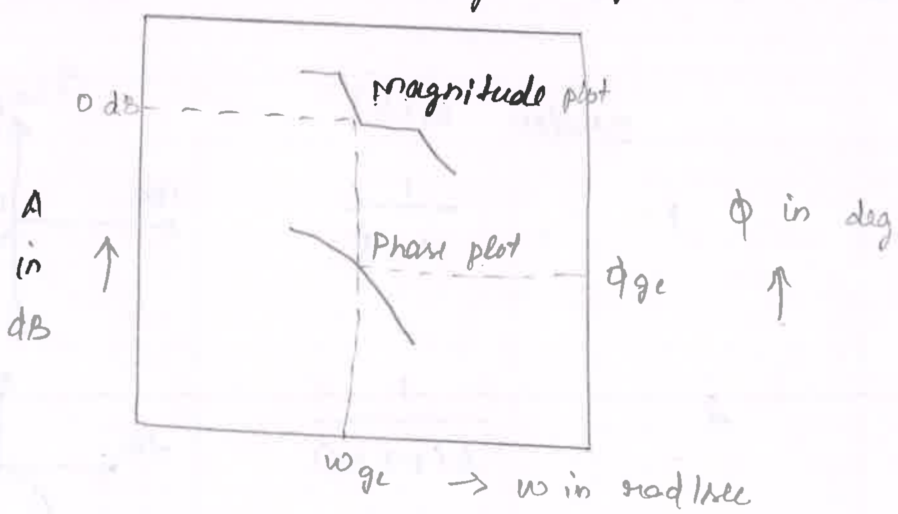
$$\gamma = 180^\circ + \phi_{gc}$$

$$\phi_{gc} = \lfloor G(j\omega) \rfloor_{\omega = \omega_{gc}}$$

Phase cross over frequency (ω_{pc}) :



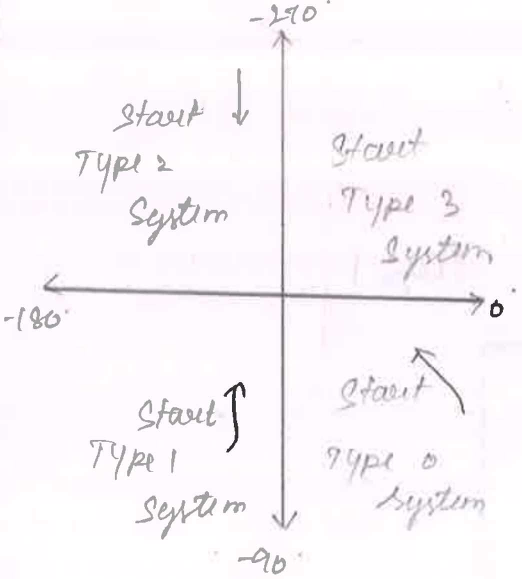
Gain Cross over Frequency (ω_{gc}) :



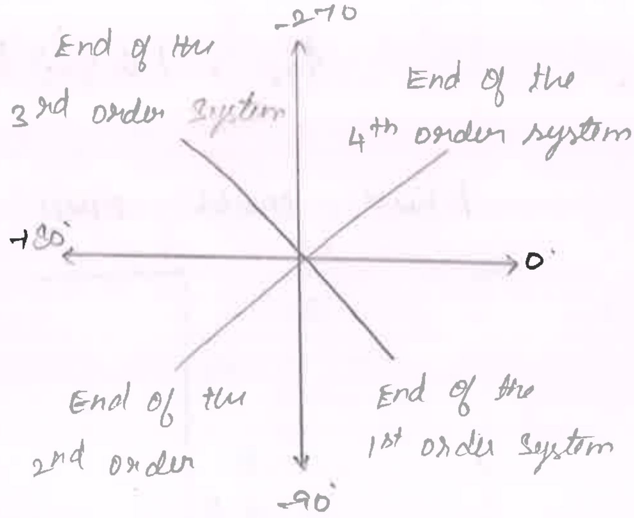
2. POLAR PLOT, (OR) NYQUIST PLOT (PROBLEMS)

Polar plot of a sinusoidal T.F $G(j\omega)$ is a plot of the magnitude of $|G(j\omega)|$ vs the phase angle of $\angle G(j\omega)$ on polar co-ordinates as ω is varied from 0 to ∞

1. Type no. of system (0 – 3)



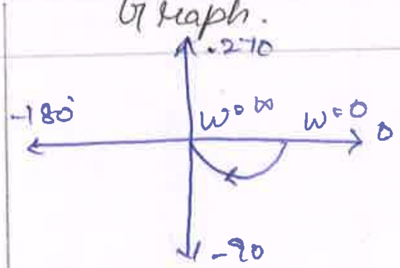
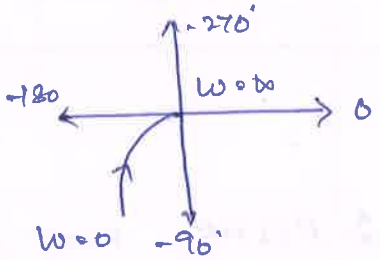
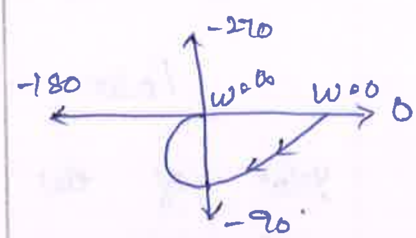
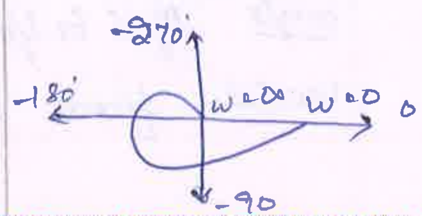
2. Order of the system (1 – 4)

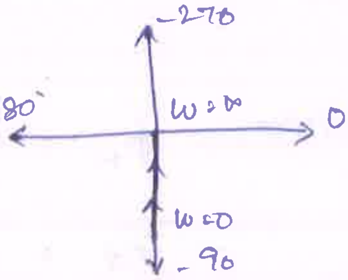


3. The change in shape of polar plot can be predicted due to addition of a pole or zero.

pole = $-90°$

Zero = $+90°$

Type	order	G(s)	Graph.
0	1	$\frac{1}{1+sT}$	
1	2	$\frac{1}{s(1+sT)}$	
0	2	$\frac{1}{(1+sT_1)(1+sT_2)}$	
0	3	$\frac{1}{(1+sT_1)(1+sT_2)(1+sT_3)}$	

1	1	$\frac{1}{s}$	

Find $|G(j\omega)|$ and $\angle G(j\omega)$.

Plot $|G(j\omega)|$ & $\angle G(j\omega)$ in Polar graph sheet.

3. NICHOL'S CHART : (PROBLEM)

Find

Magnitude in dB $|G(j\omega)| = \underline{20 \log} \left[\frac{1}{(sT_1+1)(1+sT_2)}\right]$

Phase angle or $\angle G(j\omega)$.

Plot the graph in Nichol's chart.

Gain margin, $K_g = -|G(j\omega_{pc})|$ in db.

Phase margin $\gamma = 180 + \phi_{gc}$.

4. LEAD COMPENSATOR : (PROCEDURE, PROBLEM)

A compensator having a characteristics of a lead network is called lead compensator. If a sinusoidal sign is applied to a lead compensator then in steady state the output will have a phase lead with respect to input.

Procedure :

1. Determine K.
2. Draw Bode plot for uncompensated system.
3. Determine the phase margin for uncompensated system.
4. Find ϕ_m. or $\phi_m = \gamma_d - \gamma + \varepsilon$

where, ϕ_m = Maximum phase lead angle.

γ_d = Desired phase margin.

γ = Phase margin of uncompensated system.

ε = Additional phase lead angle. ($\varepsilon = 5°$)

5. Determine the T.F of lead compensator.

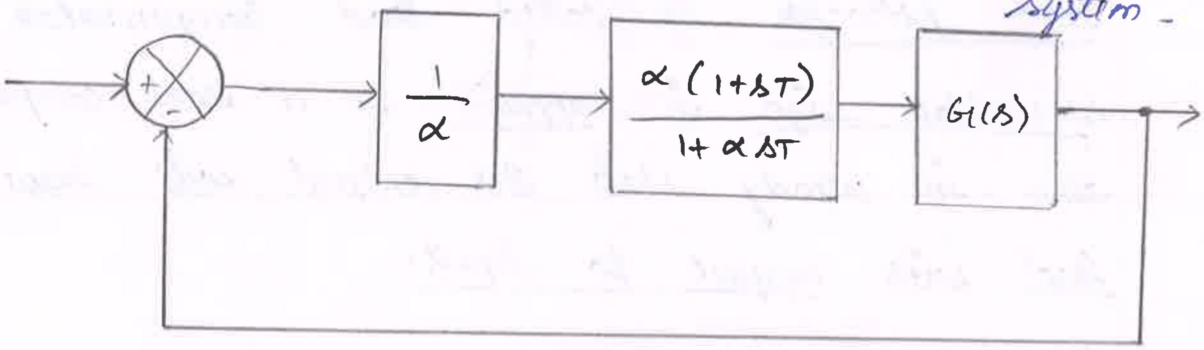
$$\alpha = \frac{1 - \sin\phi_m}{1 + \sin\phi_m}$$

$$\omega_m = -20 \log \frac{1}{\sqrt{\alpha}}$$

$$T = \frac{1}{\omega_m \sqrt{\alpha}}$$

$$G_c(s) = \frac{\alpha(1 + sT)}{1 + \alpha sT}$$

6. Determine the open loop T.F of compensated system.



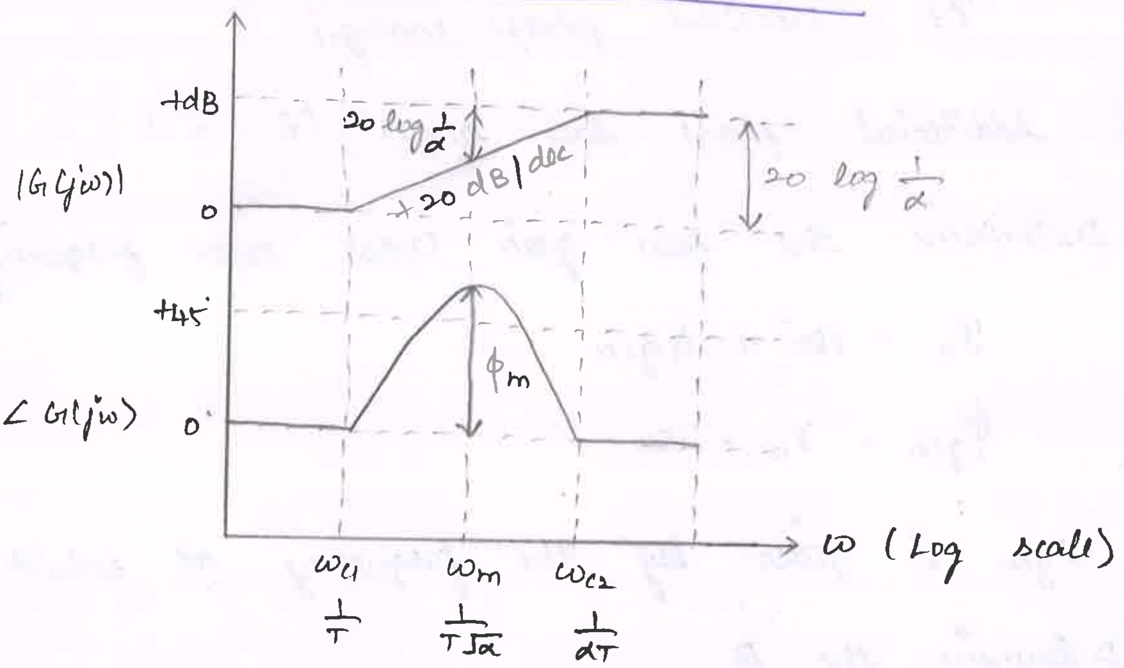
For OLTF, $G_o(s) = \frac{1}{\alpha} \cdot \frac{\alpha(1+sT)}{1+\alpha sT} \cdot G(s)$

$$\boxed{G_o(s) = G(s) \frac{(1+sT)}{1+\alpha sT}}$$

7. Draw the Bode plot of compensated system.

8. Verify the given specification.

Bode plot for Lead Compensator:



5. LAG COMPENSATOR : (PROBLEM, PROCEDURE)

A Compensator having the characteristics of a lag network is called lag Compensator. If a sinusoidal sign is applied to a lag compensator then in steady state the output will have a phase lag with respect to input.

Procedure :

1. Determine K.

2. Draw Bode plot of uncompensated system.

3. Determine the phase margin of uncompensated system.

4. Choose a suitable value for the phase margin of compensated system.

$$\gamma_n = \gamma_d + \varepsilon$$

where

γ_n = new phase margin

γ_d = Desired phase margin

ε - Additional phase lag angle ($\varepsilon = 5^\circ$)

5. Determine the new gain cross over frequency (ω_{gcn})

$$\gamma_n = 180^\circ + \phi_{gcn}$$

$$\phi_{gcn} = \gamma_n - 180^\circ$$

ω_{gcn} is given by the frequency at which ϕ_{gcn}.

6. Determine the β.

$$A_{gcn} = 20 \log \beta$$

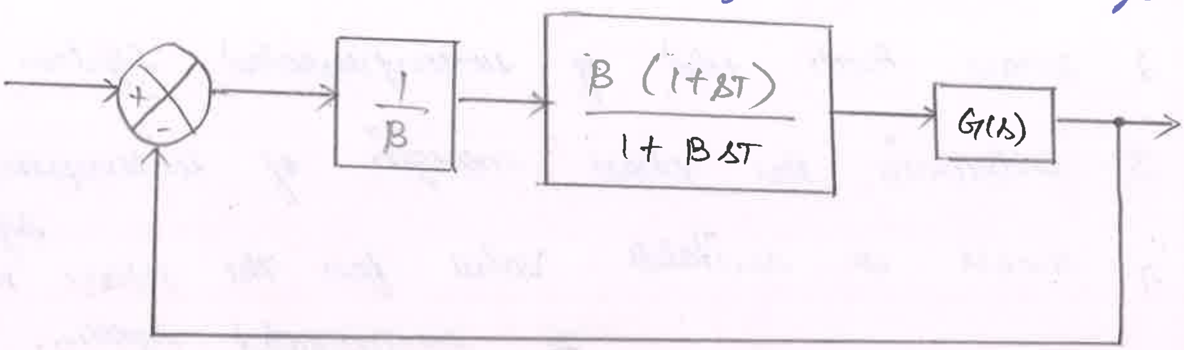
$$\beta = 10^{(A_{gcn}/20)}$$

7. Determine the T.F of lag compensator.

$$G_c(s) = \frac{\beta(1+sT)}{1+\beta sT}$$

$$\frac{1}{T} = \frac{\omega_{gcn}}{10}$$

8. Determine open loop T.F of compensated system.



OLTF.
$$G_o(s) = \frac{1}{\beta} \cdot \frac{\beta(1+sT)}{1+\beta sT} \cdot G(s)$$

$$G_o(s) = \frac{(1+sT)}{1+\beta sT} \cdot G(s)$$

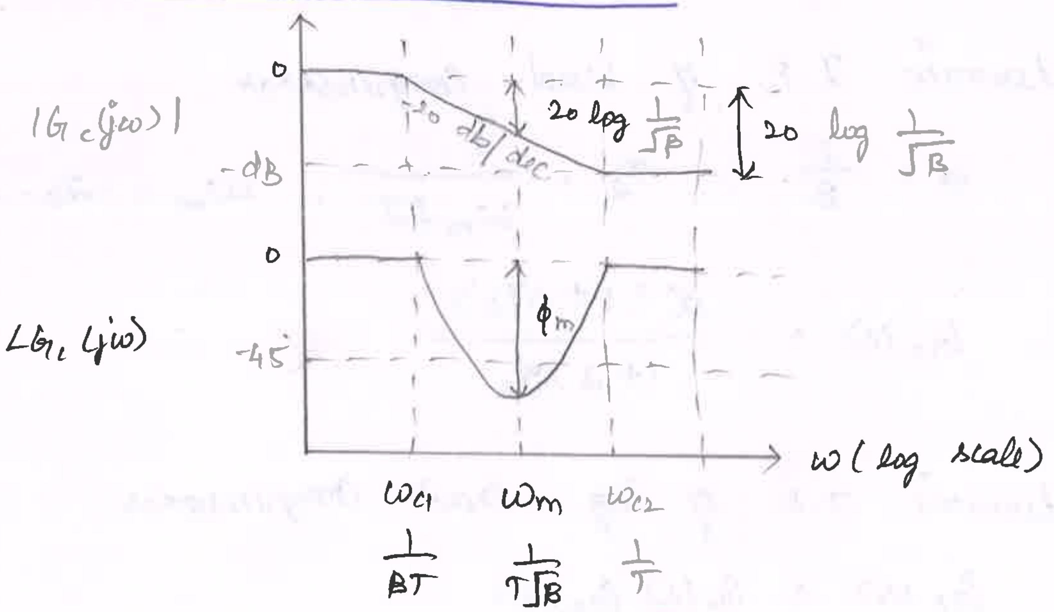
9. Determine the actual phase margin of compensated system

$$\gamma_o = 180^\circ + \phi_{gco}$$

where $\phi_{gco} = G_o(j\omega)$ at $\omega = \omega_{gcn}$

10. Verify the given specification

<u>Bode plot of lag compensation:</u>



6. LAG - LEAD COMPENSATOR : (PROCEDURE & PROBLEM).

Procedure :

1. Determine K

2. Draw Bode plot for uncompensated system.

3. Determine phase margin of uncompensated system

4: Choose a new phase margin

$$\gamma_n = \gamma_d + \varepsilon$$

γ_d - Desired phase margin , $\varepsilon = 5^\circ$

5. Determine new gain cross-over frequency ω_{gcn}.

$$\gamma_n = 180^\circ + \phi_{gcn} \quad \Rightarrow \quad \phi_{gcn} = \gamma - 180^\circ$$

6. Determine , β

$$\beta = 10^{(A_{gcn}/20)}$$

7. Determine T.F of Lag Compensator.

$$\frac{1}{T_1} = \frac{\omega_{gcn}}{10}$$

$$G_1(s) = \frac{\beta(1+sT_1)}{1+\beta sT_1}$$

8. Determine T.F of lead Compensator.

$$\alpha = \frac{1}{\beta}, \quad T_2 = \frac{1}{\omega_m\sqrt{\alpha}}, \quad \omega_m = -20-\log\frac{1}{\sqrt{\alpha}}$$

$$G_2(s) = \frac{\alpha(1+sT_2)}{1+\alpha sT_2}$$

9. Determine T.F of lag - lead Compensator.

$$G_c(s) = G_1(s)\,G_2(s)$$

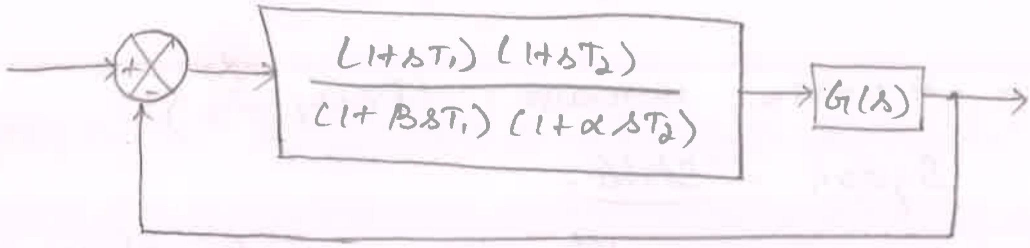
$$G_c(s) = \left[\frac{\beta(1+sT_1)}{1+\beta sT_1}\right]\left[\frac{\alpha(1+sT_2)}{1+\alpha sT_2}\right]$$

$$\alpha = \frac{1}{\beta}$$

$$G_c(s) = \frac{\beta(1+sT_1)}{1+\beta sT_1} \times \frac{1}{\beta}\cdot\frac{(1+sT_2)}{(1+\alpha sT_2)}$$

$$\boxed{G_c(s) = \frac{(1+sT_1)(1+sT_2)}{(1+\beta sT_1)(1+\alpha sT_2)}}$$

10. Determine the open loop T.F of compensated system.

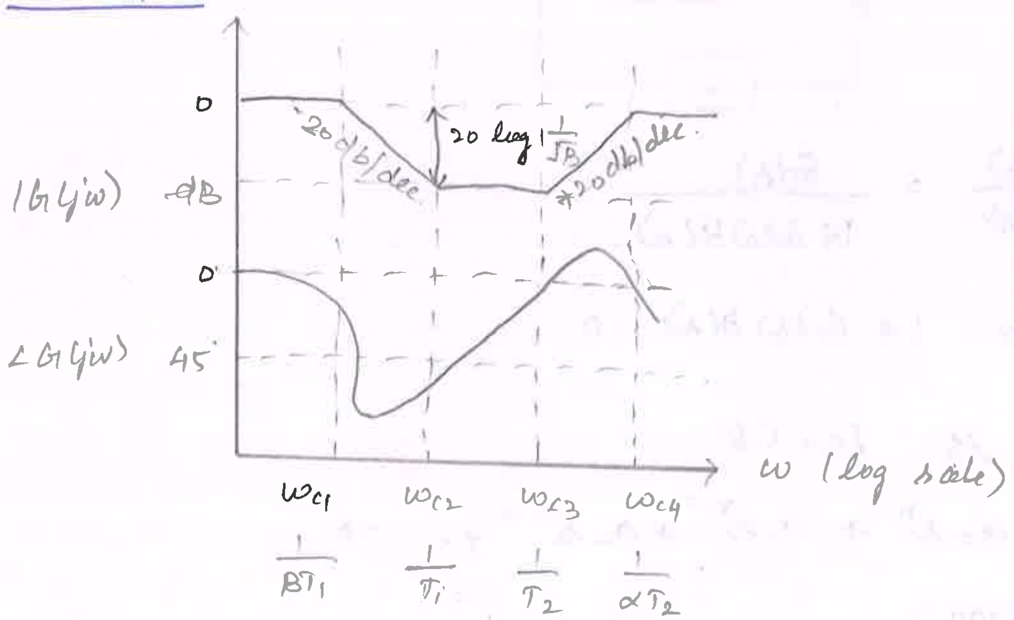


OLTF, $G_o(s) = G(s)\,\dfrac{(1+sT_1)(1+sT_2)}{(1+\beta sT_1)(1+\alpha sT_2)}$

11. Draw the Bode plot for compensated system.

12. Verify the given specification.

Bode plot:



UNIT - IV STABILITY ANALYSIS

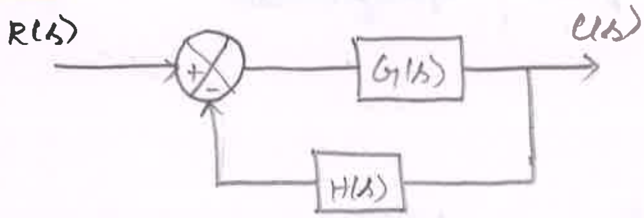
1. ROUTH - HURWITZ CRITERION : (PROBLEMS)

System - Stable.

Each term 1^{st} column Routh array of characteristic eq. → +ve.

This condition is not meet → system - Unstable.

Formation :



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

C.E ⇒ $1 + G(s)H(s) = 0.$

General eq. for C.E

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots = 0$$

Routh array :

$$\begin{array}{c|cccc}
s^n & a_0 & a_2 & a_4 & a_6 \\
s^{n-1} & a_1 & a_3 & a_5 & a_7 \\
s^{n-2} & b_0 & b_1 & b_2 & \\
s^{n-3} & c_0 & c_1 & c_2 &
\end{array}$$

where $b_0 = \dfrac{a_1 a_2 - a_0 a_3}{a_1}$

$b_1 = \dfrac{a_3 a_4 - a_0 a_5}{a_1}$, $b_2 = \dfrac{a_1 a_6 - a_0 a_7}{a_1}$

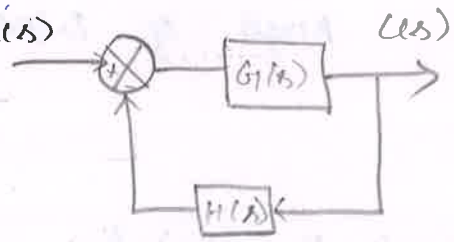
$$C_0 = \frac{b_0 a_3 - a_1 b_1}{b_0}$$

$$C_1 = \frac{b_0 a_5 - a_1 b_2}{b_0}$$

$$C_2 = \frac{b_0 a_7}{b_0}$$

2. ROOT LOCUS : (PROBLEM , PROCEDURE)

Root of CE → open loop gain K → varied from 0 to ∞ ⇒ Root Locus.



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\,H(s)}$$

C.E, $1+G(s)\,H(s) = 0$

$G(s)\,H(s) = 1$

Magnitude criterion : $|G(s)\,H(s)| = 1$

Angle criterion : $\angle G(s)\,H(s) = \pm 180^\circ (2q+1)$

$\{ q = 0, 1, 2 \ldots \}$

Procedure ;

1. Locate the poles and zeros for given T.F
2. Find Root locus on real axis
3. Calculate angle of asymptotes and centroid.

$$\text{centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m}$$

$$\angle \text{Asymptotes} = \pm \frac{180^\circ (2q+1)}{n-m}.$$

$(q = 0, 1, 2 \ldots). \; n-m).$

Step 4 : Find the breakaway and break in points

$$\frac{dk}{ds} = 0$$

5. Find angle of Departure & angle of arrival.

Angle of Departure : (Breakaway)

Complex pole => Find Angle of Departure from Complex pole.

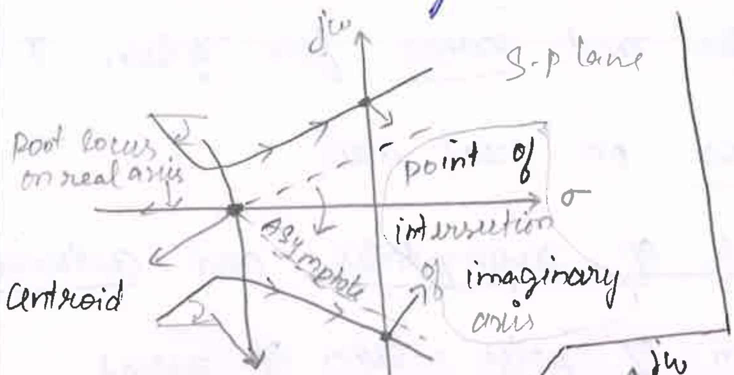
Angle of Departure = 180° - Σ angle from other poles + Σ angle from other zeros.

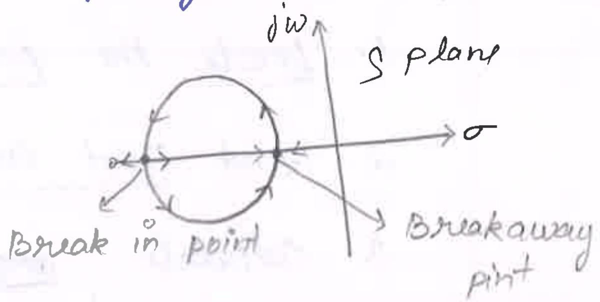
Angle of arrival : (Break in points)

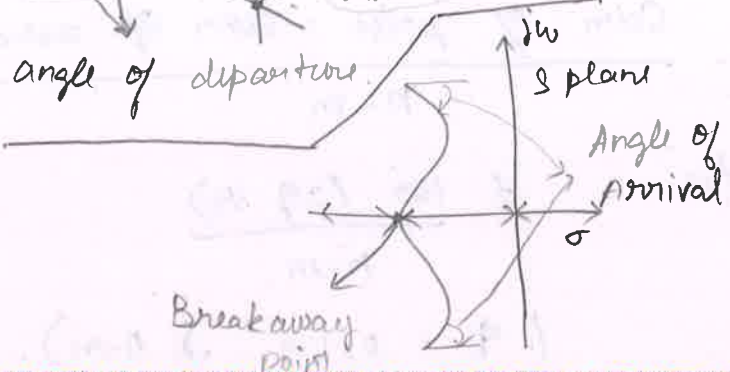
Complex zeros => Find angle of arrival.

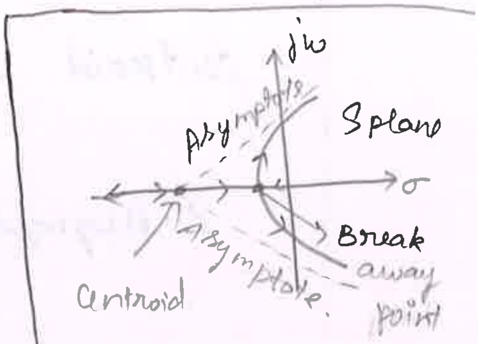
Angle of arrival = 180 - Σ angle from other zeros + Σ angle from other poles.

6. Find crossing point of Imaginary axis.









3. NYQUIST STABILITY CRITERION : (PROBLEMS)

If G(s) H(s) counter → G(s) H(s) plane corresponding to nyquist counter in s-plane, in circle (-1 +j0) in anticlockwise direction many times → no. of right half s-plane ⇒ closed loop system - stable.

Stability :

1. No encirclement (-1 +j0)

   Stable - No poles of G(s) H(s) in Right half of s-plane.

   Unstable - poles on Right half of s-plane.

2. Anticlockwise encirclement (-1 +j0)

   Stable - No. of ACW-E = No. of poles.

   Unstable - No. of ACW-E ≠ No. of poles.

3. Clockwise encirclement (-1 +j0)

   Always stable. → No poles, no. clockwise = No. of poles of closed loop system on right half s-plane.

Sketch the Nyquist contour:

I Nyquist contour.

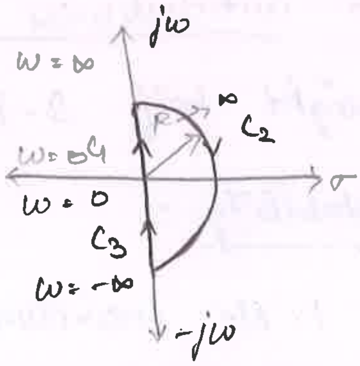
No pole on imaginary axis.

$C_1 \rightarrow s = j\omega \quad \omega = 0$ to ∞

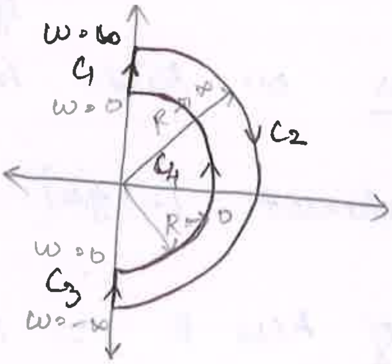
$C_2 \rightarrow 1 + sT = sT$

$$s = \lim_{R \to \infty} R e^{j\theta} \Rightarrow \theta = \frac{\pi}{2} \text{ to } -\frac{\pi}{2}$$

$C_3 \rightarrow s = j\omega, \quad \omega = -\infty$ to 0



II Nyquist contour

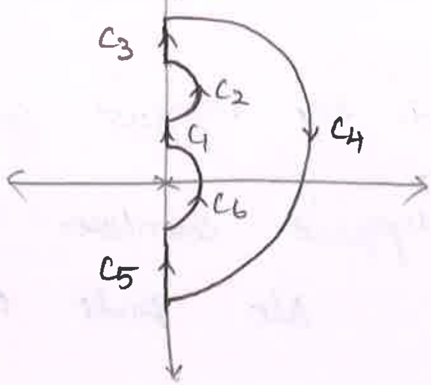


Poles at origin $[1 + sT \simeq 1]$

$$s = \lim_{R \to 0} R e^{j\theta}, \quad \theta = \frac{-\pi}{2} \text{ to } \frac{\pi}{2}$$

III Nyquist contour :

poles at origin and imaginary axis



Imaginary pole $[1 + sT \simeq 1]$

$$s = \lim_{R \to 0} R e^{j\theta},$$

$$\theta = \frac{-\pi}{2} \text{ to } \frac{\pi}{2}$$

UNIT - V STATE VARIABLE ANALYSIS

1. SOLUTIONS OF THE STATE EQUATIONS : (PROBLEMS)

State transition matrix

State transformation model.

STATE TRANSITION MATRIX :

The state transition matrix (e^{At}) can be computed by any one of the following two methods.

(i) computation of e^{At} using matrix exponential

$$e^{AT} = I + At + \frac{1}{2!}A^2t^2 + \frac{1}{3!}A^3t^3 + \dots + \frac{1}{i!}A^it^i + \dots$$

(ii) computation of e^{At} using Laplace Transform.

$$e^{At} = \phi(t)$$

$$\Rightarrow e^{At} = L^{-1}[(sI - A)^{-1}]$$

(or)

$$L[e^{AT}] = (sI - A)^{-1}$$

TRANSFORMATION OF STATE MODEL :

Transformation of state model can be done by two methods.

(i) canonical form of state model

(ii) Jordan canonical form of state model

4. TRANSFER FUNCTION FROM STATE VARIABLE REPRESENTATION : (PROBLEMS)

$$\dot{X}(t) = A\,X(t) + B\,u(t)$$

$$Y(t) = C\,X(t) + D\,u(t)$$

Taking L.T,

$$s\,X(s) = A\,X(s) + B\,U(s)$$

$$S\,X(s) - A\,X(s) = B\,U(s)$$

$$X(s)\cdot[SI - A] = B\,U(s)$$

$$X(s) = [SI - A]^{-1}\,B\,U(s)$$

Taking L.T,

$$Y(s) = C\,X(s) + D\,U(s)$$

$$= C\,[SI - A]^{-1}\,B\,U(s) + D\,U(s)$$

$$= [C\,(SI - A)^{-1}\,B + D]\;U(s)$$

$$\boxed{\frac{Y(s)}{U(s)} = C\,[SI - A]^{-1}B + D.}$$

5. STATE SPACE REPRESENTATION OF DISCRETE TIME SYSTEM : (PROBLEMS).

It is same as the continuous time system but instead of 't' is defined with respect to 'k' and the 1st order difference equation will be k+1.

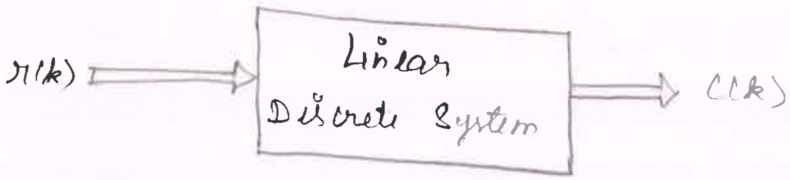
The state model is

$$X(k+1) = A\,X(k) + B\,U(k)$$

$$Y(s) = C\,X(k) + D\,U(k)$$

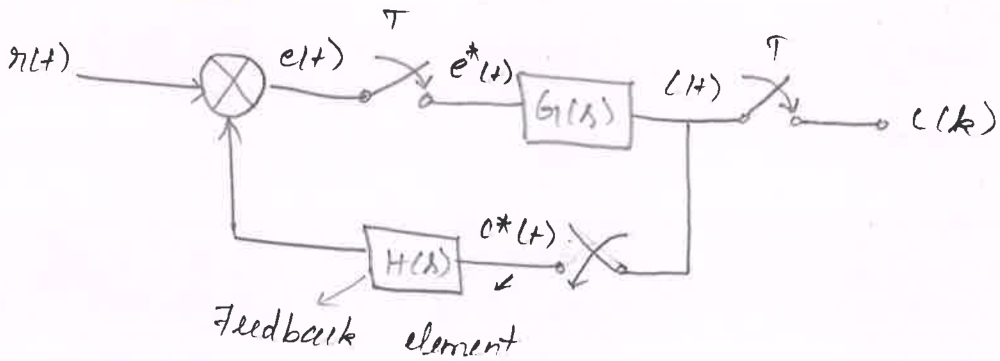
6. OPEN AND CLOSED LOOP SAMPLED DATA SYSTEM :

Open loop :



$$G(z) = \frac{G(z)}{R(z)} = \frac{z\,[c(k)]}{z\,[r(k)]}$$

Closed loop :



$$T(z) = \frac{C^*(s)}{R^*(s)} = \frac{C(z)}{R(z)} = \frac{G(z)}{1 + G(z)\,H(z)}$$

ST.ANNE'S

COLLEGE OF ENGINEERING AND TECHNOLOGY

ANGUCHETTYPALAYAM, PANRUTI – 607106.

QUESTION BANK

JULY 2018 - DECEMBER 2018 / ODD SEMESTER

BRANCH: ECE **YR/SEM:** II/III **BATCH:** 2017 - 2021

SUB CODE/NAME: EC 8391 CONTROL SYSTEMS ENGINEERING

UNIT I

SYSTEMS COMPONENTS AND THEIR REPRESENTATION

PART - A

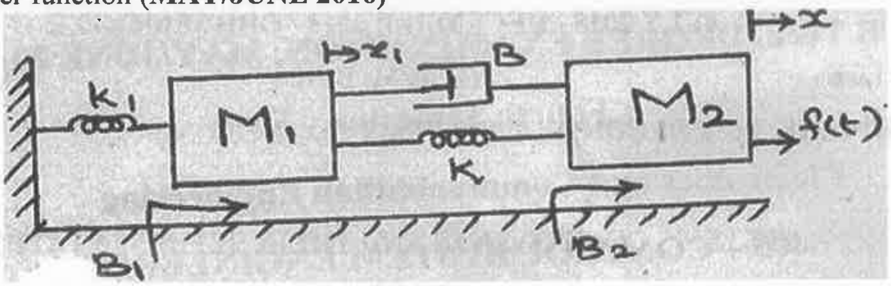
1. What is meant by a system?
2. Write Mason's Gain formula. **(NOV/DEC 2011, NOV/DEC 2010, MAY/JUNE 2013, MAY/JUNE 2014, NOV/DEC 2014, MAY/JUNE 2015, MAY/JUNE 2016)**
3. What are the three basic elements in electrical and mechanical system? **(NOV/DEC 2010)**
4. How will you get closed loop frequency response from open loop response? **(NOV/DEC 2010)**
5. List out the advantages of closed loop and open loop control system. **(APR/MAY 2010, MAY/JUNE 2012, NOV/DEC 2012, MAY/JUNE 2014, APR/MAY 2015, APR/MAY 2017)**
6. State ''transfer function'' of a system. **(APR/MAY 2010, NOV/DEC.2010, NOV/DEC 2013, NOV/DEC 2014, APR/MAY 2017)**
7. What is analogous systems? **(NOV/DEC 2013)**
8. What are the basic elements of a control systems? **(NOV/DEC 2014, MAY/JUNE 2016, NOV/DEC 2016)**
9. Write the force balance equation for ideal dashpot and ideal spring. **(APR/MAY 2015)**
10. What is Control Systems? **(NOV/DEC 2016)**
11. Differentiate between open loop and closed loop control systems. **(NOV/DEC 2010, NOV/DEC 2014, MAY/JUNE 2016, APR/MAY 2017)**
12. Prove the rule for eliminating negative and positive feedback loop.
13. Name any two dynamic models used to represent control systems. **(MAY/JUNE 2013)**
14. What are the characteristics of negative feedback? **(MAY/JUNE 2014, APR/MAY 2017)**
15. What is translational system?
16. Give the types of friction.
17. What is block diagram?(**APR/MAY 2017)**
18. What is signal flow graph?
19. What is the need for signal flow graph?

20. Why negative feedback is preferred over positive feedback system? **(NOV/DEC 2016)**

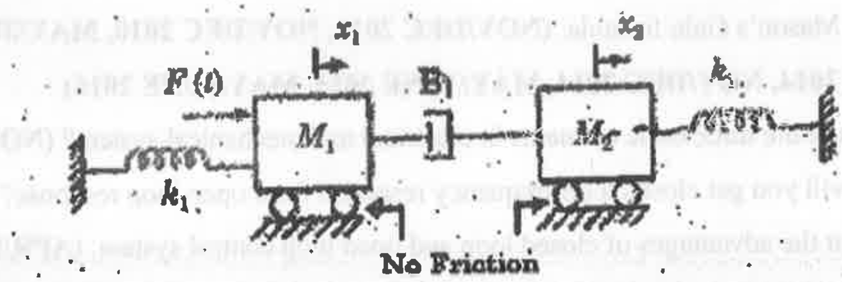
PART - B

Mechanical Translational System (16 Marks)

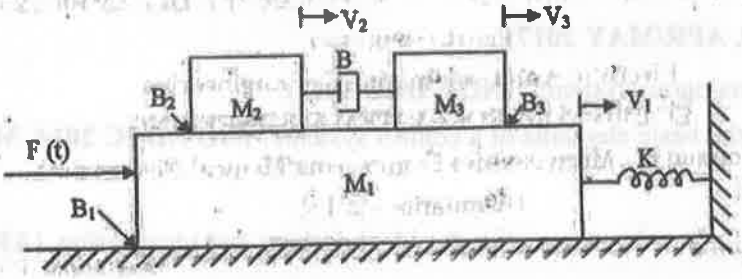
1. Write the differential equation governing the mechanical translational systems shown in the figure and find the transfer function **(MAY/JUNE 2016)**



2. Write the differential equation governing the mechanical translational systems shown in the figure and find the transfer function **(NOV/DEC 2013)**

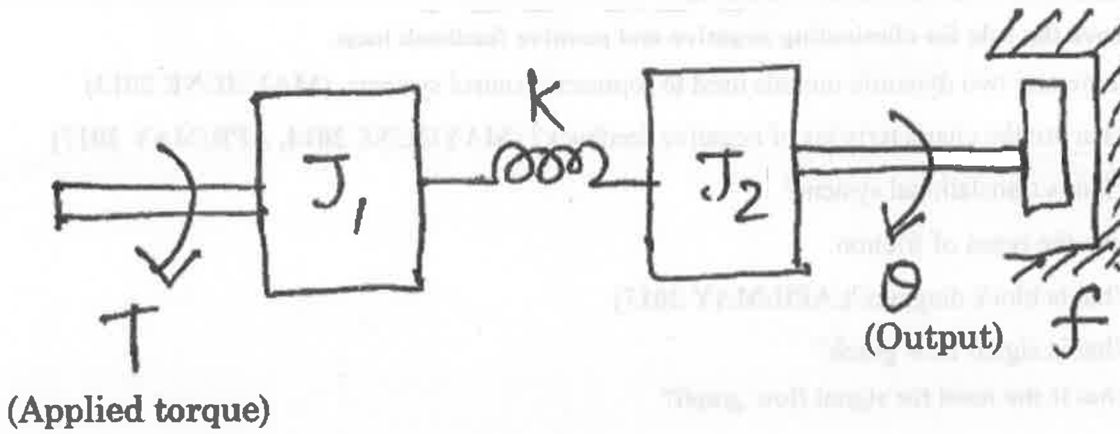


3. Write the differential equation governing the mechanical translational systems shown in the figure and find the transfer function **(APR/MAY 2017)**



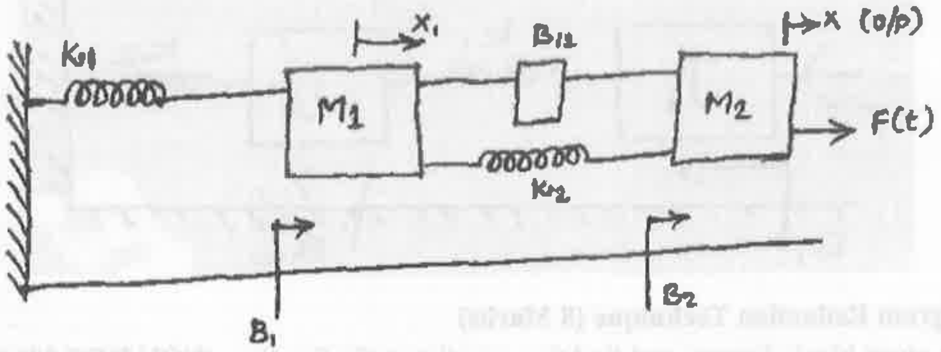
Mechanical Rotational System (16 Marks)

1. Write the differential equation governing the mechanical rotational systems shown in the figure and find the transfer function. Consider the angular displacement in J_1 as θ_1 **(MAY/JUNE 2013)**

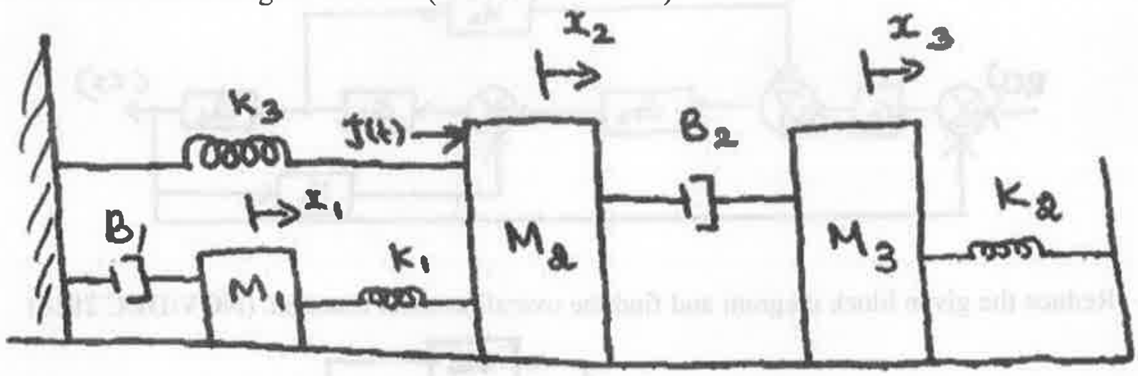


Electrical Analogous Circuits (MTS) (16 Marks)

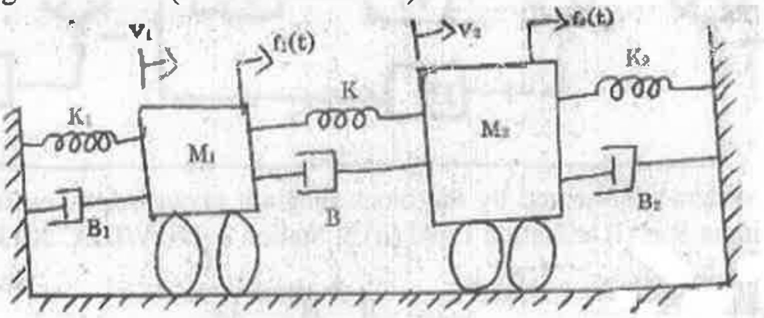
1. Obtain the transfer function of the given mechanical system. Hence the draw electrical analogous circuits. **(NOV/DEC 2014)**



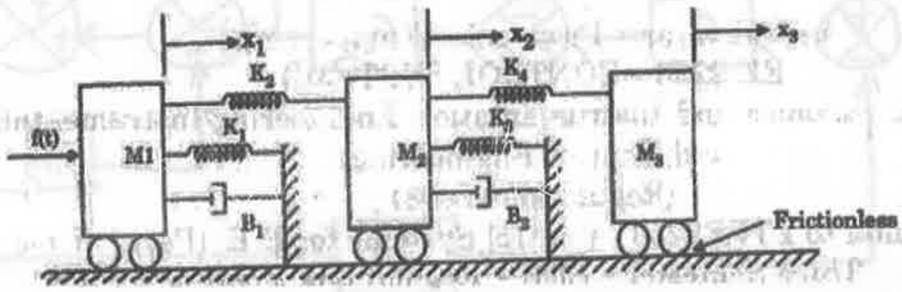
2. Write the differential equation governing the mechanical translational systems shown in the figure. Draw the Electrical analogous circuits. **(MAY/JUNE 2015)**



3. Write the differential equation governing the mechanical translational systems shown in the figure. Draw the Electrical analogous circuits. **(MAY/JUNE 2013)**

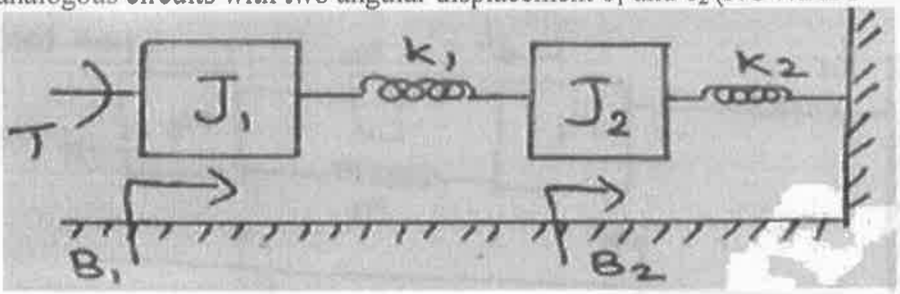


4. **Write the differential equation governing the mechanical translational systems shown in the figure.** Draw the Electrical analogous circuits. **(MAY/JUNE 2013)**



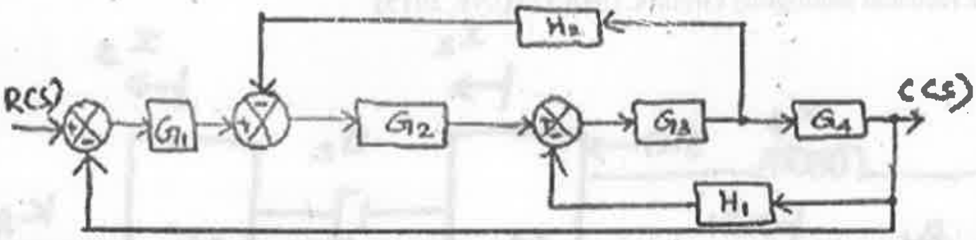
Electrical Analogous Circuits (MRS) (16 Marks)

1. Write the differential equation governing the mechanical rotational systems shown in the figure. Draw the Electrical analogous circuits with two angular displacement θ_1 and θ_2 **(NOV/DEC 2016)**

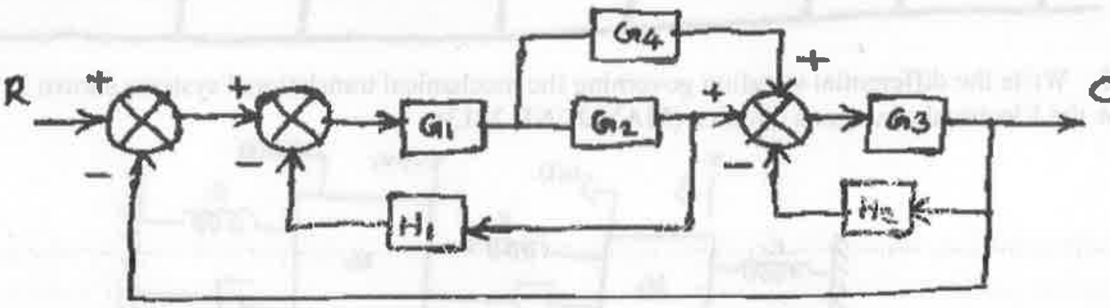


Block Diagram Reduction Technique (8 Marks)

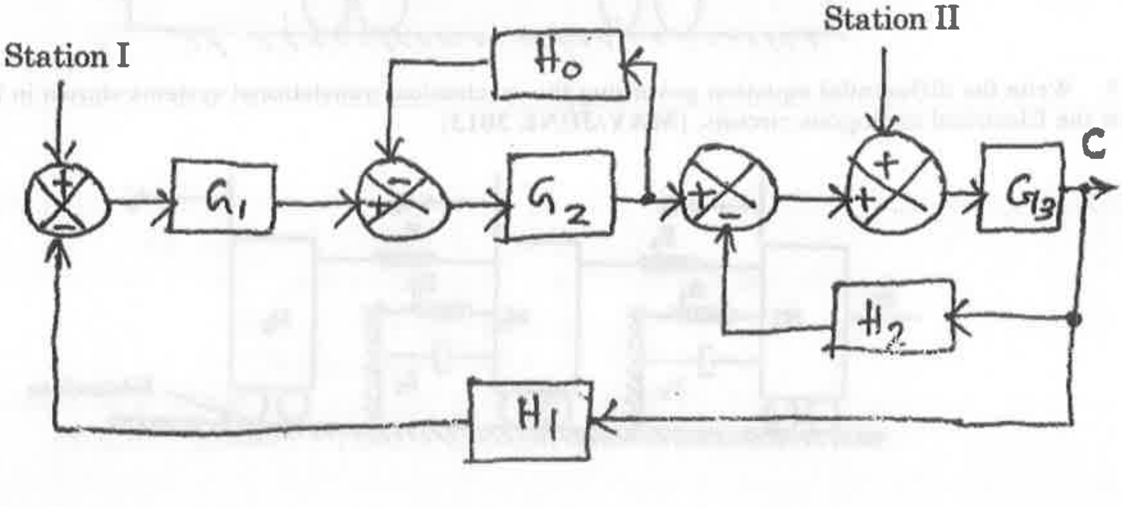
1. Reduce the given block diagram and find the overall transfer function. **(NOV/DEC 2016)**



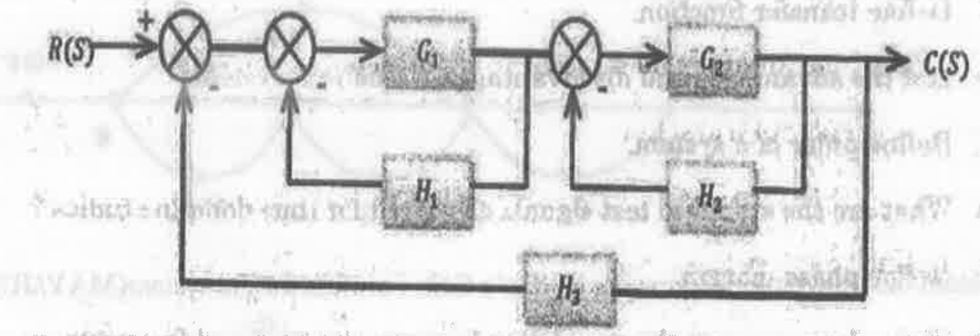
2. Reduce the given block diagram and find the overall transfer function. **(NOV/DEC 2014)**



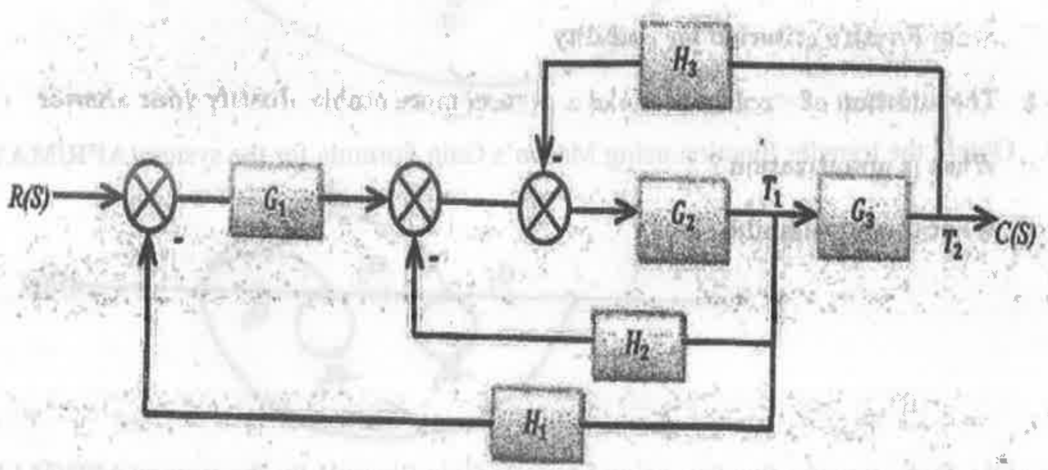
3. For the system represented by the block diagram given below, evaluate the closed loop transfer function, when input R is (i) at Station 1 and (ii) at Station 2 **(NOV/DEC 2013)** **(16 Marks)**



4. Reduce the given block diagram and find the overall transfer function. **(APR/MAY 2017) (8 MARKS)**

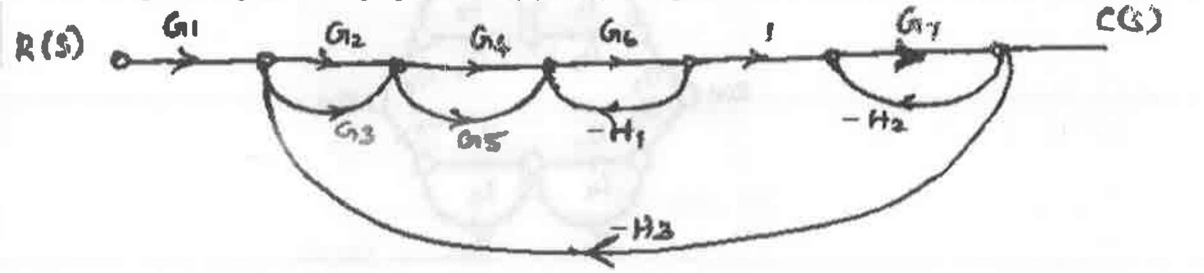


5. Reduce the given block diagram and find the overall transfer function. **(APR/MAY 2017) (8 MARKS)**

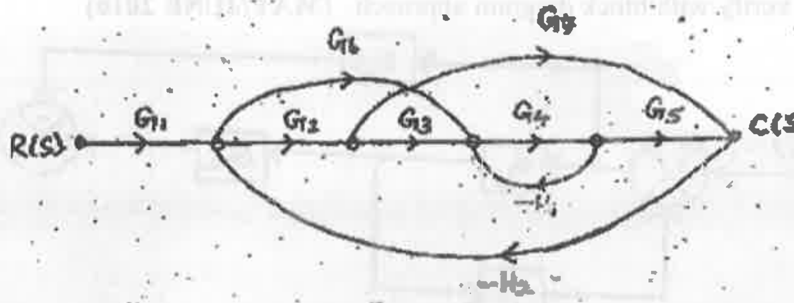


Signal Flow Graph Technique (8 Marks)

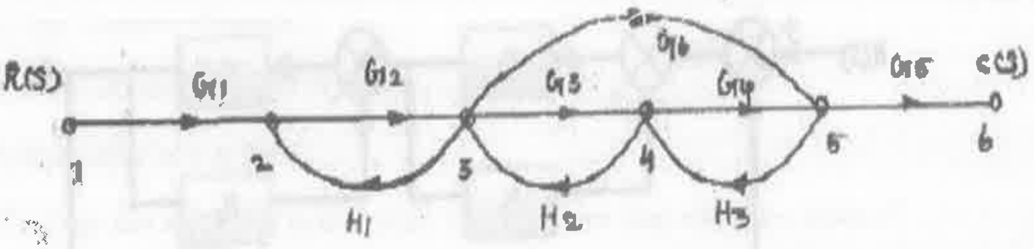
1. For the given signal flow graph find C(S)/R(S) using Mason's Gain formula. **(NOV/DEC 2014)**



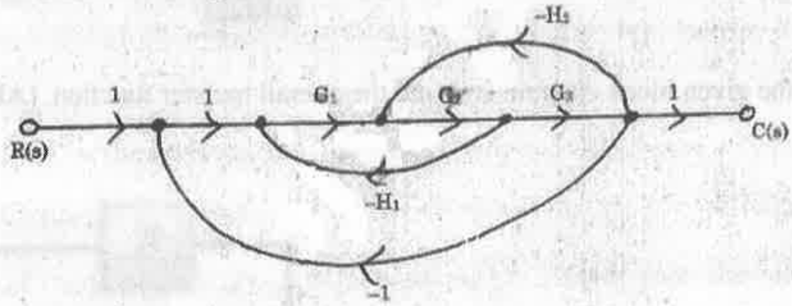
2. For the given signal flow graph find C(S)/R(S) using Mason's Gain formula. **(NOV/DEC 2013, APR/MAY 2017)**



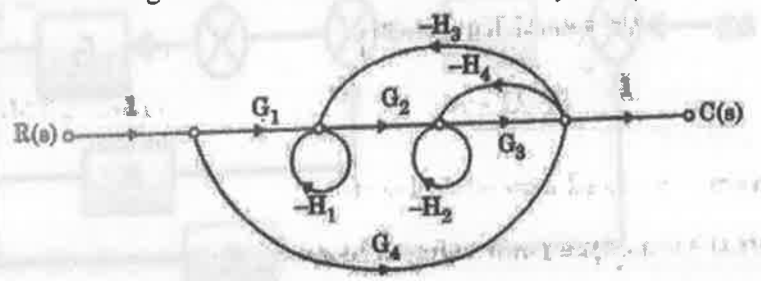
3. For the given signal flow graph find C(S)/R(S) using Mason's Gain formula. **(NOV/DEC 2013)**



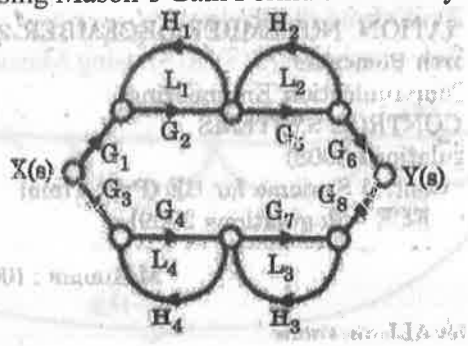
4. Obtain the transfer function using Mason's Gain Formula for the system**(MAY/JUNE 2013)**



5. Obtain the transfer function using Mason's Gain Formula for the system**(APR/MAY 2017)**

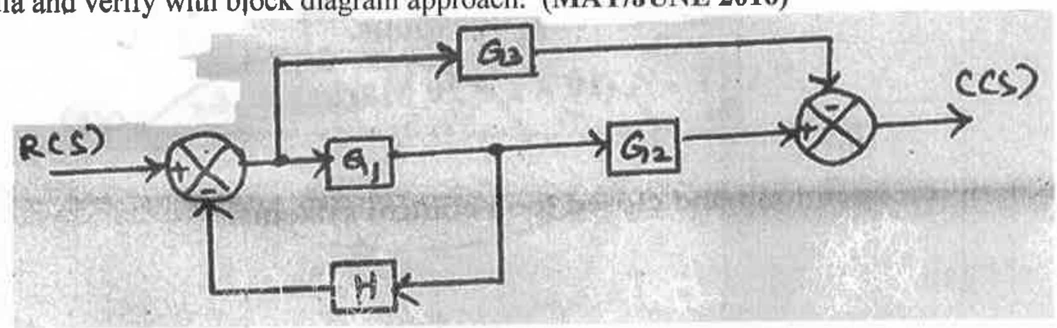


6. Obtain the transfer function using Mason's Gain Formula for the system**(APR/MAY 2017)**

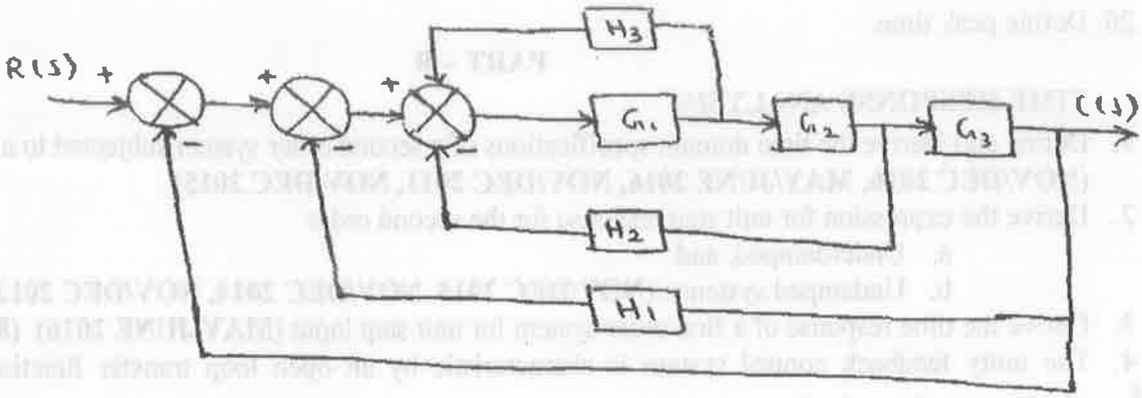


Block Diagram Reduction to Signal Flow Graph Technique (16 Marks)

1. Convert the given block diagram to signal flow graph and find the transfer function using mason's gain formula and verify with block diagram approach. **(MAY/JUNE 2016)**



2. Convert the given block diagram to signal flow graph and find the transfer function using mason's gain formula and verify with block diagram approach. **(MAY/JUNE 2016)**



Servomotor (8 Marks)

1. Describe the features and characteristics of an armature controlled d.c. servomotor. **(MAY 2007, 2009)**
2. Explain the working of d.c. servomotors. **(DEC 2014)**
3. Derive the transfer function of d.c. servomotor under field controlled. **(MAY 2005)**
4. Derive the transfer function of an armature controlled d.c. motor. **(MAY 2009, DEC 2015)**
5. Explain the working of a.c. servomotors. **(DEC 2014)**
6. Explain the principle of working of synchronous with relevant sketches. How are synchros useful in control system engineering?

UNIT II
TIME RESPONSE ANALYSIS
PART - A

1. Specify the time domain Specification? **(NOV/DEC 2016, MAY/JUNE 2016)**
2. What is meant by steady state error? **(NOV/DEC 2016, NOV/DEC 2015)**
3. List the Standard test signal used in time domain analysis. **(MAY/JUNE 2016, MAY/JUNE 2014, NOV/DEC 2015)**
4. State the effect of PI Compensation in system performance. **(MAY/JUNE 2016)**
5. How do you find the type of the system? **(MAY/JUNE 2015)**
6. Find the unit impulse response of the system H(s) = 5s/(s+2) with zero initial conditions. **(MAY/JUNE 2015)**
7. For the system described by $\frac{C(S)}{R(S)} = \frac{16}{S^2+8S+16}$;find the nature of the time response? **(NOV/DEC 2015)**
8. Why is the derivative control not used in control system? **(NOV/DEC 2015)**
9. Give the relation between static and dynamic error coeffcients. **(NOV/DEC 2016)**
10. What is type and order of the system? **(NOV/DEC 2014, MAY/JUNE 2015)**
11. What are the advantages of generalized error series? **(NOV/DEC 2014)**
12. Give the transfer function of the PID Controller. **(NOV/DEC 2013)**
13. State the effect of PD Compensation in system performance. **(MAY/JUNE 2014)**
14. What do you mean by peak over shoot? **(NOV/DEC 2010)**
15. Define settling time. **(MAY/JUNE 2010)**
16. Differentiate between steady state and transient response of the system? **(MAY/JUNE 2010)**
17. What is the effect of system performance when a proportional controller is introduced in a system? **(MAY/JUNE 2015)**

18. What is type and order of the given system $G(S) = \frac{K}{S(ST+1)}$? **(NOV/DEC 2014)**
19. Define Rise time.
20. Define peak time.

PART – B

TIME RESPONSE ANALYSIS

1. Define and Derive the time domain specifications of a second order system subjected to a step input **(NOV/DEC 2016, MAY/JUNE 2016, NOV/DEC 2011, NOV/DEC 2015)**
2. Derive the expression for unit step response for the second order
   a. Underdamped, and
   b. Undamped systems. **(NOV/DEC 2015, NOV/DEC 2014, NOV/DEC 2013)**
3. Derive the time response of a first order system for unit step input **(MAY/JUNE 2016) (8 Marks)**
4. The unity feedback control system is characteristic by an open loop transfer function $G(S) = \frac{K}{S(S+10)}$. Determine the gain K, so that the system will have damping ratio of 0.5 for this value of K. Determine the peak overshoot and peak time for a unit step input. **(MAY/JUNE 2016, MAY/JUNE 2015, MAY/JUNE 2014, NOV/DEC 2014)**
5. The overall transfer function of a control system is given by $\frac{C(S)}{R(S)} = \frac{16}{S^2+1.6S+16}$. It is desired that the damping ratio be 0.8. Determine the derivative rate feedback constant K_1 and compare rise time, peak time, maximum overshoot and steady state error for unit ramp input function without and with derivative feedback control. **(NOV/DEC 2016)**

STATIC AND DYNAMIC ERROR

1. For a unity feedback control system the open loop transfer function is given by $G(s) = \frac{10(S+2)}{S^2(S+1)}$, find
   a. The position, velocity, acceleration error constants
   b. The steady state error when $R(s) = \frac{3}{S} - \frac{2}{S^2} + \frac{1}{3S^3}$ **(NOV/DEC 2016, MAY/JUNE 2016)**
2. The open loop transfer function of a unity feedback control system is given by $G(S) = \frac{K}{S(S+1)}$. The input of the system is described by r(t)=4+6t. Find the generalized error coefficients and steady state error. **(NOV/DEC 2015)**
3. The unity feedback control system has the forward transfer function $G(S) = \frac{KS}{(S+1)^2}$. For the input r(t)=1+5t. Find the Minimum value of K so that the steady state error is less than 0.1. **(MAY/JUNE 2015)**
4. The open loop transfer function of a servo system with unity feedback is $G(S) = \frac{10}{S(0.1S+1)}$. Evaluate the static error constants (Kp, Kv, Ka) for the system. Obtain the steaty stae error of the system when subjected to an input given by the polynomial $r(t) = a_0 + a_1 t + \frac{a_2}{2}t^2$ **(MAY/JUNE 2015, MAY/JUNE 2014, NOV/DEC 2010)**
5. For the open loop system with $G(S) = \frac{1}{(S+1)}$ and $H(S) = 5$, Calculate the generalized error coefficients and error series **(NOV/DEC 2013) (8 Marks)**

CONTROLLERS (8)

1. Write short notes on : i) PD controller ii) PI controller.**(DEC 2014, 2016, MAY 2008)**
2. Discuss the effect on the performance of a second order control system of the proportional derivative control. **(MAY 2009, 2012)**
3. Explain about the following controllers. I) P controller ii) PI controller iii) PID controller **(DEC 2009, 2015, 2017, MAY 2016)**
4. With the help of diagrams and equations, explain the following controllers : i) proportional ii) Integral iii) Derivative **(DEC 2012)**

UNIT III

FREQUENCY RESPONSE AND SYSTEM ANALYSIS

PART - A

1. What is bode plot? **(APR/MAY 2016)**
2. Define gain margin and phase margin. **(NOV/DEC 2011, APR/MAY 2010, NOV/DEC 2014, APR/MAY 2015)**
3. Define Resonant Peak and Resonant Frequency. **(MAY/JUNE 2014, NOV/DEC 2014)**
4. Mention any four frequency response specifications. **(NOV/DEC 2010)**
5. What are m & n circles? **(NOV/DEC 2011, MAY/JUNE 2014)**
6. Define Corner Frequency. **(NOV/DEC 2012)**
7. What is Nichol's chart? **(NOV/DEC 2012)**
8. What is Gain and Phase Crossover Frequency? **(NOV/DEC 2013)**
9. List the advantages of Nichol's chart? **(NOV/DEC 2010).**
10. What are the Frequency Domain Specifications. **(NOV/DEC 2016).**
11. Define –Resonant Peak
12. What is bandwidth?
13. Define Cut-off rate?
14. What are the main advantages of Bode plot?
15. Define Phase cross over?
16. Define Gain cross over?
17. What is a polar plot?
18. What are compensators?
19. What are the two types of compensation techniques write short notes on them?
20. Define Lead compensator.
21. What is a lag compensator?
22. What is a lag lead compensator?
23. What is the need for compensator? **(NOV/DEC 2011, MAY/JUNE 2014, MAY/JUNE 2015, NOV/DEC 2014, NOV/DEC 2010)**
24. Sketch the electrical circuit of a Lag, Lead, lag-lead compensator. **(NOV/DEC 2011, NOV/DEC 2010)**
25. Write the transfer function and pole zero plot of lag. lead and lag-lead compensator. **(MAY/JUNE 2014, NOV/DEC 2010)**
26. What is the relation between φ_m *and* α? **(APR/ MAY 2010)**
27. What type of compensator suitable for high frequency noisy environment? **(APR/ MAY 2010)**
28. What is desired performance criteria specified in compensator design?

PART B

BODE PLOT

1. Plot the Bode diagram for the following transfer function and obtain the gain and phase cross over frequencies. **(APR/MAY 2011, NOV/DEC 2014)**

$$G(S) = \frac{10}{S\,(1 + 0.4S)(1 + 0.1S)}$$

2. Sketch the Bode plot and hence find Gain cross over frequency, Phase cross over frequency, Gain margin and Phase margin. **(NOV/DEC 2011, APR/MAY 2010, APR/MAY 2013, NOV/DEC 2014, APR/MAY 2015)**

$$G(S) = \frac{0.75(1 + 0.2S)}{S\,(1 + 0.5S)(1 + 0.1S)}$$

3. Sketch the Bode plot and hence find Gain cross over frequency, Phase cross over frequency, Gain margin and Phase margin. **(NOV/DEC 2013, NOV/DEC 2016)**

$$G(S) = \frac{10(S+3)}{S(S+2)(S^2+4S+100)}$$

4. Plot the Bode diagram for the following transfer function and obtain the gain and phase cross over frequencies

$$G(S) = \frac{KS^2}{(1+0.2S)(1+0.02S)}$$

Determine the value of K for a gain cross over frequency of 20 rad/sec.

5. Sketch the Bode plot and hence find Gain cross over frequency, Phase cross over frequency, Gain margin and Phase margin. **(NOV/DEC 2010, APR/MAY 2015)**

$$G(S) = \frac{10(1+0.1S)}{S(1+0.01S)(1+S).}$$

POLAR PLOT

1. The open loop transfer function of a unity feedback system is

$$G(S) = \frac{1}{S(1+S)(1+2S)}$$

Sketch the Polar plot and determine the Gain margin and Phase margin. **(NOV/DEC 2010, NOV/DEC 2014)**

2. Sketch the polar plot for the following transfer function and find Gain cross over frequency, Phase cross over frequency, Gain margin and Phase margin. **(APR/MAY 2010, NOV/DEC 2016)**

$$G(S) = \frac{10(S+2)(S+4)}{S(S^2+3S+10)}$$

3. Sketch the polar plot for the following transfer function .and find Gain cross over frequency, Phase cross over frequency, Gain margin and Phase margin. **(APR/MAY 2015)**

$$G(S) = \frac{400}{S(S+2)(S+10)}$$

NICHOL'S PLOT

1. A unity feedback system has open loop transfer function

$$G(S) = \frac{20}{S(S+2)(S+5)}.$$

Using Nichol's chart. Determine the closed loop frequency response and estimate all the frequency domain specifications. **(NOV/DEC 2013, APR/MAY 2014)**

2. Draw the Nichol's plot for the system whose open loop transfer function is G(S) H(S) =K / S (S+2) (S+10). Determine the range of K for which closed loop system is stable.
3. Construct Nichol's plot for a feedback control system whose open loop transfer function is given by G(S) H(S) =5/ S(1+S).Comment on the stability of open loop and closed loop transfer function.
4. Sketch the Nichol's plot for a system with the open loop transfer function G(S) H(S) =K (1+0.5S) (0.01+S) / (1+10S) (S+1). Determine the range of values of K for which the system is stable.

LEAD COMPENSATOR (13 MARKS)

1. Explain the design procedure of a lead compensator with suitable example. **(NOV/DEC 2011)**

2. The open loop transfer function of the uncompensated system is $G(s) = \frac{5}{S(S+2)}$. Design a suitable compensator for the system so that the static velocity error constant $K_V = 20sec^{-1}$, the phase margin is atleast 55° and the gain margin is atleast 12dB **(NOV/DEC 2013, APR/MAY 2010, MAY/JUNE 2016)**
3. Consider the unity Feedback system has an open loop transfer function is

$$G(s) = \frac{K}{S(0.1S+1)(0.2S+1)}$$

The system is compensated using a suitable lead compensator to meet the following specifications:

i. Phase Margin of atleast 50°;
ii. Bandwidth = 12 rad/sec;
iii. Velocity error constant $K_V \geq 30sec^{-1}$ **(MAY/JUNE 2016)**

LAG COMPENSATOR (13 MARKS)

1. Explain the design procedure of a lag compensator with suitable example. **(MAY/JUNE 2014, MAY/JUNE 2015)**
2. To open transfer function of a system is given below $G(s) = \frac{K}{S(S+1)(S+4)}$. Design a suitable lag compensator to meet the following specifications. Phase Margin = 43°; Bandwidth = 1.2 rad/sec; Velocity error constant $K_V \geq 5sec^{-1}$ **(NOV/DEC 2011)**
3. A Unity Feedback system has an open loop transfer function $G(s) = \frac{K}{S(S+1)(0.5S+1)}$. Design a suitable lag compensator to maintain the Phase Margin of atleast 40°; Bandwidth = 1.2 rad/sec; Velocity error constant $K_V \geq 5sec^{-1}$ **(MAY/JUNE 2016)**

LAG - LEAD COMPENSATOR (13 MARKS)

1. **Explain the design procedure of a Lag- Lead compensator with suitable example. (MAY/JUNE 2010, MAY/JUNE 2015)**
2. The open loop transfer function of the uncompensated system is $G(s) = \frac{1}{S(S+1)(S+2)}$. Compensate the system by cascading suitable lag – lead compensator for the system so that the static velocity error constant $K_V = 10sec^{-1}$, the phase margin is atleast 45° and the gain margin is atleast 10dB or more. **(NOV/DEC 2013)**

UNIT IV

CONCEPTS OF STABILITY ANALYSIS

PART - A

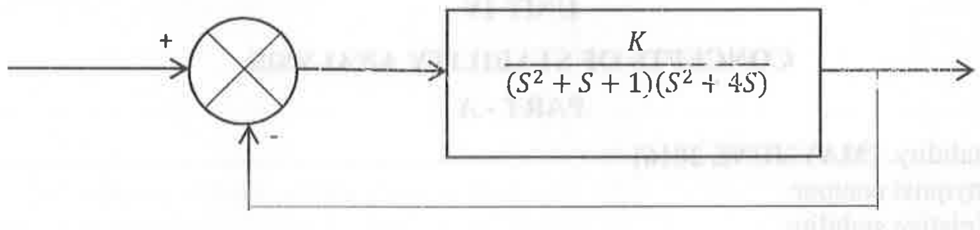
1. Define stability. **(MAY/JUNE 2016)**
2. What is nyquist contour
3. Define Relative stability.
4. What will be the nature of impulse response when the roots of characteristic equation are lying on imaginary axis?
5. What is the relationship between Stability and coefficient of characteristic polynomial?
6. What is limitedly stable system?
7. In routh array what conclusion you can make when there is a row of all zeros?
8. **What are the two segments of Nyquist contour?**
9. State any two limitations of routh stability criterion. **(NOV/DEC 2012)**
10. State the advantages of nyquist stability criterion over routh criterion. **(NOV/DEC 2012)**
11. What is BIBO stability criterion? **(NOV/DEC 2011)**

12. State Nyquist Stability Criterion. **(APR/ MAY 2010, NOV/DEC 2013, MAY/JUNE 2014, NOV/DEC 2010)**
13. How are the location of roots of the characteristic equation related to stability? **(MAY/JUNE 2014, MAY/JUNE 2015, NOV/DEC 2014)**
14. Define Routh Stability Criterion? **(MAY/JUNE 2014, MAY/JUNE 2015, NOV/DEC 2014)**
15. What is dominant pole? **(NOV/DEC 2016, NOV/DEC 2015, MAY/JUNE 2015)**
16. How will you find the root locus on real axis? **(MAY/JUNE 2016)**
17. State the basic properties of root locus. **(NOV/DEC 2016)**
18. What is the condition for the system $G(s) = \frac{K(s+a)}{s(s+b)}$ to have a circle in root locus? **(NOV/DEC 2013)**
19. What is the value of K at any given point in root locus? **(MAY/JUNE 2015, NOV/DEC 2010)**
20. What will be the nature of Step response when the roots of characteristic equation are lying on imaginary axis?

PART – B

ROUTH HURWITZ CRITERION (8 MARKS)

1. Determine the range of K for stability of unity feedback system whose open loop transfer function in $G(s) = \frac{K}{s(s+1)(s+2)}$ using Routh Stability Criterion. **(NOV/DEC 2012)**
2. Construct Routh Array and determine the stability of the system whose characteristic equation is $S^6+2S^5+8S^4+12S^3+20S^2+16S+16=0$. Also determine the number of roots lying on right half of s – plane, left half of s-plane and on imaginary axis. **(NOV/DEC 2011)**
3. Determine the stability of the given system of the given characteristic equation using Routh-Hurwitz Criterion
   i. $S^5+4S^4+8S^3+8S^2+7S+4=0$
   ii. $S^6+S^5+3S^4+3S^3+3S^2+2S+1=0$ **(16 Marks) (MAY/JUNE 2015)**
4. The open loop transfer function of a unity feedback control system is given by $G(s) = \frac{K}{(s+2)(S+4)(S^2+6S+25)}$. By applying Routh criterion, dicuss the stability of the closed loop system as a function of K. Determine the values of K which will cause sustained oscillations in the closed loop system. What are the corresponding oscillation frequencies? **(16 Marks) (MAY/JUNE 2014)**
5. Consider the closed – loop system shown in the figure, determine the range of K for which the stable. **(MAY/JUNE 2016)**



NYQUIST STABILITY CRITERION (13 MARKS)

1. The open loop transfer function of a unity feedback system is given by $G(s)H(s) = \frac{5}{S(S+1)(S+2)}$. Draw the nyquist plot and hence find out whether the system is stable or not. **(NOV/DEC 2013)**

ROOT LOCUS (13 MARKS)

1. **With neat steps write down the procedure for construction of root locus. Each rule give a example. (NOV/DEC 2016, NOV/DEC 2015)**

2. A unity Feedback Control system has an open loop transfer function

$$G(s) = \frac{K}{S(S^2 + 4S + 13)}$$

Sketch the root locus. **(MAY/JUNE 2016, NOV/DEC 2010)**

3. A single loop negative feedback system has a transfer function $G_c(s)G(s) = \frac{K(S+6)^2}{S(S^2+1)(S+4)}$. Skectch the root locus as a function of K. Find the range of K for which the system is stable. **(MAY/JUNE 2015)**

4. Draw the root locus of the system is given by $G(s) = \frac{K(S+1)}{S(S^2+5S+20)}$. **(NOV/DEC 2016)**

5. Plot the root locus for a unity feedback closed loop system whose open loop transfer function is $G(s) = \frac{K}{S(S+4)(S^2+2S+2)}$. **(NOV/DEC 2011)**

6. Sketch the root locus of a unity feedback system with an open loop transfer function $G(s) = \frac{K}{S(S+2)(S+4)}$. Find the value of K so that the damping ratio of the closed loop system is 0.5. **(MAY/JUNE 2015, MAY/JUNE 2014)**

7. Sketch the root locus of a unity feedback system with an open loop transfer function $G(s) = \frac{K(S+0.5)}{S^2(S+4.5)}$. **(NOV/DEC 2013)**

8. The open loop transfer function of a unity feedback system is given by

$$G(s) = \frac{K(S+9)}{S(S^2 + 4S + 11)}$$

Sketch the root locus. **(NOV/DEC 2015)**

9. Sketch the root locus of a unity feedback system with an open loop transfer function $G(s) = \frac{K}{S(S+1)(S+2)}$. **(NOV/DEC 2014)**

UNIT V

CONTROL SYSTEM ANALYSIS USING STATE VARIABLE METHODS

PART - A

1. Define State and State Variable. **(NOV/DEC 2012, NOV/DEC 2015, APR/MAY 2016)**
2. What is controllability? **(APR/MAY 2015)**
3. What is observability? **(APR/MAY 2015)**
4. Write the properties of state transition matrix.
5. What is modal matrix?
6. State the duality between controllability and observability.
7. What are the methods available for the stability analysis of sampled data control system?
8. What is the necessary condition to be satisfied for design using state feedback?
9. What is similarity transformation?
10. What is meant by diagonalization?
11. What is the need for controllability test?
12. What is the need for observability test?
13. What is the need for state observer?
14. What is the pole placement by state feedback?
15. How control system design is carried in state space?
16. What is state transition matrix? **(NOV/DEC 2016, NOV/DEC 2015)**
17. When do you say the system is completely controllable? **(NOV/DEC 2015)**
18. State the limitations of state variable feedback? **(NOV/DEC 2016)**
19. Define Sampling Theorem? **(NOV/DEC 2012, NOV/DEC 2015, APR/MAY 2016)**
20. Draw Sample and hold circuits.

PART - B

Controllability and Observability (16 Marks)

1. Determine whether the system described by the following state model is completely controllable and observable **(NOV/DEC 2016, APR/MAY 2016)**

$$[\dot{x}(t)] = \begin{bmatrix} 0 & 0 & 1 \\ -2 & -3 & 0 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

2. Determine whether the system described by the following state model is completely controllable and observable **(APR/MAY 2015)**

$$[\dot{x}] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 5 \\ -24 \end{bmatrix} u$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

3. Consider the system is defined by X = Ax+Bu and Y = Cx, Where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}; B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}; C = \begin{bmatrix} 10 & 5 & 1 \end{bmatrix}$$

Check the controllability and observability of the system. **(NOV/DEC 2015, APR/MAY 2016)**

State Model Transformation (16 Marks)

4. Consider the the following system with differential equation given by

$$\dddot{y} + 6\ddot{y} + 11\dot{y} + 6y = 6u$$

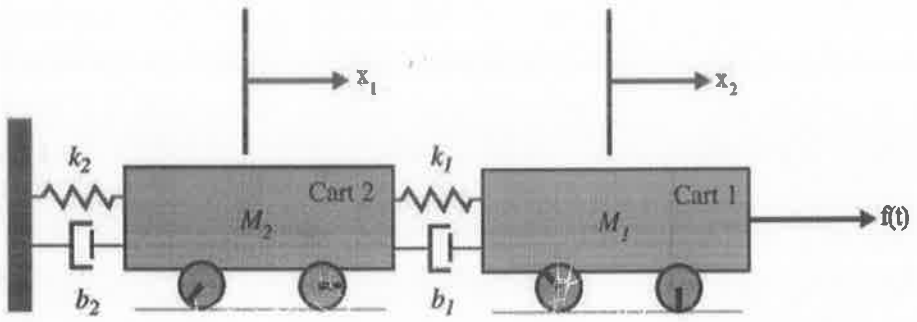
Obtain the state model in diagonal canonical form. **(NOV/DEC 2015)**

5. **Construct the state model for the system characterized by the differential equation $\frac{d^3x}{dt^3} + 6\frac{d^2x}{dt^2} + 11\frac{dx}{dt} + 6x + u = 0$ (APR/MAY 2016)**
6. Consider the system is defined by X = Ax+Bu and Y = Cx, Where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}; B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}; C = \begin{bmatrix} 10 & 5 & 1 \end{bmatrix}$$

Obtain the diagonal canonical form of the state model by a suitable transformation matrix. **(APR/MAY 2016)**

7. Obtain the state model of the mechanical translational system in which f(t) is input and x2(t) is output. **(NOV/DEC 2015)**



State Model to Transfer Function (16 Marks)

8. Convert the given state model to transfer function

$$[\dot{x}] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -14 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y(t) = \begin{bmatrix} 15 & 5 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

9. Consider the system is defined by X = Ax+Bu and Y = Cx, Where

$$A = \begin{bmatrix} -7 & 1 & 0 \\ -14 & 0 & 1 \\ -8 & 0 & 0 \end{bmatrix}; B = \begin{bmatrix} 20 \\ 125 \\ 185 \end{bmatrix}; C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

Obtain the transfer function of the state model.

10. Convert the given state model to transfer function

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y(t) = \begin{bmatrix} 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Digital Control System

11. State the advantages of digital control systems. (4) **(MAY 2016, DEC 2017)**
12. Draw and explain the block diagram of sampled data system and digital data system. (8) **(MAY 2015, DEC 2017)**